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On Bitopological -open sets

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Abstract: *The purpose of this paper is to introduce and study the properties of (1,2) γ- open set in bitopological space .In this paper we also study (1, 2) γ-locally closed set, (1,2) γ-t-set, (1,2) γ-B-set and relationship between (1,2) b-open set and (1,2) γ-open set .*

Keywords- *(1, 2) b-open set , (1,2) B-set, (1,2) t-set (1,2) locally closed , (1,2) γ- open set .*

I. Introduction

D. Andrijevic and M. Ganster **[3]** introduced a class of generalised open sets in a topological spaces, the so called γ-open set .The class of γ-open sets contains all semi-open sets, pre-open sets, and b-open sets. Further Tong **[4]** introduced the concept of t-set and B-set in topological spaces. In 1963 J.C Kelly **[5]** introduced the concept of bitopological spaces .The purpose of this paper is to introduce the γ -open set in bitopological space, study the properties of this set and investigate the relationship between (1,2) γ-open set, (1,2) locally -closed set , (1,2) γ-locally closed set and (1, 2) b-open set . In this paper lastly we study comparison of (1, 2) γ-open set with $(1,2)^*$ -γ-open set .

II. Preliminaries

Throughout this paper by X we mean bitopological space (X, τ_1, τ_2)

Definition 2.1 A subset A of X is $\tau_1\tau_2$ -open ([6]) if A ∈ $\tau_1 \bigcup \tau_2$ and $\tau_1\tau_2$ -closed if its complement is $\tau_1\tau_2$ -open in X. The $\tau_1\tau_2$ -closure of A is denoted by $\tau_1\tau_2$ -cl (A) and $\tau_1\tau_2$-cl(A) = $\bigcap$ { F:A⊂F and F^C is a $\tau_1\tau_2$ -open}

Remark 2.2 [2] Notice that $\tau_1\tau_2$-open subsets of X need not necessarily form a topology.

Now, we recall some definitions and results which are used in this paper.

Definition 2.3 [7] A subset *A* of a bitopological space X is

(i) (1,2) semi-open set if $A \subseteq \tau_2$-cl(τ_1-int(A))

(ii) (1,2) pre - open set if $A \subseteq \tau_1$-int(τ_2-cl(A))

(iii) (1,2) α - open set if $A \subseteq \tau_1$-int(τ_2-cl(τ_1-int(A)))

(iv) (1,2) b-open set if $A \subseteq \tau_1$-int (τ_2-cl(A)) $\bigcup$ τ_2-cl(τ_1-int(A)) .

(v) (1, 2) regular open set if *A* = τ_1-int (τ_2-cl(*A*)).

The complements of all the above mentioned open sets are called their respective closed sets. The family of all (1,2) open sets [(1,2) semi open, (1,2) regular open,(1,2) α-open, (1,2) b-open]of X will be denoted by (1,2) O(X)(resp. (1,2) SO(X), (1,2) RO(X), (1,2)α O(X), (1,2) b O(X)].

Proposition 2.4[7] In a bitopological space (X, τ_1, τ_2) , any open set in (X, τ_1) is (1,2) b-open set and any open set in (X, τ_2) is (2,1) b-open set .

Remark 2.5[7] In a bitopological space (1, 2) b-open set and (1, 2) locally closed are independent.

Proposition 2.6 [7]

(i) The union of any family of (1, 2) b-open sets is (1, 2) b-open.

(ii) The intersection of an τ_1 open set and a (1, 2) b-open set is a (1, 2) b-open set.

III. (1, 2) - open set

Definition 3.1 A subset *A* of (X, τ_1, τ_2) is said to be (1, 2) γ- open set if for any non empty (1, 2) pre-open set *B* in *X* such that $A \bigcap B \subseteq \tau_1$-int ($\tau_2$-cl ($A \bigcap B$)) .

Example 3.2 Let X={a, b, c}, τ_1 = {{a} ,{b}, {a,b}, φ ,X } , τ_2 = {{b}, φ ,X }

(1, 2) γ- O(X) = {{a}, {b}, {a, b} ,{b,c}, φ ,X}.

Remark 3.3 In a bitopological space (X, τ_1, τ_2) a τ_1 open set and a (1, 2) γ-open set are independent as seen in the following examples.

Example 3.4 Let X= {a, b, c} $\tau_1 = \{$ {a , c},φ ,X} τ_2 = {{b}, {a, b},φ ,X}

(1, 2)PO(X) = {{a, c}, {b, c}, {a, b}, {b}, φ , X} (1,2) γ -O(X) = {{a,b}, {b}, φ ,X

Here, {a, c} is τ_1 open set but not a (1, 2) γ-open set .

Example 3.5 Let X= {a, b, c} $\tau_1 = \{$ {a }, {b}, {a, b} φ ,X} τ_2 = {{b}, { c },{b ,c}, φ ,X}

(1, 2)PO(X) = {{a }, {b }, {a, b}, {b, c}, φ ,X}

(1, 2) γ -O(X) = {{a}, {b }, {a, b}, {b, c}, φ ,X }

Here {b,c} is a (1, 2) γ-open set but not a τ_1 -open set.

Definition 3.6 A subset A of (X, τ_1, τ_2) is called

(i) (1,2) locally closed if $A = U \bigcap V$,where $U \in \tau_1$ and V is a τ_2 - closed set.

(ii) (1,2) locally γ -closed if $A = U \bigcap V$, where $U \in \tau_1$ and V is a τ_2 - γ closed set.

Remark 3.7 (1, 2) γ- open set and (1, 2) locally closed are independent as seen from the following example.

Example 3.8 Let X = {a,b, c, d}, τ_1 ={{a}, {a,b}, {a, c, d}, φ ,X}, τ_2 = {{a},{d}, {a, d}, φ ,X}

(1, 2) γ -O(X) = {{a}, {a,b},{a, d},{a, b, d},{a, c, d}, φ ,X}.

(1,2) LC(X) = {{a}, {b},{c},{a, c},{a,b}, {a, b, c},{b, c, d}, {a, c, d}, φ ,X }.

Here {a, b, d} is (1,2) γ -open set but not (1,2) locally closed and {a, b,c} is (1,2) locally closed but not (1,2) γ -open set.

Definition 3.9 A space is called (1, 2) extremally γ-disconnected space if τ_2 - γ closure of each τ_1 - γ open set is τ_1 - γ open ,similarly τ_1 - γ closure of each τ_2 - γ open set is τ_2 - γ open.

Example 3.10 Let X = {a,b, c} , τ_1 = {{a}, {b}, {a,b}, φ ,X}. τ_2 = {{a},{c},{a, c}, φ ,X}

τ_1 - γ O(X) = {{a}, {b}, {a, b}, φ ,X}.

τ_2 - γ O(X) = {{a},{c}, {a, c}, φ , X}.

Hence every τ_2 - γ closure of each τ_1 - γ open set is τ_1 - γ open set and also every τ_1 - γ closure of each τ_2 - γ open set is τ_2 - γ open set.

Theorem 3.11 For a subset A of an (1, 2) extremely γ-disconnected space X, if A is (1, 2) γ-open set and (1, 2) locally closed set , then A is τ_1 -open.

Proof Let A be (1, 2) γ-open set and (1, 2) locally closed set. So

$(A \bigcap B) \subseteq \tau_1 \operatorname{int}(\tau_2 Cl(A \bigcap B))$ and $A = U \bigcap \tau_2 Cl(A)$ .Then

$$(A \bigcap B) \subseteq \tau_1 \operatorname{int}(\tau_2 Cl(A) \bigcap \tau_2 Cl(B))$$

$$\subseteq \tau_1 \operatorname{int}(\tau_2 Cl(U \bigcap \tau_2 Cl(A)) \bigcap \tau_2 Cl(B))$$

$$\subseteq \tau_1 \operatorname{int}(U \bigcap \tau_2 Cl(A) \bigcap \tau_2 Cl(B)) \quad \text{(Since X is (1, 2) extremely γ-disconnected)}$$

$$\subseteq \tau_1 \operatorname{int}(A \bigcap \tau_2 Cl(B))$$

$$\subseteq \tau_2 Cl(\tau_1 \operatorname{int}(A \bigcap B))$$

$$= \tau_1 \operatorname{int}(A \bigcap B)$$

Hence $(A \bigcap B) \subseteq \tau_1 \operatorname{int}(A \bigcap B)$

Therefore A is τ_1 -open.

Proposition 3.12 Let H be a subset of X, H is (1,2) locally γ- closed set iff there exist an τ_1 open set U$\subseteq$ X such that H=U$\bigcap$ (1,2) γ cl-(H).

Proof: Since H is (1,2) locally γ-closed $\Rightarrow$ H=U$\bigcap$ F, where U is τ_1 open and F is (1,2) γ-closed . So H $\subseteq$ U, H$\subseteq$F

H $\subseteq$ (1, 2) γ-cl (H) $\subseteq$(1,2) γ-cl(F)=F

Hence H$\subseteq$U$\bigcap$(1,2) γ-cl(H) $\subseteq$ U$\bigcap$(1,2) γ- cl(F)$\subseteq$U$\bigcap$F=H

Hence H = U$\bigcap$ (1,2) γ-cl(H) .

Conversely since (1,2) γ-cl(H) is (1,2) γ- closed and H=U $\bigcap$ (1,2) γ-cl(H) then H is (1,2) locally γ- closed.

Proposition 3.13 The union of any family of (1, 2) γ -open set is a (1,2) γ -open set .

Proof: Let *A* and *B* be any two (1,2) γ- open set .So there exist two non empty pre-open set C and D ,we have

$$(A\bigcap C)\subseteq\tau_1 \operatorname{int}(\tau_2 cl(A\bigcap C)).$$

$$(B\bigcap D)\subseteq\tau_1 \operatorname{int}(\tau_2 cl(B\bigcap D)).$$

Now, $(A\bigcup B)\bigcap(C\bigcup D)$

$= (A\bigcap C)\bigcup(B\bigcap D)$

$\subseteq\tau_1 \operatorname{int}(\tau_2 cl(A\bigcap C))\bigcup\tau_1 \operatorname{int}(\tau_2 cl(B\bigcap D))$

$\subseteq\tau_1 \operatorname{int}(\tau_2 cl(A\bigcup B))\bigcap\tau_1 \operatorname{int}(\tau_2 cl(B\bigcup D))$

$\subseteq\tau_1 \operatorname{int}(\tau_2 cl((A\bigcup B)\bigcap(C\bigcup D)))$.

Hence the union of any family of (1,2) γ -open set is a (1,2) γ -open set.

Proposition 3.14 The intersection of any two (1, 2) γ-open set is a (1, 2) γ-open set .

Proof Let *A* and *B* be any two (1,2) γ- open set .

So, for any (1, 2) pre open set *C*

$(A\bigcap C)\subseteq\tau_1 \operatorname{int}(\tau_2 cl(A\bigcap B))$ and

$(B\bigcap C)\subseteq\tau_1 \operatorname{int}(\tau_2 cl(B\bigcap C)$

Now, $(A\bigcap B) = A\bigcap(B\bigcap C)$

By the definition of (1, 2) γ- open set $(A\bigcap(B\bigcap C))$ is a (1, 2) pre open set. Hence $A\bigcap B$ is a (1,2) γ- open set.

Proposition 3.15 Let *A* be a subset of (X,τ_1,τ_2) and if *A* is (1,2) locally γ -closed then

(i) (1,2) γ cl(*A*)- *A* is (1,2) γ -closed.

(ii) [*A*$\bigcup$ (X-γ cl(*A*)] is (1,2) γ -open set.

(iii) $A\subseteq\gamma\operatorname{int}(A\bigcup(X-\gamma cl(A)))$

Proof (i) If *A* is an (1,2) locally γ -closed, then there exist an τ_1 open set U such that

$A = U\bigcap(1,2)\gamma cl(A)$

Now, $(1,2)\gamma cl(A)$ - *A*

= (1, 2) γ *c l*(*A*) – [$U\bigcap(1,2)\gamma cl(A)$]

= (1, 2) γ cl (*A)* $\bigcap$ [(X-U)$\bigcup$ (X-(1,2) cl(*A*))].

= (1, 2) γ cl (*A)* $\bigcap$ (X-U) ,which is (1,2) γ-closed.(by proposition 3.8)

(ii)Since (1,2) cl(*A*)-*A* is (1,2) γ - closed, then [X-((1,2)γ cl(*A*)-*A*] is (1,2) γ -open .

And

{[X-((1, 2)cl(*A*)-*A*)]

=(X-(1, 2) cl(*A*))$\bigcup$ (X$\bigcap A$)

= *A*$\bigcup$ [X-(1, 2)cl(*A*)]}.

Hence [*A*$\bigcup$ (X- cl (*A*))] is (1, 2) γ -open.

(iii) It is clear that

$A\subseteq[A\bigcup(X-(1,2)\gamma cl(A)]$

= (1, 2) γ int [(A$\bigcup$ (X- γ cl (A))].Hence the proof.

Remark 3.16 (1, 2) α -open set and (1, 2) locally γ-closed are independent as seen from the following example.

Example 3.17 Let X={a ,b, c, d}, τ_1={{a}, {a,b}, {a, c, d} , φ,X}, τ_2={{a}, {a, d}, φ,X}.

(1,2) α O(X)={{a},{a,b},{a, c},{a, d},{a, b,c},{a ,b ,d},{a, c, d}, φ X}

Locally (1,2) γ C(X) = {{a},{b},{c},{d},{a,b},{b,c},{b, d},{c, d},{b, c ,d},{a, c, d}, φ ,X}

It is obvious that {a, b, c} is a (1, 2) α -open set but not (1,2) locally γ -closed . Also {b, c, d} is (1,2) locally γ -closed but not (1,2) α -open set.

Proposition 3.18 A subset A of (X,τ_1,τ_2) is τ_1-open iff it is (1, 2) pre-open and (1,2) B-set.

Proof If A is τ_1 open then A is (1,2) pre-open.

And $A \subseteq \tau_1 \operatorname{int}(\tau_2 \operatorname{cl}(A))$

So $A = U \cap V$, where $U \in \tau_1$ and V is a (1,2) t-set.

Hence A is a (1, 2) pre-open and (1,2) B-set.

Conversely A is a (1, 2) pre-open and (1,2) B-set.

$\Rightarrow A \subseteq \tau_1 \operatorname{int}(\tau_2 cl(A))$ and $A = U \cap V$, where $U \in \tau_1$ and V is a (1,2) t-set.

Therefore $\tau_1 \operatorname{int} A = \tau_1 \operatorname{int}(\tau_2 cl(A))$

Hence $\tau_1 \operatorname{int} A \subseteq A$

Also $\tau_1 \operatorname{int} A \subseteq A$

Hence A is τ_1 open.

Definition 3.19 A subset A of a space (X,τ_1,τ_2) is said to be

(i) (1, 2) γ- semi open if $A \subseteq \tau_2 cl(\tau_1 \gamma \operatorname{int}(A))$

(ii) (1, 2) γ -pre open if $A \subseteq \tau_1 \operatorname{int}(\tau_2\ \gamma cl(A))$

Proposition 3.20 For subsets A and B of a space (X,τ_1,τ_2) the following properties are hold.

(i) A is (1, 2) γ-t set iff it is (1, 2) γ-semi closed.

(ii) If A and B are (1, 2) γ-t sets then $(A \cap B)$ is a (1, 2) γ-t set.

(iii) If A is τ_2-γ closed then it is a (1,2) γ-t set.

Proof (i) Let A be (1, 2) γ-t set.

So $\tau_1 \operatorname{int} A = \tau_1 \operatorname{int}(\tau_2\ \gamma\text{-cl}(A))$

Therefore, $\tau_1 \operatorname{int}(\tau_2 (\gamma\text{-cl}(A)) \subseteq \tau_1 \operatorname{int} A \subseteq A$ and A is (1,2) γ- semi closed.

Conversely, If A is (1, 2) γ-semi closed then

$\tau_1 \operatorname{int}(\tau_2\ \gamma\text{-cl}(A)) \subseteq A$

Thus $\tau_1 \operatorname{int}(\tau_2\ \gamma\text{-cl}(A)) \subseteq \tau_1 \operatorname{int} A$

Also $A \subseteq \tau_2 \gamma\text{-cl}(A)$

$\Rightarrow \tau_1 \operatorname{int} A \subseteq \tau_1 \operatorname{int}(\tau_2 \gamma\text{-cl}(A))$

Hence $\tau_1 \operatorname{int} A = \tau_1 \operatorname{int}(\tau_2 \gamma\text{-cl}(A))$.

(ii) Let A and B be (1, 2) γ- t –sets. Then we have

$$\tau_1 \operatorname{int}(A \cap B) \subseteq \tau_1 \operatorname{int}(\tau_2 cl(A \cap B))$$
$$\subseteq \tau_1 \operatorname{int}(\tau_2 cl(A)) \cap \tau_1 \operatorname{int} \tau_2 cl(B))$$
$$= \tau_1 \operatorname{int} A \cap \tau_1 \operatorname{int} B$$

Then $\tau_1 \operatorname{int}(A \cap B) = \tau_1 \operatorname{int}(\ \tau_2 cl(A \cap B))$

Hence $A \cap B$ is (1, 2) γ- t –set.

(iii) Let A be a τ_2-γ closed .

A = τ_2-γ clA

$\Rightarrow \tau_1 \operatorname{int} A = \tau_1 \operatorname{int}(\tau_2 \gamma\text{-cl}(A))$

Hence A is a (1,2) γ-t set.

Remark 3.21 In a bitopological space, if A is a (1, 2) γ-t set then it is may not be (1, 2) γ -closed as seen in the following example.

Example 3.22 Let X ={a,b, c} , τ_1 = {{a},{b},{a,b} φ ,X}, τ_2 = {{b}, φ ,X}

(1, 2) PO(X) = {{a}, {b}, {a,b},{b,c}, φ ,X}. (`1, 2) γ O(X) ={{a},{b},{a,b},{b,c}, φ, X}

(1, 2) γ -cl(X) = {{b, c}, {a, c},{c},{a}, φ ,X}.

Here {b} is (1, 2) γ -t set but it is not (1, 2) γ -closed.

Remark 3.23 In bitopological space every (1, 2) γ-open set is (1, 2) b -open set but converse is not true as seen in the following example.

Example 3.24 Let X = {a, b, c, d} , τ_1 = { {a}, {a, c , d}, {a, b}, φ,X}, τ_2 = {{a}, {d}, {a , d} , φ ,X}
(1, 2) γ O(X) = {{a}, {a, b}, {a, d}, {a, b, d}, {a, c, d}, φ , X}
(1,2) b O(X) ={{a}, {a,b},{a, d},{a,b ,d},{a, c, d },{a, c},{a,b, c}, φ,X}
It is obvious that {a, c} is (1, 2) b- open set but not (1, 2) γ - open set.

IV . (1, 2)*- γ - Open Set

Now we define (1, 2)* γ-open set using the $\tau_{1,2}$ open set [8] and $\tau_{1,2}$ pre open set [8] .

Definition 4.1 Let A be a subset of X. Then A is called $\tau_{1,2}$ - open [8] if A= $A_i \bigcup B_i$
Where $A_i \in \tau_1$, $B_i \in \tau_2$.
The complement of $\tau_{1,2}$-open set [8] is $\tau_{1,2}$ -closed set. The family of all $\tau_{1,2}$-open set and $\tau_{1,2}$-closed set is denoted by (1, 2)*-O(X), (1,2)*-C (X).

Definition 4.2 Let A be a subset of a bitopological space X. Then
1) $\tau_{1,2}$-closure of *A* [8] denoted by $\tau_{1,2}$-cl(*A*) is defined as the intersection of all $\tau_{1,2}$ - closed sets containing *A*.
2) $\tau_{1,2}$ -interior of *A* [8] denoted by $\tau_{1,2}$ -int (*A*) is defined as the union of all $\tau_{1,2}$ - open sets contained in *A* .

Definition 4.3 A subset *A* of a space (X, τ_1, τ_2) is called (1,2)*- γ - open set if there exist a non empty (1,2)*- pre open set *B* such that
$$(A \bigcap B) \subseteq \tau_{1,2} - \text{int}(\tau_{1,2} - cl(A \bigcap B)).$$

Example 4.4 Let X = {a, b, c} τ_1 = {{a}, {a, b}, φ, X} τ_2 = {{c}, φ, X}
$\tau_{1,2}$-open = {{a}, {a, b}, {c}, {a, c}, φ, X} .(1, 2)* -pre -open set = {{a}, {a, b}, {c}, {a, c}, φ , X }
(1, 2)* - γ -open set = {{a}, {a, b}, {c}, {a, c}, φ , X }

Remark 4.5 Every (1, 2) γ open set is a (1, 2)*-γ-open set but converse may not be true as seen in the following example.

Example 4.6 Let X = {a, b, c} τ_1 = {{a}, {a, b}, {b}, φ, X} τ_2 = {{c}, {a,b}, φ, X}
(1, 2) pre -open = {{a}, {a, b}, {b} , {b, c}, {a, c}, φ, X}
(1, 2) γ -open set = {{a}, {a, b}, {b} , φ, X}
(1,2)* -O(X) = {{a}, {a, b}, {b , {c}, {b,c}, {a ,c}, φ, X}
(1,2)* - PO(X) = {{a}, {a, b}, {b} , {c}, {b,c}, {a ,c}, φ, X}
(1, 2)* - γ -O(X) = {{a}, {a, b}, {b} {c}, {b,c}, {a ,c}, φ, X}

V . Conclusion

We know that in bitopological space collection of open set need not necessarily form a topology [2] and violate the topological properties. But from the present study we conclude that collection of γ -open set form a topology in bitopological space .Also in this paper we have proved (1,2) γ -O(X) $\subseteq$ (1,2)*- γ-O(X) and we will use the collection (1,2)*- γ-open set for more results .

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