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An Inventory Model for Constant Demand with Shortages under Permissible Delay in Payments

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Abstract : *This paper presents an inventory model for deteriorating products with constant demand and time varying deteriorating rate, under permissible delay in payments. It discussed in two cases whether the permissible periods are less than or equal to or greater than replenishment cycle. During the permissible period both supplier and retailer got some benefits. Shortages are allowed and are completely backlogged. This model is explained with numerical example and sensitivity analysis.*
Keywords - *Deteriorating Products, Economic Order Quantity, Holding Cost, Permissible delay in periods.*

I. Introduction

In Recent periods, researchers are developing inventory models for deteriorating items by introducing new concepts. For example there are few inventory models in association with salvage Value or disposal cost or sales revenue etc. Similarly researchers have developed trade credit with new ideas. This trade credit means that the supplier offers the customer (Case I) a permissible delay in payments to attract new customers to increase sales (Case II) and cash discount to motivate quick payment and reduce credit expenses. Case (I) has been derived under two different situations, (a) the permissible delay period less than or equal to replenishment cycle period for setting the account and (b) the permissible delay period is greater than replenishment cycle period for settling the account. Ouyang L-Y, C-T Chang and J-T Teng [14] developed an economic order quantity model under trade Credit .Mukesh Kumar, Anand Chauhan, Rajat Kumar [12] extended Ajantha Roy [1]model with trade credit for the purpose of maximizing the profit. Chaitanya Kumar Tripathy and Umakanta Mishra [2] and Shital S.Patel and R.D.Patel [17] gave inventory models with permissible delay in periods which include purchase cost and selling price with shortages and without shortages respectively. Kun –Shan Wu [8] made an attempt in his paper to obtain the optimal ordering quantity of deteriorating items for two parameters of Weibull distribution deterioration with shortages under permissible delay in payments. Manish pal and Sanjay Jain, Mukesh Kumar and Priya Advani [16], Tripathi R.P [20] discussed permissible delay in payments without deterioration. It means that they obtain an optimal ordering policy only for demand rate and not for any deterioration rate. Nita H.Shah, Poonam Pandey [13] prepared inventory model under delay in payment is permissible. Tripathy P.K, S.Pradhan [22] and Hesam Saiedy, Mohammed Bamani Moghadam [4] discussed partial backlogging in trade credit. Hardik Soni, Nita H.Shan and Chandra K.Jaggi [3] presented a review about trade credit. Trailokyanath Singh, Hadibandhu Pattanayak [19] formulated a model for exponentially declining demand under permissible delay in payments without shortages. Ravi Gor, Nita H Shah [15] developed a selling price dependent demand rate in which selling price is exponentially increasing with respect to time.

In our proposed model an attempt is made to develop an EOQ model for deteriorating items under permissible delay in payment with constant demand and time proportional deterioration rate. Shortages are allowed and are completely backlogged. Mathematical model has been developed in two cases. (I) The Credit period is less than or equal to the cycle time for settling the account and (ii) the credit period is greater than the cycle time for settling the account. Our aim is to minimize the total cost. Numerical examples are given and sensitivity analysis is carried out for changes in.

II. Assumptions and Notations

- *The inventory system involves only one item*
- *Lead time is zero*
- *Shortages are not allowed*
- *Replenishment is instantaneous*
- *Delay in payment is allowed.*
- *Time horizon is infinite*
- *During time $T_1$the inventory level becomes Zero due to demand and deterioration. At time $T_1$the shortages start occurring.*
- *The demand for the item is constant with time.*

- *A = set up cost*
- *Q = the initial order quantity*
- *h = holding cost per unit*
- *$\theta = \theta t$, time proportional deterioration rate.*
- *C = the unit purchasing cost*
- *I_e = The Interest earned per unit time*
- *I_c= the interest Charged per unit time.*
- *I(t) = Inventory level at any instant of time, $0 \le t \le T$*
- *T = Time interval (in Years) between two successive orders.*
- *M = Permissible period of delay in selling the accounts with the supplier.*
- *C_3 = The Inventory shortage cost per unit time*

III. MATHEMATICAL MODEL

THE DIFFERENTIAL EQUATION THAT GOVERNS THE VARIATION OF INVENTORY WITH RESPECT TO TIME IS

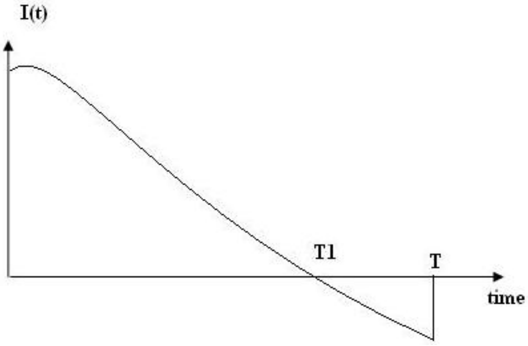

Fig 1

$$\frac{dI(t)}{dt} + \theta t I(t) = -a, \; 0 \le t \le T_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -a, \; T_1 \le t \le T \tag{2}$$

With boundary condition I (T_1) = 0 and I (0) = Q.

Solving equation (1)

$$I(t) = a(T_1 - t) + a\,\theta\left[\frac{T_1^3}{6} - \frac{t^3}{6} - \frac{T_1 t^2}{2} + \frac{t^3}{2}\right] + a\theta^2\left[\frac{T_1^5}{40} - \frac{t^5}{40} - \frac{T_1^5 t^2}{6} + \frac{t^5}{6} + \frac{T_1 t^4}{8} - \frac{t^5}{8}\right] \tag{3}$$

Solving equation (2)

$$I(t) = a(T_1 - t) \tag{4}$$

Shortage cost

$$SC = \int_{T_1}^{T} I(t)dt$$

$$= -C_3 \int_{T_1}^{T} a(T_1 - t)dt$$

$$= \frac{-aC_3}{2}\left[2TT_1 - T^2 - T_1^2\right] \tag{5}$$

Put I (0) = Q in (3)

$$Q = aT_1 + \frac{a\theta T_1^3}{6} + \frac{a\theta^2 T_1 5}{40} \tag{6}$$

Deterioration cost

$$DC = C\left(Q - \int_{0}^{T_1} D(t)dt\right)$$

$= -C\,(Q - \int_0^{T_1} a\,dt\,)$

$$DC = C\,(\frac{a\theta T_1^3}{6} + \frac{a\theta^2 T_1 5}{40}) \quad (7)$$

Inventory holding cost

$HC = h\int_0^{T_1} I(t)dt$

$= h\int_0^{T_1}[a(T_1 - t) + a\theta(\frac{T_1^3}{6} - \frac{t^3}{6} - \frac{T_1 t^2}{2} + \frac{t^3}{2}) + a\,\theta^2(\frac{T_1^5}{40} - \frac{t^5}{40} - \frac{T_1^3 t^2}{60} + \frac{t^5}{6} + \frac{T_1 t^4}{8} - \frac{t^5}{40})]\,dt$

$$= \frac{h}{T}\,[\frac{aT_1^2}{2} + \frac{a\theta T_1^4}{12} + \frac{11a\theta^2 T_1^6}{720}] \quad (8)$$

The setup cost per cycle : $\frac{A}{T}$ (9)

Case I: $M \le T_1$ (Payment at or before total depletion of inventory)

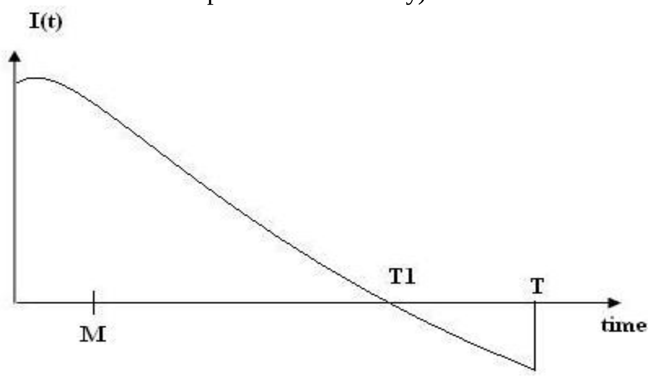

Fig. 2

In this case, the credit time expires on or before the inventory depleted completely to zero. The interest payable per cycle for the inventory not being sold after the due date M is Interest payable in the horizon when $M < t \le T_1$

$IE_1 = CI_e\int_0^{T_1}(T_1 - t)dt$

$$= \frac{aCI_e T_1^2}{2} \quad (10)$$

$IC_1 = CI_c\int_M^{T_1} a\,dt$

$= CI_c\,(T_1 - M).$ (11)

Total average cost of the system per unit time is given by

$K_1(T, T_1) = \frac{1}{T}$ *(setup cost + Deterioration cost + holding cost + shortages cost +* $IC_1 - IE_1$*)*

$$K_1(T, T_1) = \frac{1}{T}[A + C(\frac{a\theta T_1^3}{6} + \frac{a\theta^2 T_1^5}{40}) + h(\frac{aT_1^2}{2} + \frac{a\theta T_1^4}{12} + \frac{11a\theta^2 T_1^6}{720}) - \frac{ac_3}{2}(2TT_1 - T^2 - T_1^2) + \frac{aCI_e T_1^2}{2} - CI_c\,a(T_1 - M)]. \quad (12)$$

Our objective is to minimize the total cost. The necessary conditions for minimizing the total cost are

$\frac{\partial K_1(T,T_1)}{\partial T_1}$ = 0 and $\frac{\partial K_1(T,T_1)}{\partial T}$ = 0

$$\frac{\partial K_1(T,T_1)}{\partial T_1} = \frac{1}{T}[C(\frac{a\theta T_1^2}{2}+\frac{a\theta^2 T_1^4}{8}) + h(aT_1+\frac{a\theta T_1^3}{3}+\frac{11a\theta^2 T_1^5}{120}) - \frac{aC_3}{2}(2T-2T_1) + \frac{aCI_e T_1}{2} - CI_c a] = 0 \quad (13)$$

$$\frac{\partial K_1(T,T_1)}{\partial T} = \frac{-1}{T^2}[A+C(\frac{a\theta T_1^3}{6}+\frac{a\theta^2 T_1^5}{40})h(\frac{aT_1^2}{2}+\frac{a\theta T_1^4}{12}+\frac{11a\theta^2 T_1^6}{720})] - \frac{aC_3}{2}(0-1+\frac{T_1^2}{T^2}) - (\frac{aCI_e T_1^2}{2T^2}+\frac{CI_c a(T_1-M)}{T^2}\} = 0 \quad (14)$$

Provided they (equations (13) and (14)) satisfy the conditions

$$(\frac{\partial^2 K_1(T,T_1)}{\partial T^2}) > 0, (\frac{\partial^2 K_1(T,T_1)}{\partial T_1^2}) > 0 \ \& \ (\frac{\partial^2 K_1(T,T_1)}{\partial T^2})(\frac{\partial^2 K_1(T,T_1)}{\partial T_1^2}) - (\frac{\partial^2 K_1(T,T_1)}{\partial T \partial T_1}) > 0$$

Case II: $M > T_1$ (Payment after depletion)

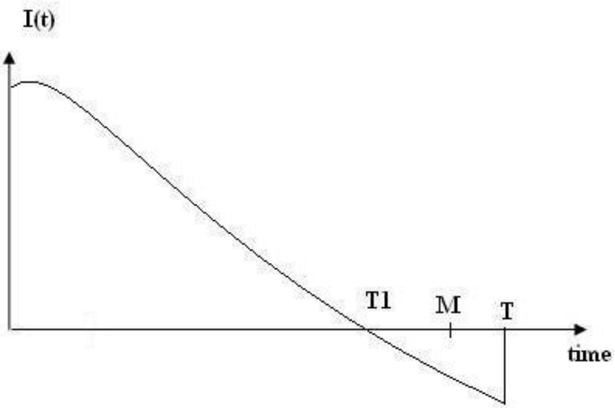

Fig.3

In this case, the interest payable per cycle is zero, $IC_2 = 0$, when $T_1 < M \le T$

$$IE_2 = CI_e[\int_0^{T_1}(T_1-t)(a)dt + (M-T_1)\int_0^{T}(a)dt]$$

$$= CI_e[aMT_1 - \frac{aT_1^2}{2}] \quad (15)$$

Total average cost of the system per unit time is given by

$K_2(T,T_1) = \frac{1}{T}$ [set *up cost + Deterioration cost + holding cost + Shortage cost – IE₂]*

$$K_2(T,T_1) = \frac{1}{T}[A + C(\frac{a\theta T_1^3}{6}+\frac{a\theta^2 T_1^5}{40}) + h(\frac{aT_1^2}{2}+\frac{a\theta T_1^4}{12}+\frac{11a\theta^2 T_1^6}{720}) - \frac{ac_3}{2}(2TT_1 - T^2 - T_1^2) - CI_e(aMT_1 - \frac{aT_1^2}{2})] \quad (16)$$

Our objective is to minimize the total cost. The necessary conditions for minimizing the total cost are

$\frac{\partial K_2(T,T_1)}{\partial T_1}$ = 0 and $\frac{\partial K_2(T,T_1)}{\partial T}$ = 0

$$\frac{\partial K_2(T,T_1)}{\partial T_1} = \frac{1}{T}[C(\frac{a\theta T_1^2}{2}+\frac{a\theta^2 T_1^4}{8}) + h(aT_1+\frac{a\theta T_1^3}{3}+\frac{11a\theta^2 T_1^5}{120}) - \frac{aC_3}{2}(2T-2T_1) - CI_e(aM - aT_1)] = 0 \quad (17)$$

$$\frac{\partial K_2(T,T_1)}{\partial T} = \frac{-1}{T^2}[A+C(\frac{a\theta T_1^3}{6}+\frac{a\theta^2 T_1^5}{40})\,h\,(\frac{aT_1^2}{2}+\frac{a\theta T_1^4}{12}+\frac{11a\theta^2 T_1^6}{720})] - \frac{aC_3}{2}(-1+\frac{T_1^2}{T^2}) +$$

$$(\frac{CI_e(aMT_1)}{T^2} - \frac{CI_e aT_1^2)}{2T^2}) = 0 \qquad (18)$$

Provided they (equations (17) & (18)) satisfy the conditions

$$(\frac{\partial^2 K_2(T,T_1)}{\partial T_1^2}) > 0,\ (\frac{\partial^2 K_2(T,T_1)}{\partial T^2}) > 0 \text{and} (\frac{\partial^2 K_2(T,T_1)}{\partial T^2})(\frac{\partial^2 K_2(T,T_1)}{\partial T_1^2}) - (\frac{\partial^2 K_2(T,T_1)}{\partial T_1 \partial T}) > 0.$$

IV. NUMERICAL EXAMPLES

Case I: M ≤ T_1

Let us take A = 1000, a = 300, h= 5, θ = 0.6, C_3=0.8, I_e = 0.09, M = 0.08, I_c = 0.12, C= 20 in the respective proper units. Then we get T_1=0.4801 is greater than M = 0.08. Also T = 2.9532, TC = 593.5394 and Q = 247.4187

Case II: M > T_1

Let us take A = 1000, a = 300, h= 5, θ = 0.6, C_3=0.8, I_e = 0.09, M = 0.35, C= 20 in the respective proper units. Then we get T_1=0.3143 is less than M = 0.35. Also T = 2.9899, TC = 642.1459 and Q = 95.2297

V. SENSITIVITY ANALYSIS

Sensitivity analysis has been carried out by keeping one variable changing while all other variables as constant.

Table 1

h	T_1	T	TC	Q
6	0.4456	2.9588	603.1786	136.3818
7	0.4148	2.9627	611.5131	126.6143
8	0.3873	2.9655	618.7733	117.9564
9	0.3627	2.9675	625.1404	110.2584
θ	T_1	T	TC	Q
0.7	0.4615	2.9475	596.6431	141.9671
0.8	0.4452	2.9430	599.4622	137.1735
0.9	0.4308	2.9393	602.0433	132.9280
1.0	0.4178	2.9362	604.4248	129.0820
a	T_1	T	TC	Q
350	0.4667	2.7278	633.0839	166.9725
400	0.4557	2.5453	668.6476	186.1360
450	0.4464	2.3935	700.9201	204.9548
500	0.4385	2.2645	730.4170	223.5387
C_3	T_1	T	TC	Q
0.7	0.4715	3.1414	560.6751	144.6575
0.6	0.4619	3.3757	524.4924	141.5832
0.5	0.4510	3.6785	484.1174	138.1024
0.4	0.4385	4.0903	438.2180	134.1232

Table - 2

h	T_1	T	TC	Q
6	0.2886	2.9833	646.7447	87.3065
7	0.2662	2.9772	650.6418	80.4295
8	0.2467	2.9716	653.9757	74.4629
9	0.2295	2.9664	656.8530	69.2144
θ	T_1	T	TC	Q
0.7	0.3054	2.9854	643.1991	92.6267
0.8	0.2973	2.9814	644.1701	90.2523
0.9	0.2900	2.9778	645.0703	88.1100
1.0	0.2832	2.9745	645.9087	86.1093

a	T_1	T	TC	Q
350	0.2991	2.7680	691.2791	105.6291
400	0.2866	2.5888	736.6964	115.5886
450	0.2761	2.4402	779.0716	125.1986
500	0.2671	2.3143	818.8957	134.5089
C_3	T_1	T	TC	Q
0.7	0.3029	3.1824	604.6992	91.7106
0.6	0.2901	3.4220	563.7346	87.7680
0.5	0.2758	3.7314	518.3469	83.3737
0.4	0.2592	4.1521	467.1546	78.2856

VI. Conclusion

This model developed in this paper assumes that demand of the product is constant and deteriorating rate is time proportional. The benefit of the trade credit for the retailer is illustrated with numerical example and sensitivity analysis. It can be noted down that increase of holding cost, deterioration rate and demand give increase in total cost. Similarly decrease in shortage cost decreases the total cost for case (I). For Case (II) also increase in the parameters increases the total cost and decreases causes decrease in the total cost.

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