

## Gracefulness of $^N c_4$ Merging With Paths

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**Abstract:** Gracefulness of  $nc_4$  merging with paths

### I. Introduction:

Most graph labeling methods trace their origin to one introduced by Rosa [2] or one given Graham and Sloane [1]. Rosa defined a function  $f$ , a  $\beta$ -valuation of a graph with  $q$  edges if  $f$  is an injective map from the vertices of  $G$  to the set  $\{0, 1, 2, \dots, q\}$  such that when each edge  $xy$  is assigned the label  $|f(x)-f(y)|$ , the resulting edge labels are distinct.

A. Solairaju and K. Chitra [3] first introduced the concept of edge-odd graceful labeling of graphs, and edge-odd graceful graphs.

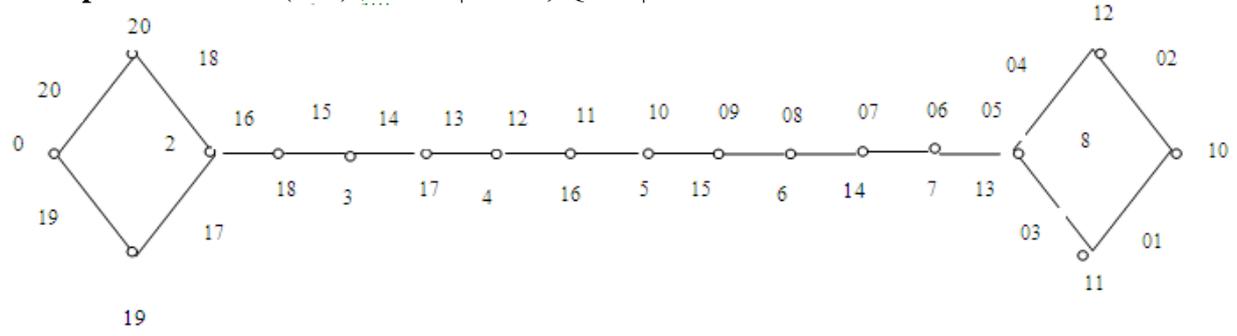
A. Solairaju and others [5,6,7,8,9] proved the results that(1) the Gracefulness of a spanning tree of the graph of Cartesian product of  $P_m$  and  $C_n$ , was obtained (2) the Gracefulness of a spanning tree of the graph of cartesian product of  $S_m$  and  $S_n$ , was obtained (3) edge-odd Gracefulness of a spanning tree of Cartesian product of  $P_2$  and  $C_n$  was obtained (4) Even -edge Gracefulness of the Graphs was obtained (5) ladder  $P_2 \times P_n$  is even-edge graceful, and (6) the even-edge gracefulness of  $P_n \circ nC_5$  is obtained.

### II. Section – I: Preliminaries

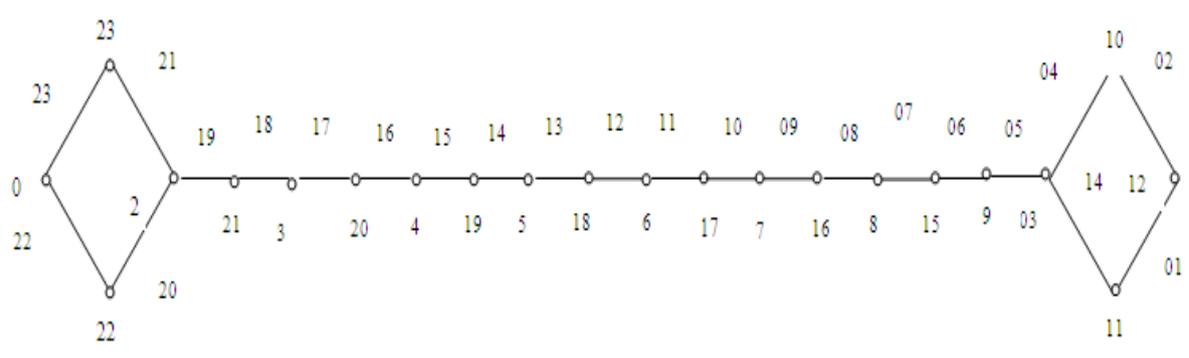
**Definition 1.1** Let  $G = (V,E)$  be a simple graph with  $p$  vertices and  $q$  edges. A map  $f:V(G) \rightarrow \{0,1,2,\dots,q\}$  is called a graceful labeling if

- i)  $F$  is one-to-one
- ii) The edges receive all the labels (numbers) from 1 to  $q$  where the label of an edge is the absolute value of the difference between the vertex labels at its end, a graph having a graceful labeling is called a graceful graph.

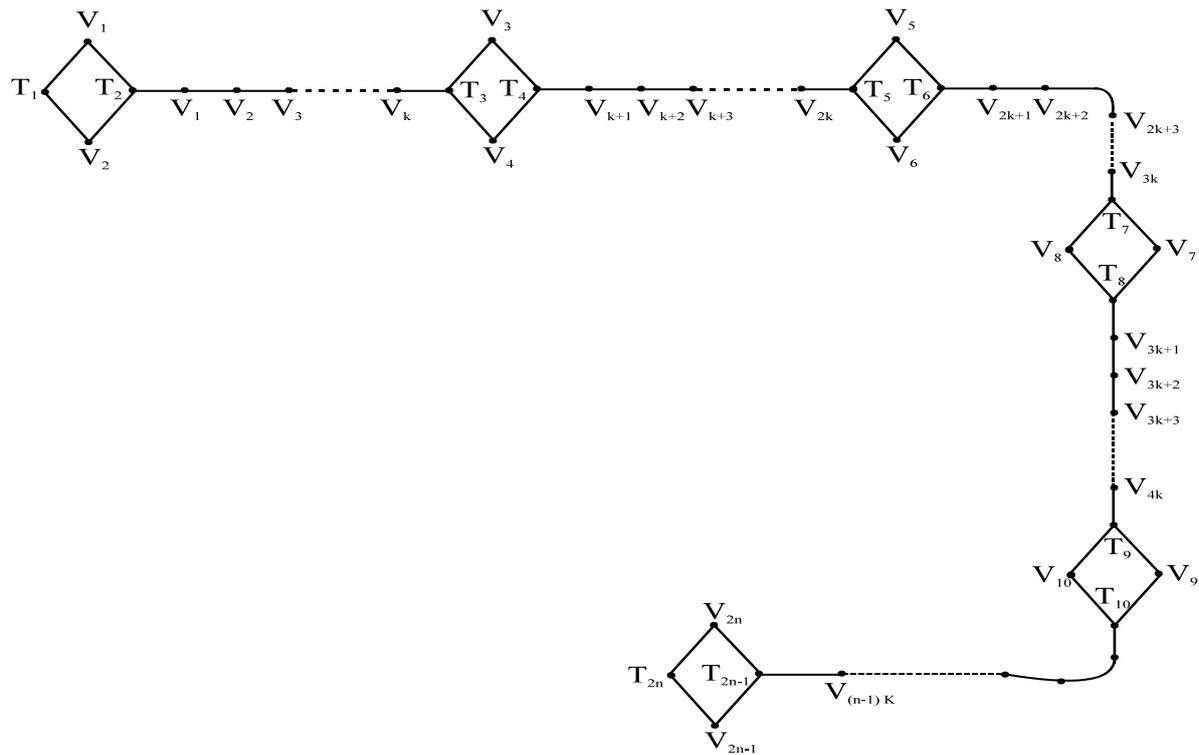
**Example 1.1:**  $k = 11$  (odd) ;  $P: V \mapsto 19; Q: e \mapsto 20$



**Example 2.2:**  $k = 14$  (even) ;  $P: V \mapsto 22; Q: e \mapsto 23$



Gracefulness of  $nc_4$  merging with path generalization



**Case I**

**K is odd**

**n is copies;**  $p = V(G), q = e(G)$

Define :  $f : V(G) \rightarrow \{0, \dots, q\}$  by

$$f(T_1) = 0,$$

$$f(T_2) = 2,$$

$$f(V_1) = q$$

$$f(V_2) = (q) - 1$$

$$f(V_i) = f(V_1) - \text{If } i \text{ is odd.}$$

$$f(V_i) = f(V_1) - \{f(V_{i-1})\} - 1; \text{ if } i = 4l + 2$$

$$f(V_i) = f(V_{i-1}) + 1; \text{ if } i = 4l$$

$$f(T_i) = f(T_1) + K; \text{ if } i = (4l + 3, 4l + 1)$$

$$f(T_i) = f(T_{i-1}) + 2; \text{ if } i = (4l, 4l + 2)$$

**Case II**

**K is even**

**n is copies;**  $p = V(G), q = e(G)$

Define :  $f : V(G) \rightarrow \{0, \dots, q\}$  by

$$f(V_0) = q,$$

$$f(V_1) = q-1,$$

$$f(T_1) = 0,$$

$$f(T_2) = 2.$$

$$f(V_3) = 6,$$

$$f(V_4) = 7.$$

$$\begin{aligned}
 f(V_1) &= \\
 f(V_i) &= \\
 f(T_i) &= f(T_{i-1}) + 11; & \text{if } i = (4l + 1, 4l + 2) \\
 f(T_3) &= q - (K/2 + 2); & \text{if } i = 5, 9, 13, \dots \\
 f(T_4) &= f(T_3) + 2; & \text{if } i = 6, 10, 14, \dots \\
 f(T_i) &= f(T_{i-1}) - 13; & \text{if } i = 4l + 3, \\
 & & \text{if } i = 7, 11, 15, \dots \\
 f(T_i) &= f(T_{i-1}) - 2; & \text{if } i = 4l, \\
 & & \text{if } i = 8, 12, 16, \dots
 \end{aligned}$$

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