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# T-Pre –Operators

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***Abstract**: The main object of this paper to introduce T-pre-operator,T-pre-open set, T-pre-monotone,pre-subaditive operator and pre-regular operator. As well as we introduce (T,L)pre-continuity.*
***Keywords**: T-pre-operator, T-pre-open set, (T,L)pre-continuity*

## I. Introduction

Mashhour [2] introduced pre-open sets in a topological space and studied some of their properties. In 1979 kasahara [1] defined the operator $\alpha$ on atopological space $(X,\Gamma)$ as a map from P(X) to P(X) such that $U\subseteq\alpha(U)$ for every $U \in \Gamma$. In 1991 Ogata[4] called the operation $\alpha$ as $\gamma$ operation and introduced the notion of $\tau\gamma$ which is the collection of all $\gamma$- open sets in atopological space $(X ,\tau)$. Several research papers published in recent years using $\gamma$ operator due to Ogata[4].

In1999 Rossas and Vielma [5] modified the definition by allowing the operator $\alpha$ to be defined in P(X) as a map $\alpha$ from P(X) to P(X). In 2006 Mansur and Ibrahim [3] introduced the concept of an operation T on $\alpha$-open set in a topological space $(X,\Gamma)$ namely T- *$\alpha$-operator* and studied some of their properties .

In this paper we introduce the concept of an operation T on a family of pre-open sets in atopological space $(X,\Gamma)$. Asubset S of X is called pre-open set if $S \subset int(cl(S))$. In §2 Using the operation T, we introduce the concept T-pre-open sets, T-pre-monotone, pre-subaditive operator and pre-regular operator. We study some of their properties and optained new results. In §3 we present and study new types of function by using the operations T and L. say (T,L)pre-continuous, (T,L)pre-irresolute continuous and (T,L)strongly pre-continuous.

## II. T-Pre-Operators

***2.1 Definition:***

Let $(X,\Gamma)$ be a topological space and T be an operator from $\Gamma_{pre}$ to P(X), i.e., $T : \Gamma_{pre} \longrightarrow P(X)$. We say that T is a pre-operator associated with $\Gamma_{pre}$ if:

$U \subseteq T(U)$, for every $U \in \Gamma_{pre}$

and the triple $(X,\Gamma,T)$ is called T-pre-operator topological space.

The following example shows that T is a pre-operator.

***2.2 Example:***

Let $X = \{a,b,c\}$, $\Gamma = \{\emptyset,X,\{a\},\{a,b\}\}$, $\Gamma_{pre} = \{\emptyset,X,\{a\},\{a,b\}, \{a,c\}\}$ and $T : \Gamma_{pre} \longrightarrow P(X)$, defined as:

$$T(U) = \begin{cases} U, & \text{if } U = \{a\} \\ cl(U), & \text{if } U \neq \{a\} \end{cases}$$

Clearly $U \subseteq T(U)$, for every $U \in \Gamma_{pre}$
Hence, T is a pre-operator.
In the following theorem, we present the relationship between T-operator and T-pre-operator.

***2.3 Theorem:***

Every T-operator is T-pre-operator.

***Proof:***

Suppose that $(X,\Gamma,T)$ is an operator topological space
Therefore, $U \subseteq T(U)$, for every $U \in \Gamma$
Since every open set is a pre-open set
Hence, each open set in T(U) is a pre-open set
Therefore, T is a pre operator. ■

The Converse of the theorem is not true in general, as the following example shows:

***2.4 Example:***

The operator T in example (2.2) is a pre-operator, but not an operator, since {a,c} is pre-open set but not open.

In the next, the relationship between T-$\alpha$-operator and T-pre-operator will be given.

***2.5 Theorem:***

Every T-$\alpha$-operator is T-pre-operator.

***Proof:***

Suppose that $(X,\Gamma,T)$ be an $\alpha$-operator topological space
Therefore, $U \subseteq T(U)$ for every $U \in \Gamma^{\alpha}$
Therefore, since every $\alpha$-open set is a pre-open set
Hence, $\alpha$-open set in T(U) is a pre-open set
Hence T is a pre-operator. ■

Now, we will define T-pre-monotone operators.

***2.6 Definition:***

Let $(X,\Gamma)$ be a topological space and T be a pre-operator associated with $\Gamma_{pre}$. T is said to be pre-monotone operator if for every pair of pre-open sets U and V such that $U \subseteq V$, then $T(U) \subseteq T(V)$.

The following example shows that T is a pre-monotone operator.

***2.7 Example:***

Let $X = \{a,b,c\}$, $\Gamma = \{\emptyset,X,\{a\},\{a,b\}\}$, $\Gamma_{pre} = \{\emptyset,X,\{a\},\{a,b\}, \{a,c\}\}$ and let $T : \Gamma_{pre} \longrightarrow P(X)$, be defined as follows:

$T(U) = \text{cl int cl}(U)$

Since for any U, V in $\Gamma_{pre}$, such that $U \subseteq V$, then $T(U) = \text{cl int cl }(U)$, $T(V) = \text{cl int cl}(V)$
Clearly $T(U) = T(V)$
Hence, T is a pre-monotone operator.

Now, the relationship between monotone operators, pre-monotone operators and $\alpha$-monotone operators will be discussed.

***2.8 Theorem:***

Every monotone operator is a pre-monotone operator.

***Proof:***

Suppose that $(X,\Gamma,T)$ be an operator topological space and T be a monotone operator
Let U, V be two open sets, such that $U \subseteq V$
Since T is a monotone operator
Hence, $T(U) \subseteq T(V)$
Since every open set is pre-open set
Therefore, U and V are pre-open sets
Therefore, we have two pre open sets U and V such that $T(U) \subseteq T(V)$
Hence, T is a pre-monotone operator. ■

***2.9 Theorem:***

Every $\alpha$-monotone operator is a pre-monotone operator.

***Proof:***

Let $(X,\Gamma,T)$ be an operator topological space and let T be an $\alpha$-monotone operator
Hence, for every pair of $\alpha$-open sets U and V, such that $U \subseteq V$, then $T(U) \subseteq T(V)$
Since every $\alpha$-open set is pre-open set
Therefore, U and V are pre-open sets, such that $U \subseteq V$ then $T(U) \subseteq T(V)$
Hence, T is a pre-monotone operator. ■
The converse of the above theorem is not true, as the following example shows:

***2.10 Example:***

Let $X = \{a,b,c\}$, $\Gamma = \{\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\}\}$, $\Gamma^{\alpha} = \{\emptyset,X, \{a,b\}\}$, $\Gamma_{pre} = \{\emptyset,X,\{a\},\{b\}, \{a,b\},\{a,c\},\{b,c\}\}$ and $T : P(X) \longrightarrow P(X)$, defined as:

$$T(U) = \begin{cases} \{b\}, & \text{if } b \notin U \\ \text{int}(U), & \text{if } b \in U \end{cases}$$

Then {a},{a,b} are pre-open sets, such that $\{a\} \subseteq \{a,b\}$ and $T(\{a\}) \subseteq T(\{a,b\})$, but {a} is not $\alpha$-open set.
Now, we are in a position to give the definition of T-pre-open sets:

***2.11 Definition:***

Let $(X,\Gamma)$ be a topological space and T be a pre-operator on $\Gamma$. A subset A of X is said to be T-pre-open set if for each $x \in A$, there exist a pre-open set U containing x such that $T(U) \subseteq A$. We denote the set of all T-pre-open sets by $T_{\Gamma_{pre}}$.

A subset B of X is said to be T-pre-closed set if its complement is T-pre-open set.

***2.12 Example:***

Let $X = \{1,2,3\}$, $\Gamma = \{\emptyset,X,\{1\},\{2\},\{1,2\},\{1,3\}\}$, $\Gamma_{pre} = \{\emptyset,X, \{1\},\{2\},\{1,2\},\{1,3\}\}$ and $T : \Gamma_{pre} \longrightarrow P(X)$, be defined as follows:

$$T(U) = \begin{cases} \emptyset, & \text{if } U = \emptyset \\ U, & \text{if } 1 \in U \\ \{1,2\}, & \text{if } 1 \notin U \end{cases}$$

Hence, $T_{\Gamma_{pre}} = \{\emptyset,X,\{1\},\{1,2\}\}$.

In the next theorem, we will give the relationship between T-open sets and T-pre-open sets.

***2.13 Theorem:***

Every T-open set is a T-pre-open set.

***Proof:***

Suppose that $(X,\Gamma,T)$ be an operator topological space and let $A \subseteq X$ be T-open set
Hence for each $x \in A$, there exists an open set U in X containing x, such that $T(U) \subseteq A$
Since every open set is a pre-open set
Therefore, we have U is a pre-open set containing x and $T(U) \subseteq A$
Hence, A is a T-pre-open set. ■

As a consequence from the last theorem, we can give and prove the next corollary:

***2.14 Corollary:***

Every T-closed set is T-pre-closed set

***Proof:***

Suppose that $(X,\Gamma,T)$ is an operator topological space and let F be T-closed set in X
Hence $F^c$ is T-open set in X
Since every T-open set is T-pre-open set
Therefore, $F^c$ is T-pre-open set
Hence, F is T-pre-closed set. ■

The converse of theorem (2.14) is not true in general, as the following example illustrate:

***2.15 Example:***

Let $X = \{a,b,c\}$, $\Gamma = \{\emptyset,X,\{a,b\}\}$, $\Gamma_{pre} = \{\emptyset,X,\{a\},\{b\},\{a,b\}, \{a,c\},\{b,c\}\}$ and $T : \Gamma_{pre} \longrightarrow P(X)$ is an operator defined by:

$$T(U) = \begin{cases} U, & \text{if } a \in U \\ cl(U), & \text{if } a \notin U \end{cases}$$

Let $A = \{a,c\}$, then A is T-pre-open set, but not T-open.

***2.16 Theorem:***

Every T-$\alpha$-open set is T-pre-open set.

***Proof:***

Suppose that $(X,\Gamma,T)$ be an $\alpha$-operator topological space and let A be any T-$\alpha$-open set in X
Hence for each $x \in A$, there exist $\alpha$-open set U in X containing x and $T(U) \subseteq A$
Since every $\alpha$-open set is pre-open set
Then, U is pre-open set in X containing x and $T(U) \subseteq A$
Therefore, A is a T-pre-open set. ■

The converse of the above theorem is not true, as it is seen from the following example:

***2.17 Example:***

$X$ ,$\Gamma$ ,$\Gamma_{pre}$ ,and T is same as In example (2.7), let A = {a}, then A is T-pre-open set but not T-$\alpha$-open set.

Now, we will define a pre-subadditive operators.

***2.18 Definition:***

Let $(X,\Gamma)$ be a topological space, a pre-operator T on $\Gamma_{pre}$ is said to be pre-subadditive if for every collection of pre-open sets $\{U_\beta\}$:

$$T(\bigcup U_\beta) \subseteq \bigcup T(U_\beta)$$

***2.19 Theorem:***

If the operator T is pre-subadditive, then the union of any collection of T-pre-open sets is T-pre-open set.

***Proof:***

Let $\{A_i : i \in \Omega\}$ be a collection of T-pre-open sets, where $\Omega$ is any index set

Let $x \in \bigcup \{A_i : i \in \Omega\}$

Then, $x \in A_i$ for some $i \in \Omega$

Since Ai is T-pre-open set

Hence there exist a pre-open set $U_i$, $x \in U_i$, such that $T(U_i) \subseteq A_i$, and so $\bigcup T(U_i) \subseteq \bigcup \{A_i : i \in \Omega\}$

Since, T is a pre-subadditive, hence:

$$T(\bigcup U_i) \subseteq \bigcup T(U_i)$$

Therefore, we have pre-open set $U_i$ such that $x \in U_i$ and $T(U_i) \subseteq \bigcup \{A_i : i \in \Omega\}$

Hence, $\bigcup \{A_i : i \in \Omega\}$ is a T-pre-open set. ■

***2.20 Remark:***

The intersection of two T-pre-open sets is not necessary T-pre-open set in general.

***2.21 Example:***

$X = \{a,b,c\}$, $\Gamma = \{\varnothing,X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\}\}$, $\Gamma_{pre} = \{\varnothing,X,\{a\}, \{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $T : \Gamma_{pre} \longrightarrow P(X)$, defined as:

$$T(U) = \begin{cases} \operatorname{int} cl(U), & \text{if } a \in U \\ \{b\}, & \text{if } a \notin U \\ \varnothing, & \text{if } U = \varnothing \end{cases}$$

for each $U \in \Gamma_{pre}$. Then:

$$T_{\Gamma_{pre}} = \{\varnothing,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$$

But: $\{a,c\} \cap \{b,c\} = \{c\} \notin T_{\Gamma_{pre}}$

Now, we will define a pre-regular operator.

***2.22 Definition:***

A pre-operator $T : P(X) \longrightarrow P(X)$ is said to be pre-regular if for each point $x \in X$ and for every pair of pre-open sets U and V containing x, there exist a pre-open set W, such that $x \in W$ and $T(W) \subseteq T(U) \cap T(V)$.

***2.23 Proposition:***

If $T : P(X) \longrightarrow P(X)$ is a pre-regular operator, then the intersection of two T-pre-open sets is T-pre-open set.

***Proof:***

Let A and B be T-pre-open sets in a pre-regular operator topological space $(X,\Gamma,T)$

Let $x \in A \cap B$, since A and B are T-pre-open sets

Then there exist pre-open sets U and V such that $x \in U$, $x \in V$ and $T(U) \subseteq A$, $T(V) \subseteq B$. Hence:

$T(U) \cap T(V) \subseteq A \cap B$
Since T is a pre-regular operator
Then, there exist a pre-open set W containing x, such that:
$T(W) \subseteq T(U) \cap T(V)$
Since $T(U) \cap T(V) \subseteq A \cap B$, hence $T(W) \subseteq A \cap B$
Therefore, we have a pre-open set W, such that $x \in W$ and $T(W) \subseteq A \cap B$
Which implies to $A \cap B$ is T-pre-open set. ■

***2.24 Corollary:***

If T is a pre-regular operator, then the collection of all T-pre-open sets $T_{\Gamma_{pre}}$ form a topology on $(X,\Gamma_{pre})$.

In the following, we present the definition of T-pre-regular open set.

***2.25 Definition:***

Let $(X,\Gamma)$ be a topological space and T be a pre-operator on $\Gamma_{pre}$, a subset A of X is said to be T-pre-regular open if $A = int(T_{pre}(A))$.

***2.26 Example:***

Let $X = \{a,b,c\}$, $\Gamma = \{\varnothing,X,\{a\},\{a,b\}\}$, $\Gamma_{pre} = \{\varnothing,X,\{a\},\{a,b\}, \{a,c\}\}$ and
$T : \Gamma_{pre} \longrightarrow P(X)$, defined as:

$$T(U) = \begin{cases} U, & \text{if } a \in U \\ \{c\}, & \text{if } a \notin U \text{ or } U = X \\ \varnothing, & \text{if } U = \varnothing \end{cases}$$

Let $U \subseteq X$
Then, U is T-regular open set if $U = \{a\}, \{a,b\}, \{a,c\}$

***2.27 Theorem:***

Every T-regular open set is T-pre-regular open set.

***Proof:***

Suppose that $(X,\Gamma,T)$ be an operator topological space and let $A \subseteq X$ be T-regular open set
Hence $A = int(T(A))$
Since every open set is pre-open set. Hence, $A = int(T_{pre}(A))$
Therefore, A is T-pre-regular open set. ■

The converse of the above theorem is not true, as it is shown in the next example:

***2.28 Example:***

In example (2.26), if $A = \{a,c\}$, then A is T-pre-regular open set but not T-regular open.

***2.29 Theorem:***

Every T-$\alpha$-regular open set is T-pre-regular open.

***Proof:***

Suppose that$(X,\Gamma,T)$ be an $\alpha$-operator topological space and let
$A \subseteq X$ be T-$\alpha$-regular open set
Hence, $A = int(T^{\alpha}(A))$
Since every $\alpha$-open such that is pre-open set
Hence, $A = int(T_{pre}(A))$
Therefore, A is T-pre-regular open set. ■

***Definition:-***

Let X be a space and Y subspace of X. Then the class of T-open sets in Y is defined in a natural as:

$\Gamma{*}pre = \{Y \cap O : O \in T_{\Gamma_{pre}}\}$

where $T_{\Gamma_{pre}}$ is the class of T-pre-open sets of X. That is U is T-pre-open set in Y if and only if U = Y ∩ O, where O is a T-pre-open set in X.

## III. PRE-Operator of Continuous Functions

***3.1 Definition:[3]***

Let (X,Γ) and (Y,σ) be two topological spaces and T, L be an operators on Γ and σ, respectively. A function f : (X,Γ,T) ⟶ (Y,σ,L) is said to be (T,L) α-continuous if and only if for every L-open set U in Y, $f^{-1}(U)$ is T-α-open set in X.

The following example shows that the function f is (T,L) α-continuous.

***3.2 Example:***

Let X = {1,2,3}, Γ = {∅,X,{1},{1,3}}, $\Gamma^\alpha$ = {∅,X,{1},{1,2}, {1,3}} and
T : P(X) ⟶ P(X) defined as:

$$T(U) = \begin{cases} U, & \text{if } 1 \in U \\ cl(U), & \text{if } 1 \notin U \end{cases}$$

for each U ∈ $\Gamma^\alpha$. Then:

$T_\Gamma$ = {∅,X,{1},{1,3}}

$T_{\Gamma^\alpha}$ = {∅,X,{1},{1,2},{1,3}}

and let Y = {a,b,c}, σ = {∅,Y,{a,b}}, L : P(X) ⟶ P(X) defined as:

$$L(U) = \begin{cases} U, & \text{if } a \in U \\ \operatorname{int} cl(U), & \text{if } a \notin U \end{cases}$$

for each U ∈ σ. Then:

$L_\sigma$ = {∅,Y,{a,b}}

and let f : (X,Γ,T) ⟶ (Y,σ,L) be a function defined as:

f(1) = a, f(2) = b, f(3) = c

Then f is (T,L) α-continuous function.

In the following theorem we present the relationship between (T,L) continuous and (T,L) α-continuous functions.

***3.3 Theorem:***

Every (T,L) continuous function is (T,L) α-continuous function.

Now, we will define (T,L) pre-continuous function.

***3.4 Definition:***

Let (X,Γ) and (Y,σ) be two topological spaces and T and L be an operator on Γ and σ, respectively. A function f : (X,Γ,T) ⟶ (Y,σ,L) is (T,L) pre-continuous if and only if for every L-open set U in Y, $f^{-1}(U)$ is T-pre-open set in X.

The following example shows that the function is (T,L) pre-continuous.

***3.5 Example:***

Let X = {a,b,c,}, Γ = {∅,X,{a,b}} and T : P(X) ⟶ P(X) is defined by:

$$T(U) = \begin{cases} U, & \text{if } a \in U \\ cl(U), & \text{if } a \notin U \end{cases}$$

for each U ∈ Γ. Then:

$T_\Gamma$ = {∅,X,{a,b}}

and let Y ={1,2,3}, σ = {∅,Y,{1},{3},{1,3},{2,3}}, $\sigma_{pre}$ = {∅,Y,{1}, {3},{1,3}} and L : P(X) ⟶ P(X) is defined by:

$$L(U) = \begin{cases} U, & \text{if } 2 \in U \\ cl(U), & \text{if } 2 \notin U \end{cases}$$

for each $U \in \sigma_{pre}$. Then:

$L_\sigma = \{\emptyset, Y, \{1\}, \{2,3\}\}$, $L_{\sigma pre} = \{\emptyset, Y, \{1\}\}$

and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be a function defined as:

$f(a) = 2$, $f(b) = 1$, $f(c) = 3$

Then, f is (T,L) pre-continuous.

The next theorem is explaining the relationship between (T,L)continuous and (T,L)pre-continuous function.

***3.6 Theorem:***

Every (T,L) continuous function is (T,L) pre-continuous function.

***Proof:***

Suppose that $(X,\Gamma,T)$ and $(Y,\sigma,L)$ be two operators topological spaces and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be (T,L) continuous function.

Let U be L-open set in Y

Since f is (T,L) continuous

Hence, $f^{-1}(U)$ is T-open set in X

Since every T-open set is T-pre-open set in X.

Hence, $f^{-1}(U)$ is T-pre-open set in X

Hence f is (T,L)pre-continuous function.

The converse of the above theorem is not true in general, as the following example show:

***3.7 Example:***

Let X, $\Gamma$, $T_\Gamma$, $L_\sigma$, $L_{\sigma_{pre}}$ and f be the same as in example (3.5) and:

$f(a) = 2$, $f(b) = 1$, $f(c) = 3$

$f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ is (T,L) pre-continuous function, but not (T,L) continuous function.

***3.8 Theorem:***

Every (T,L) $\alpha$-continuous function is (T,L) pre-continuous function.

***Proof:***

Suppose that $(X,\Gamma,T)$ and $(Y,\sigma,L)$ be two operators topological spaces and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be (T,L) $\alpha$-continuous function.

Let U be L-open set in Y

Since f is (T,L) $\alpha$-continuous

Hence, $f^{-1}(U)$ is T-$\alpha$-open set in X

Since every T-$\alpha$-open set is T-pre-open set

Hence, $f^{-1}(U)$ is T-pre-open set in X, for each L-open set U in Y

Hence, f is (T,L) pre-continuous function. ■

The converse of theorem (3.8) is not true in general, as the following example illustrate:

***3.9 Example:***

Let X and $\Gamma$ be the same as in example (3.5), let $\Gamma^\alpha = \{\emptyset, X, \{a,b\}\}$ and

$T_{\Gamma^\alpha} = \{\emptyset, X, \{a,b\}\}$. Then the function $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ defined by :

$f(a) = 2$, $f(b) = 1$, $f(c) = 3$

is (T,L) pre-continuous function, but not (T,L) $\alpha$-continuous.

Now, we will define (T,L) pre-irresolute continuous function.

***3.10 Definition:***

Let $(X,\Gamma)$ and $(Y,\sigma)$ be two topological space and T, L be operators on $\Gamma$ and $\sigma$, respectively. A function $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ is said to be (T,L) pre-irresolute continuous if and only if for every L-pre-open set U in Y, $f^{-1}(U)$ is T-pre-open set in X.

The following example on (T,L)-pre-irresolute continuous function.

***3.11 Example:***

Let $X = \{a,b,c\}$, $\Gamma = \{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\}\}$, $\Gamma_{pre}= \{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and defined $T : P(X) \longrightarrow P(X)$ by:

$$T(U) = \begin{cases} \operatorname{int} cl(U) & \text{if } \ a \in U \\ \{b\} & \text{if } \ a \notin U \end{cases}$$

for each $U \in \Gamma_{pre}$. Then:

$T_{\Gamma\text{-}pre} = \{\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$

and let $Y =\{1,2,3\}$, $\sigma = \{\emptyset,Y,\{1\},\{2\},\{3\},\{1,2\},\{1,3\}\}$, $\sigma_{pre}=\{\emptyset,Y,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$ and $L : P(X) \longrightarrow P(X)$ is defined by:

$$L(U) = \begin{cases} \{1,3\}, & \text{if } \ 2 \in U \\ cl\operatorname{int}(U), & \text{if } \ 2 \notin U \end{cases}$$

for each $U \in \sigma_{pre}$. Then:

$L_{\sigma\text{-}pre} = \{\emptyset,Y,\{1\},\{3\},\{1,3\}\}$

and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be a function defied as:

$f(a) = 1$, $f(b) = 3$, $f(c) = 2$

Then, f is (T,L) pre-irresolute continuous function.

Now, we will introduce the relationship between (T,L) pre-irresolute continuous and (T,L) pre-continuous functions.

***3.12 Theorem:***

Every (T,L) pre-irresolute continuous function is (T,L) pre-continuous function.

***Proof:***

Suppose that $(X,\Gamma,T)$ and $(Y,\sigma,L)$ be two operators topological spaces and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be (T,L) pre-irresolute continuous function.

Let U be L-open set in Y

Since every T-open set is T-pre-open set

Hence U is L-pre-open set in Y

Since f is (T,L) pre-irresolute continuous function

Hence, $f^{-1}(U)$ is T-pre-open set in X

Hence, f is (T,L) pre-continuous function. ■

The converse of the above theorem is not true, as the following example shows:

***3.13 Example:***

$X$, $\Gamma$, $\Gamma_{pre}$, $T_{\Gamma\text{-}pre}$, $\sigma_{pre}$ and $L_{\sigma\text{-}pre}$ be the same as in example(3.2),

the function $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ is defined by:

$f(1) = a$, $f(2) = b$, $f(3) = c$

is (T,L) pre-continuous function, but not (T,L)-pre irresolute continuous function.

***3.14 Definition:***

Let $(X,\Gamma)$ and $(Y,\sigma)$ be two topological spaces and T, L be operators on $\Gamma$ and $\sigma$, respectively. A function $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ is said to be (T,L) strongly pre-continuous if and only if for every L-pre-open set U in Y, $f^{-1}(U)$ is T-open set in X.

The following example shows that the function f is (T,L) strongly pre-continuous.

***3.15 Example:***

Let $X = \{1,2,3\}$, $\Gamma = \{\emptyset,X,\{1\},\{1,3\}\}$ and $T : P(X) \longrightarrow P(X)$ defined as:

$$T(U) = \begin{cases} U, & \text{if } 1 \in U \\ \{3\}, & \text{if } 1 \notin U \text{ or } U = X \end{cases}$$

for each $U \in \Gamma$. Then:

$T_\Gamma = \{\emptyset,X,\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$

and let $Y = \{a,b,c\}$, $\sigma = \{\emptyset,Y,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$, $\sigma_{pre} = \{\emptyset,Y,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and $L : P(Y) \longrightarrow P(Y)$ defined as:

$$L(U) = \begin{cases} \operatorname{int} cl(U), & \text{if } a \in U \\ \{b\}, & \text{if } a \notin U \\ \emptyset, & \text{if } U = \emptyset \end{cases}$$

for each $U \in \sigma_{pre}$. Then:

$L_{\sigma\text{-pre}} = \{\emptyset,Y,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$

and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be a function defined as:

$f(1) = a$, $f(2) = c$, $f(3) = b$

Then f is (T,L) strongly pre-continuous function.

The following theorem give us the relationship between (T,L) strongly pre-continuous and (T,L) pre-irresolute continuous functions.

***3.16 Theorem:***

Every (T,L) strongly pre-continuous function is (T,L) pre-irresolute continuous function.

***Proof:***

Suppose that $(X,\Gamma,T)$ and $(Y,\sigma,L)$ be two operators topological spaces and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be (T,L) strongly pre-continuous function.
Let U be L-pre-open set in Y
Since f is (T,L) strongly pre-continuous function
Hence, $f^{-1}(U)$ is T-open set in X
Since every T-open set is T-pre-open set
Hence, $f^{-1}(U)$ is T-pre-open set in X
Therefore, we have $f^{-1}(U)$ is T-pre-open set in X for every L-pre-open set U in Y
Hence, f is (T,L) pre-irresolute continuous function. ■

***3.17 Corollary:***

Every (T,L) strongly pre-continuous function is (T,L) pre-continuous function.

***Proof:***

Suppose that $(X,\Gamma,T)$ and $(Y,\sigma,L)$ be two operators topological spaces and let $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ be (T,L) strongly pre-continuous function.
Since every (T,L) strongly pre-continuous function is (T,L) pre-irresolute continuous function (by theorem (3.16))
Then, f is (T,L) pre-irresolute continuous function and since every (T,L) pre-irresolute continuous function is (T,L) pre-continuous function (by theorem (3.12))
Hence f is (T,L) pre-continuous function. ■

The converse of corollary (3.17) is not true in general, as shown by the following example:

***3.18 Example:***

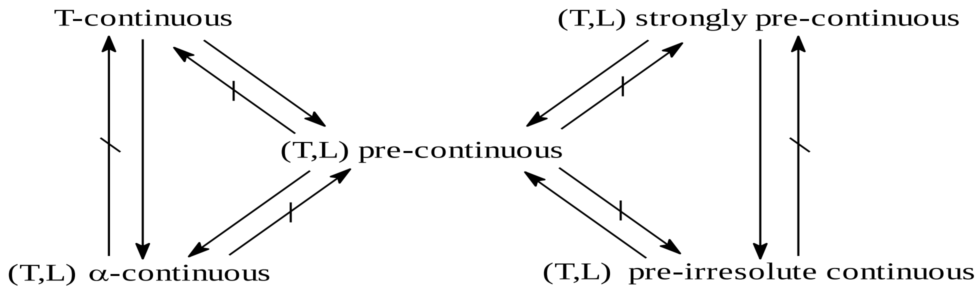
In example (3.5) if $\Gamma_{pre} = \{\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$ and $T_{\Gamma\text{-pre}} = \{\emptyset, X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$ and the function $f : (X,\Gamma,T) \longrightarrow (Y,\sigma,L)$ is defined as:

$f(a) = 2$, $f(b) = 1$, $f(c) = 3$

Then f is (T,L) pre-continuous function, but not (T,L) strongly pre-continuous.

***3.19 Remark:***

From the last theorems and examples, we have the following diagram:




## References


[1] S. Kasahara, Operation- compact spaces,*Math. Japonica,* **24** (1979), 97.

[2] A.S. Mashhour, M.E. Abd EI-Monsef and S.N.EI-Deep, "On Precontinuous and Weak Continuous Mappings" ,*Proc., Math., Phys., Soc., Egypt*,**53**(1982),47-53.

[3] N.G.Mansur, A.M.Ibrahim "T-α -operator" ,Journal of college of education, No.3(2006), p 118-124.

[4] H. Ogata,"Operation on topological spaces and associated topology", Math. Japonica, 36 (1991), 175-184.

[5] E. Rosas, J. Vielma, "Operator decomposition of continuous mappings", Divu'aciones Matem´aticas, 7, No. 1 (1999), 29-33.