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$(\tau_i, \tau_j)-$ RGB Closed Sets in Bitopological Spaces

Bushra Jaralla Tawfeeq, Dunya Mohamed Hammed
Mathematic Department,College of Education ,Al – Mustansirya University

Abstract: *In this paper we introduce and study the concept of a new class of closed sets called $(\tau_i, \tau_j)-$ regular generalized b- closed sets (briefly$(\tau_i, \tau_j)-$ rgb-closed) in bitopological spaces.Further we define and study new neighborhood namely $(\tau_i, \tau_j)-$ rgb- neighbourhood (briefly$(\tau_i, \tau_j)-$ rgb-nhd) and discuss some of their properties in bitopological spaces. Also, we give some characterizations and applications of it.*

I. Introduction

In 1963, Kelley J. C. [16] was first introduced the concept of bitopological spaces , where X is a nonempty set and τ_i, τ_j are two topologies on X.1970 ,M.K.Signal[28] introduced some more separation axioms theses consider with bitopological spaces.1977,V.Popo.[26]introduced some properties of bitopological semi separation spaces.

In (1985), Fukutake [7] introduced and the studied the notions of generalized closed (g-closed) sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Sundaram, P. and Shiek John[29], El- Tantawy and Abu-Donia [6]introduced the concept of *w*-closed sets and generalized semi-closed (gs-closed) sets in bitopological spaces respectively.

Sheik John and Sundaram (2004),[27] introduced g*- closed sets in bitopological spaces in 2004. Jafara,S.,M.Lellis Thivagar and S.Athisaya Ponmani ,(2007)[11] studied some new separation axioms using the (1,2)α-open sets in bitopological spaces.In 2007,[2] S.S.Benchalli and R.S.Wail introduced new class of closed sets called regular-weakly –closed in bitopological spaces.In (2013),[23],K.Mariappa and S.Seker introduced and the studied the notions of regular generalized b- closed sets in topological spaces.

In §2 we recollect the basic definitions which are used in this paper.

In §3 we find basic properties and characteristics of $(\tau_i, \tau_j)-$ rgb closed sets ,also we provide several properties of above concept and to investigate its relationships with certain types of closed sets with some new results and examples.

In §4We provide several properties of characterizations of $(\tau_i, \tau_j)-$rgb-closed sets $(\tau_i, \tau_j)-$ rgb-open sets and$(\tau_i, \tau_j)-$ rgb –nhd of a point as well as some propositions and examples that are included throughout the section.

II. Introduction And Preliminaries

If A is a subset of a topological space X with a topology τ, with then the closure of A is denoted by τ -cl(A) or cl(A), the interior of A is denoted by τ -int(A) or int(A),semi-closure (resp. pre-closure) of A is denoted by τ -scl(A) or scl(A) (resp. τ -pcl(A) or pcl(A)), semi-interior of A is denoted by $\tau -$ sint(A)or sint(A) and the complement of A is denoted by A^c.

Before entering into our work we recall the following definitions:

Definition 2.1. A subset A of a topological space (X, τ) is called:
1) an α-open set[18]if A ⊆ int(cl(int(A))).

2) a semi-open set [12] if A ⊆ cl(int(A)).

3) a pre-open set [13]if A⊆ int(cl(A)).

4) a semi –pre-open set (β-open set)[5] if A ⊆ cl(int(cl(A))).

5) a regular open set [9] if A = I nt(cl(A)).

6)a b-open set [1]if A⊆ int(cl(A)) ∪ cl(int(A))..

The semi closure [4](resp α -closure [20]) of a subset A of X denoted by scl(A)(α cl(A)) is defned to be the intersection of all semi-closed (α -closed) sets containingA. The semi interior [4] of A denoted by sint(A) is defned to be the union of all semi-open sets contained in A. If A⊆B⊆X then Cl_B (A) and Int_B (A) denote the closure of A relative to B and interior of A relative to B.

Definition 2.2 Let (X,τ) a topological space and A be a subset of X,then A is called
1) a generalized closed set [18](abbreviated g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
2) a generalized α-closed set [21](abbreviated gα-closed) if αcl(A) ⊆ U whenever A ⊆ Uand U is α-open in X.
3) α- generalized closed set [21](abbreviated αg-closed) if αcl(A) ⊆ U whenever A ⊆ Uand U is open in X.
4) a generalized b-closed set [22](abbreviated gb-closed) if bcl(A) ⊆ U whenever A ⊆ Uand U is open in X.
5) semi- generalized closed set [5](abbreviated sg-closed) if scl(A) ⊆ U whenever A ⊆Uand U is semi-open in X.
6) a generalized semi-closed set [5](abbreviated gs-closed) if scl(A) ⊆ U whenever A ⊆ Uand U is open in X.
7) *w*-closed set [24] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
8) a weakly generalized closed set [25](abbreviated wg-closed) if cl(int(A)) ⊆ U whenever A ⊆ Uand U is open in X.
9) a semi- generalized b- closed set [10](abbreviated sgb-closed) if bcl(A) ⊆ U whenever A ⊆Uand U is semi-open in X.
11)a strongly generalized closed set [27] (abbreviated g^*-closed) if cl(A) ⊆ U whenever A ⊆U and U is g-open in X.
12) a generalized gαb-closed set [30](abbreviated gαb closed) if bcl(A) ⊆ U whenever A ⊆ U and U is α -open in X.
13) a regular generalized b-closed set [23](abbreviated rgb- closed) if bcl(A) ⊆ U whenever A ⊆ U and U is regular -open in X.

The complements of the above mentioned sets are called their respective open sets.
Definition 2.3. A subset A of a bitopological space (X, τ_i, τ_j) is called a
1. (τ_i, τ_j) -pre- open [12] if A⊆ τ_i -int[τ_j-cl (A)]
2.(τ_i, τ_j)-semi open [20] if A ⊆ τ_j- cl[τ_i- int (A)]
3. (τ_i, τ_j)- α- open [13] if A ⊆ τ_i-int[τ_j- cl[τ_i-int (A)]]
4. (τ_i, τ_j)-regular open [3] if A =τ_i -int[τ_j-cl(A)]

Definition 2.4. A subset A of a bitopological space (X, τ_i, τ_j) is called a
1.$\tau_i, \tau_j)-$ g-closed [7] if τ_j − cl(A) ⊆ Uwhenever A ⊆ U and U ∈ τ_i .
2.$(\tau_i, \tau_j)-$ gs-closed [6] if τ_j− scl(A) ⊆ Uwhenever A ⊆ U and U ∈ τ_i.
3. $(\tau_i, \tau_j)-$ weakly generalized closed[6]((τ_i, τ_j)−wg-closed) sets if τ_j −cl(τ_i−int(A)) ⊆U whenever A ⊆ U and U is τ_i − open in X.
4.(τ_i, τ_j) w-closed [8] if τ_j_cl(A) ⊆ U whenever A ⊆ U and U is semi-open in τ_i.
5.$(\tau_i, \tau_j)-$ g *-closed[27] if τ_j− cl(A) ⊆Uwhenever A ⊆ U and U is τ_i − g-open set.
6. $(\tau_i, \tau_j)-\alpha$g-closed [17]if τ_j−αcl(A) ⊆ U whenever A ⊆ Uand U is τ_i − open in X.
7. $(\tau_i, \tau_j)-$ gα-closed [17]if τ_j αcl(A) ⊆ U whenever A ⊆ Uand U is τ_i − α-open in X.
8. $\tau_i, \tau_j)-$ g *p-closed[30] if τ_j− pcl(A) ⊆Uwhenever A ⊆ U and U is τ_i − g -open set .
9. $(\tau_i, \tau_j)-$ rg-closed[3] if τ_j−cl(A) ⊆Uwhenever A ⊆ U and U is τ_i − regular open set .
10. $(\tau_i, \tau_j)-$ rg^{**}-closed[14] if τ_j−cl[τ_i -int(A)] ⊆ Uwhenever A ⊆ U and U is $(\tau_i, \tau_j)-$ regular open set .
11. $(\tau_i, \tau_j)-$ rw-closed[15] if τ_j−cl(A) ⊆Uwhenever A ⊆ U and U is τ_i − regular semi -open set.
12. $(\tau_i ,\tau_j)-$ regular weakly generalized closed[8].((τ_i,τ_j)−rwg-closed)if τ_j−cl[τ_i-int(A)] ⊆Uwhenever A ⊆ U and U is τ_i − regular open set.

III. $(\tau_i, \tau_j)-$ RGB Closed Sets In Bitopological Spaces

In this section we introduce $(\tau_i, \tau_j)-$ rgb-closed sets in bitopological spaces and study some of their properties.

Definition 3.1. Let i, j ∈{1, 2} be fixed integers. A subset A of a bitopological space (X, τ_i, τ_j) is said to be (τ_i, τ_j) – rgb closed(briefly (τ_i, τ_j)-rgb-closed) set if τ_j −bcl(A) ⊆U whenever A ⊆U and U is regular -open in (X, τ_i).
The family of all $(\tau_i, \tau_j)-$ rgb closed sets in a bitopological space (X, τ_i, τ_j) is denoted by D^*RGB (τ_i, τ_j)
Remark 3.2: By setting $\tau_1 = \tau_2$ in Definition 3.1, $(\tau_i, \tau_j)-$ rgb-closed set is a rgb- closed set

Proposition 3.3: If A is τ_j-closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j) –rgb - closed set.
Proof. Let A be any τ_j-closed set and U be any $\tau_i -$ regular -open set containing A.Sinceτ_j $-$bcl(A) ⊆ τ_j $-$ cl(A) ⊆U,then τ_j $-$bcl(A) ⊆U. Hence A is $(\tau_i, \tau_j)-$ rgb-closed.

The converse of the above proposition need not be true as seen from the following example.
Example 3.4: Let X = {a, b, c} and τ_i= {X, φ, {a,b} , {b} } and τ_j = {X, φ ,{a}} ,the set {b} is$(\tau_i, \tau_j)-$ rgb-closed but not τ_j-closed set.

Proposition 3.5: If A is (τ_i, τ_j)-b-closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j) –rgb - closed set.
Proof. Let A be any (τ_i, τ_j)-b-closed set in (X, τ_i, τ_j)such that A⊆U ,where U is τ_i $-$regular open set .Since A is (τ_i, τ_j)-b-closed which implise that τ_j $-$bcl(A) ⊆ τ_j $-$cl(A) ⊆ U,then τ_j $-$bcl(A) ⊆U. Hence A is $(\tau_i, \tau_j)-$ rgb-closed.

The converse of the above proposition need not be true in general, as seen from the following example.
Example 3.6: Let X = {a, b, c} and τ_i= {X, φ,{a},{a,c}} and τ_j = {X, φ ,{a,c}} .Then the set {a,c} is $(\tau_i, \tau_j)-$ rgb-closed but not (τ_i , τ_j)-b-closed .

Proposition 3.7: If A is τ_j- α -closed (resp. τ_j- semi-closed) subset of (X, τ_i, τ_j) then A is (τ_i, τ_j)- rgb-closed.
Proof . Let A be any $\tau_j - \alpha$ -closed set in(X, τ_i, τ_j)such that A⊆U ,where U is τ_i $-$regular open set. Since A is τ_j- α -closed set ,then τ_j $-$bcl(A) ⊆ τ_j $-$ αcl(A) ⊆ τ_j $-$cl(A) ⊆ U,so τ_j $-$bcl(A) ⊆U. Therefore A is (τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.8: Let X = {a, b, c} and τ_i= {X, φ,{a,}{b},{c},{a,b},{a,c},{b,c}} and τ_j = {X, φ,{b}, {b, c} } ,the set {c} is$(\tau_i, \tau_j)-$ rgb-closed but not, τ_j- α -closed .

Remark 3.9: The concept of (τ_i,τ_j)- α -closed sets and $(\tau_i, \tau_j)-$ rgb-closed sets are independent of each other as seen from the following examples.

Example 3.10: Let X = {a, b, c} and τ_i= {X, φ,{c}} and τ_j = {X, φ ,{b},{c},{b,c}} ,the set {b,c} is $(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- α -closed .

Example 3.11: Let X = {a, b, c} and τ_i= {X, φ,{a,}{b},{c},{a,b},{a,c},{b,c}} and τ_j = {X, φ, {a} ,{a,c}} ,the set {a,b} is(τ_i,τ_j)- α -closed sets but not(τ_i,τ_j)- rgb-closed set.

Remark 3.12: The concept of (τ_i,τ_j)- semi-closed sets and $(\tau_i, \tau_j)-$ rgb-closed sets are independent of each other as seen from the following examples.

Example 3.13: Let X = {a, b, c} and τ_i= {X, φ,{ b},{b,c }} and τ_j = {X, φ,{a}} .Then the set {c} is $(\tau_i, \tau_j)-$ rgb-closed but not$(\tau_i, \tau_j)-$semi-closed set.

Example 3.14: Let X = {a, b, c} and τ_i= {X, φ,{a,}{b},{c},{a,b},{a,c},{b,c}} and τ_j = {X, φ ,{b},{b,c}} ,the set { b} is(τ_i,τ_j)- semi -closed sets but not(τ_i,τ_j)- rgb-closed set.

Remark 3.15: (τ_i,τ_j)- pre-closed sets and $(\tau_i, \tau_j)-$ rgb-closed sets are independent of each other as seen from the following two examples.

Example 3.16: Let X = {a, b, c} and τ_i= {X, φ,{a}{b},{a,b}} and τ_j = {X, φ ,{a},{b,c}} ,the set {a,b} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)-pre- closed.

Example 3.17: Let X, τ_i and τ_j be as in Example 3.14.The set {b,c} is$(\tau_i, \tau_j)-$ pre-closed but not $(\tau_i, \tau_j)-$ rgb-closed.

Remark 3.18: (τ_i,τ_j)- semi –pre-closed sets (β-closed sets) and $(\tau_i, \tau_j)-$ rgb-closed sets are independent of each other as seen from the following two examples.

Example 3.19: Let X = {a, b, c} and τ_i= {X, φ,{a},{b},{c},{a,b},{a,c},{b,c}} and τ_j = {X, φ , {a},{a,b}} the set {a} is$(\tau_i, \tau_j)-$ β -closed but not(τ_i, τ_j)- rgb - closed .

Example 3.20: Let X = {a, b, c} and τ_i= {X, φ,{b,c}} and τ_j = {X, φ ,{b},{c},{b,c}} ,the set {a,c} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- β - closed.

Remark 3.21: The concept of (τ_i,τ_j)- rg^{**}-closed sets and $(\tau_i, \tau_j)-$ rgb-closed sets are independent of each other as seen from the following example.

Example 3.22: Let X = {a, b, c} and τ_i= {X, φ,{a,}{b},{a,b},{a,c}} and τ_j = {X, φ ,{a,b},{b}} . Then the set {a} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- rg^{**}- closed and{b} is$(\tau_i, \tau_j)-$ rg^{**}- closed but not (τ_i, τ_j)- rgb-closed.

Proposition 3.23: If A is $(\tau_i, \tau_j)-$ g-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$rgb-closed.
Proof . Suppose that A is $(\tau_i, \tau_j)-$ g-closed set U be any $\tau_i -$ regular -open set such that A⊆U. Since A is (τ_i, τ_j)-g –closed, then $\tau_j -$cl(A) ⊆ U, we have $\tau_j -$bcl(A) ⊆ $\tau_j -$cl(A) ⊆ U.Hence A is ((τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.24: Let X = {a, b, c} and τ_i= {X, φ,{a},{b,c}} and τ_j = {X, φ,{a},{b},{a,b}} ,the set {a}is $(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)-g-closed .

Proposition 3.25: If A is $(\tau_i, \tau_j)-$ g*-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$rgb-closed.
Proof . Let A be any $(\tau_i, \tau_j)-$ g*-closed set and U be any $\tau_i -$ regular -open set containing A . Since A is τ_j-g ∗-closed set and$\tau_j -$cl(A) ⊆ U, $\tau_j -$bcl(A) ⊆ $\tau_j -$cl(A) ⊆ U, so $\tau_j -$bcl(A) ⊆U. Therefore A is (τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.26: Let X = {a, b, c} and τ_i= {X, φ,{b},{c},{b,c}} and τ_j = {X, φ,{b,c}} ,the set {c} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- g ∗ -closed .
Proposition 3.27 : If A is $(\tau_i, \tau_j)-$ g *p -closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$gbr-closed.
Proof . Assume A is (τ_i, τ_j)- g *p -closed ,A⊆ U and U is $\tau_i -$ regular -open set. Since A is $(\tau_i, \tau_j)-$ gp* closed set , we have$\tau_j -$pcl(A) ⊆ Uand $\tau_j -$pcl(A) ⊆ $\tau_j -$bcl(A) ⊆ U , $\tau_j -$bcl(A) ⊆ U.Therefore A is (τ_i, τ_j)-gbr-closed.

The following example show that the converse of the above proposition is not true :
Example 3.28: Let X = {a, b, c} and τ_i= {X, φ,{a},{b},{a,b}} and τ_j = {X, φ,{a}} ,the set {a,c} is(τ_i, τ_j)- rgb-closed but not(τ_i, τ_j)- gp*closed .

Proposition 3.29: If A is$(\tau_i, \tau_j)-$gb-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$ rgb-closed .
Proof . Let A be any $(\tau_i, \tau_j)-$gb-closed set in(X, τ_i, τ_j)such that A⊆U ,where U is $\tau_i -$ regular -open set. Since A is $(\tau_i, \tau_j)-$gb-closed set,which implise that $\tau_j -$bcl(A) ⊆U. Therefore A is (τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.30 Let X = {a, b, c} and τ_i= {X, φ,{a,}{b},{a,b},{a,c}} and τ_j = {X, φ ,{a,b},{b}} ,the set {a,b}is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- gb-closed .

Proposition 3.31: If A is$(\tau_i, \tau_j)-$rw-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$ rgb-closed .
Proof . Let A be any $(\tau_i, \tau_j)-$rw-closed set in(X, τ_i, τ_j) and U be any τ_i- regular open set containing A. Since A is $(\tau_i, \tau_j)-$rw-closed set,then $\tau_i -$cl(A) ⊆U and $\tau_i -$bcl(A) ⊆ $\tau_i -$cl(A) ⊆U. Hence A is $(\tau_i, \tau_j)-$ rgb -closed.

The converse of the above proposition need not be true in general, as seen from the following example.
Example 3.32: Let X = {a, b, c} and τ_i= {X, φ,{a},{b},{a,b}} and τ_j = {X, φ,{a}} ,the set {b} is(τ_i, τ_j)- rgb-closed but not(τ_i, τ_j)- rw-closed .

Proposition 3.33: If A is $(\tau_i, \tau_j)- \alpha g -$closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j)–rgb-closed.
Proof . Let A be any $(\tau_i, \tau_j)- \alpha g$-closed set and U be any τ_i- regular open set containing A . Since A is $(\tau_i, \tau_j)- \alpha g$ -closed set, then $\tau_j -$bcl(A) ⊆ $\tau_j - \alpha$cl(A) ⊆ U. Therefore $\tau_j -$bcl(A) ⊆U. Hence A is (τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.34: Let X = {a, b, c} and τ_i= {X, φ,{b,c}} and τ_j = {X, φ,{b}} ,the set {b,c} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- $\alpha g -$closed .

Similarly, we prove the following Proposition**:**

Proposition 3.35: If A is $(\tau_i, \tau_j)- g\alpha -$closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j)–rgb-closed but not conversely.

Example 3.36: Let X = {a, b, c} and τ_i= {X, φ,{a,b},{c}} and τ_j = {X, φ,{a,b}} ,the set {a} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- $\alpha g -$closed .

Proposition 3.37: If A is$(\tau_i, \tau_j)-$ gαb -closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$ rgb -closed .
Proof . Let A be any $(\tau_i, \tau_j)-$ gαb -closed set in(X, τ_i, τ_j) such that A⊆U ,where U is $\tau_i -$regular open set. Since A is $(\tau_i, \tau_j)-$ gαb -closed set, $\tau_i -$bcl(A) ⊆ U. Hence A is $(\tau_i, \tau_j)-$ rgb -closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.38: Let X = {a, b, c} and τ_i= {X, φ,{a},{a,c}} and τ_j = {X, φ,{a,b}} ,the set {a,b} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- gαb − closed .

Proposition 3.39: If A is $(\tau_i, \tau_j)- gs -$closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j)–rgb-closed.
Proof . Let A be any $(\tau_i, \tau_j)- gs$-closed set and U be any $\tau_i -$ regular -open set containing A . Since A is $(\tau_i, \tau_j)- gs$-closed set,then$\tau_j - s$cl(A) ⊆ U,so $\tau_j -$bcl(A) ⊆ $\tau_j - s$cl(A) ⊆ U . Therefore A is (τ_i, τ_j)- rgb-closed.

The following example show that the converse of the above proposition is not true :

Example 3.40: Let X = {a, b, c} and τ_i= {X, φ,{a},{b},{a,b},{a,c}} and τ_j = {X, φ,{a},{b},{a,b}} ,the set {a,b} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- gs − closed .

Similarly, we prove the following Proposition**:**

Proposition 3.41: If A is $(\tau_i, \tau_j)-$ sg-closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j)–rgb-closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.42: Let X = {a, b, c} and τ_i= {X, φ,{a,}{b},{c},{a,b},{a,c},{b,c}} and τ_j = {X, φ ,{c},{a,c}} ,the set { a} is(τ_i,τ_j)- rgb -closed sets but not(τ_i,τ_j)- sg-closed set.

Proposition 3.43: If A is $(\tau_i, \tau_j)-$ rg- closed subset of (X, τ_i, τ_j) then A is (τ_i, τ_j)–rgb-closed.
Proof . Let A be any $(\tau_i, \tau_j) -$ rg-closed set and U be any $\tau_i -$ regular -open set containing A . Since A is $(\tau_i, \tau_j)-$rg-closed set, then$\tau_j -$cl(A) ⊆ U,so $\tau_j -$bcl(A) ⊆ $\tau_j -$cl(A) ⊆ U . Therefore A is (τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.44: Let X = {a, b, c} and τ_i= {X, φ,{b},{a,c}} and τ_j = {X, φ,{a},{b},{a,b}} ,the set {b} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- rg-closed .

Proposition 3.45: If A is$(\tau_i, \tau_j)-$ sgb -closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$ rgb -closed .
Proof . Let A be any $(\tau_i, \tau_j)-$ sgb -closed set in(X, τ_i, τ_j) such that A⊆U ,where U is τ_i −regular open set. Since A is $(\tau_i, \tau_j)-$ sgb -closed set, τ_i −bcl(A) ⊆ U. Hence A is $(\tau_i, \tau_j)-$ rgb -closed.

The following example show that the converse of the above proposition is not true :

Example 3.46: Let X = {a, b, c} and τ_i= {X, φ,{b},{a,b}} and τ_j = {X, φ,{c}} ,the set {a,c} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)-sgb -closed .

Proposition 3.47: If A is $(\tau_i, \tau_j)-$ w-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$rgb-closed.
Proof . Let A be any $(\tau_i, \tau_j)-$ w-closed set and U be any τ_i − regular -open set containing A . Since A is $(\tau_i, \tau_j)-$ w-closed set, then τ_j −cl(A) ⊆ U,so τ_j −bcl(A) ⊆ τ_j −cl(A) ⊆ U . Therefore A is (τ_i, τ_j)- rgb-closed.

The converse of the above proposition need not be true as seen from the following example.
Example 3.48: Let X = {a, b, c} and τ_i= {X, φ,{a},{b},{a,b},{a,c}} and τ_j = {X, φ,{a},{b,c}} ,the set {a,b} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- w − closed .

Similarly, we prove the following Proposition
Proposition 3.49: If A is $(\tau_i, \tau_j)-$ wg-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.50: Let X = {a, b, c} and τ_i= {X, φ,{b}} and τ_j = {X, φ,{b},{c},{b,c}} ,the set {a} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- wg- closed .

Proposition 3.51: If A is $(\tau_i, \tau_j)-$ rwg-closed subset of (X, τ_i, τ_j) then A is $(\tau_i, \tau_j)-$rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.52: Let X = {a, b, c} and τ_i= {X, φ,{a},{b},{c},{a,b},{a,c}} and τ_j = {X, φ , {a},{a,b}} the set {b} is$(\tau_i, \tau_j)-$ rgb-closed but not(τ_i, τ_j)- rwg- closed .

IV. Characterizations And Properties Of $(\tau_i, \tau_j)-$ RGB-Closed Sets, $(\tau_i, \tau_j)-$ RGB -Open Sets And $(\tau_i,\tau_j)-$ RGB - Neighborhoods

In this section we introduce some characterizations of(τ_i, τ_j) − rgb -closed sets and (τ_i, τ_j) − rgb -open sets, also we define and study new neighborhood namely$(\tau_i, \tau_j)-$ rgb- neighborhood (briefly $(\tau_i, \tau_j)-$ rgb-nhd) and discuss some of their properties.

Definition 4.1. A subset A of bitopological space (X, τ_i, τ_j) is called $(\tau_i, \tau_j)-$ rgb -open set if and only if its complement is $(\tau_i, \tau_j)-$ rgb -closed in X.

The family of all (τ_i, τ_j)- rgb- open subsets of X is denoted by D^* RGBO (τ_i, τ_j)

Remark 4.2 Let A and B be two $(\tau_i, \tau_j)-$ rgb - closed sets in (X, τ_i, τ_j)
1)The union A ∪ B is not generally$(\tau_i, \tau_j)-$ rgb – closed set .
2) The intersection A ∩ B is not generally$(\tau_i, \tau_j)-$ rgb -closed set as seen from the following examples.

Example 4.3. Let X = {a, b, c} and τ_i= {X, φ,{a,b},{c}} and τ_j = {X, φ,{a,b}} , the subsets{a} ,{b}is$(\tau_i, \tau_j)-$ rgb - closed sets but their union{a}∪{b}={a,b} is not $(\tau_i, \tau_j)-$ rgb - closed set.

Example 4.4. Let X = {a, b, c} and τ_i= {X, φ,{b},{c},{b,c}} and τ_j = {X, φ,{b}} , the subsets{a,c} ,{b,c}are $(\tau_i, \tau_j)-$ rgb -closed sets but their intersection{a,c}∩{ b,c}={b} is not $(\tau_i, \tau_j)-$ rgb -closed set.

Remark 4.5 Let A and B be two $(\tau_i, \tau_j)-$ rgb - open sets in (X, τ_i, τ_j)
1)The union A ∪ B is not generally$(\tau_i, \tau_j)-$ rgb – open set .
2) The intersection A ∩ B is not generally$(\tau_i, \tau_j)-$ rgb - open set as seen from the following examples.
Example 4.6. Let X={a, b, c} and τ_i= {X, φ,{b},{c},{b,c}} and τ_j = {X, φ,{b}},the subsets{a},{c}is$(\tau_i, \tau_j)-$ rgb -open sets but their union {a}∪{c}={a,c} is not $(\tau_i, \tau_j)-$ rgb - open set.

Example 4.7. Let X = {a, b, c} and τ_i= {X, φ,{a,b},{c}} and τ_j = {X, φ,{a,b}},the subsets{a,c},{b,c}is$(\tau_i, \tau_j)-$ rgb - open sets but their intersection {a,c}∪{b,c}={c} is not $(\tau_i, \tau_j)-$ rgb - open set.

Proposition 4.8: If a set G is (τ_i, τ_j)-rgb-closed set in (X, τ_i, τ_j), then τ_j–cl(A)contains no non-empty τ_i-regular -closed set.
Proof. Let G be(τ_i, τ_j)-rgb-closed and F be a τ_i- regular -closed set such that F ⊆ $(\tau_j$ −bcl(G))^c. Since G is (τ_i, τ_j)-rgb-closed, then G ∈ D^* RGB (τ_i, τ_j) which implise that τ_j–bcl(G) ⊆ F^c . Then F ⊆ τ_j–bcl(G) ∩ $(\tau_j$–bcl(G))^c . Therefore F is empty.

The converse of the above theorem need not be true as seen from the following example.

Example 4.9. Let X = {a, b, c} and τ_i={ X, φ,{b} ,{c} ,{b,c}}, τ_j = { X, φ,{b}}.If G={b} ,then τ_j–cl(G)-G={a,c} does not any non-empty τ_i- regular -closed set.But G is not a (τ_i,τ_j)–rgb-closed set.
Proposition 4.10: If A is(τ_i,τ_j)–rgb-closed set and A ⊆ B ⊆ τ_j -bcl(A), then B is $(\tau_i,\tau_j)-$ rgb-closed set**.**
Proof. Let B ⊆ U,where U is - regular open set.Since A⊆ B,so τ_j -bcl(A) ⊆ U. But B⊆ τ_j_bcl(A) **,**

We have τ_j -bc l(B) ⊆ τ_j $-$ $(\tau_j$ -c l(A))then τ_j -bc l(B) ⊆ U .Therefore B is rgb-closed in X.

Proposition 4.11: Let A ⊆ Y ⊆ X and if A is$(\tau_i,\tau_j)-$ rgb -closed in X then A is $(\tau_i,\tau_j)-$ rgb -closed relative to Y.

Proof. Let A ⊆ Y∩ G where G is τ_i $-$ regular open in X. Since A is (τ_i, τ_j)-rgb-closed .Then τ_j −bcl(A) ⊆ cl⊆G. Then Y ∩τ_j −bcl(A) ⊆Y ∩ G. Thus A is rgb -closed relative to Y.

Proposition 4.12: If A is (τ_i,τ_j)–rgb-closed set,then τ_j -bcl({x}) ∩ A ≠ φ for each x ∈ τ_j-bcl(A)
Proof. If τ_i -bcl({x}) ∩ A = φ for each x ∈ τ_j-bcl(A), then A ⊆ $(\tau_i$-bcl({x}))^c. Since A is $(\tau_i,\tau_j)-$ rgb-closed set, so τ_j -bcl(A) ⊆ $(\tau_i$-bcl({x}))^c which implise that x ∉ τ_j -bcl(A).This contradicts to the assumption.

Definition 4.13. Let (X, τ_i, τ_j) be bitopological space,and let g∈ X.A subset N of X is said to be, $(\tau_i, \tau_j)-$ rgb-neighbourhood (briefly$(\tau_i, \tau_j)-$ rgb-nhd) of a point g if and only if there exists a $(\tau_i, \tau_j)-$ rgb –open set G such that g∈G ⊆ N.

The set of all$(\tau_i, \tau_j)-$ rgb –nhd of a point g is denoted by$(\tau_i, \tau_j)-$ rgb –N(g)

Proposition 4.14: Every τ_{i-} nhd of g ∈ X is a $(\tau_i, \tau_j)-$ rgb –nhd of g ∈ X .
Proof. Since N is τ_{i-} nhd of g ∈ X ,then there exists τ_{i-} open set G such that g ∈G ⊆N. Since every τ_{i-} open set is $(\tau_i, \tau_j)-$ rgb –open set, G is$(\tau_i, \tau_j)-$ rgb –open set .By Definition 4.13. N is$(\tau_i, \tau_j)-$ rgb –nhd of x

Remark 4.15 :The converse of the above Proposition need not be true as seen from the following example.
Example 4.16. Let X = {a, b, c} and τ_i ={X, φ,{a,}{b},{a,b},{a,c}}, τ_j = { X, φ,{a},{b,c}}.

D^* RGBO (τ_i, τ_j) = {X, φ,{a,}{b},{c},{a,b},{a,c}{b,c}},the set{b,c}is$(\tau_i, \tau_j)-$ rgb –nbhd of c ,since there exists a $(\tau_i, \tau_j)-$ rgb –open set G={c}such that c∈{c}⊆{b,c}.However {b,c}is not τ_{i-} nhd of c ,since no τ_i −open set G such that c ∈G ⊆{b,c}.

Remark 4.17. The $(\tau_i, \tau_j)-$ rgb –nhd of a point g ∈ X need not be a $(\tau_i, \tau_j)-$ rgb –open set in X as seen from the following example.
Example 4.18. Let X = {a, b, c} and τ_i ={X, φ,{b},{c},{b,c}}, τ_j = { X, φ,{b}}.

D* RGBO (τ_i, τ_j) = {X, φ,{a,}{b},{c},{a,b},{b,c}},the set{a,c}is$(\tau_i, \tau_j)-$ rgb –nhd of c ,since there exists a $(\tau_i, \tau_j)-$ rgb –open set G={c}such that c∈{c}⊆{a,c}.However {a,c}is not $(\tau_i, \tau_j)-$ rgb- open set .

Proposition 4.19: If N a subset of a bitopological space (X, τ_i, τ_j) is $(\tau_i, \tau_j)-$ rgb –open set ,then N is$(\tau_i, \tau_j)-$ rgb –nhd of each of its points.

Proof. Let N be a $(\tau_i, \tau_j)-$ rgb –open set. By Definition 4.13. N is an $(\tau_i, \tau_j)-$ rgb –nhd of each of its points.

Remark 4.20. The $(\tau_i, \tau_j)-$ rgb –nhd of a point g ∈ X need not be a (τ_i, τ_j) − nhd–of x in X as seen from the following example.

Example 4.21. Let X = {a, b, c} and τ_i ={X, φ,{a,}{b},{a,b},{a,c}}, τ_j = { X, φ,{a,b},{b}}.

D* RGBO (τ_i, τ_j) = {X, φ,{a,}{b},{c},{a,b},{b,c}},the set{a,c}is$(\tau_i, \tau_j)-$ rgb –nhd of a ,since there exists a $(\tau_i, \tau_j)-$ rgb –open set G={a}such that a∈{a}⊆{a,c}.Also the set{a,c}is$(\tau_i, \tau_j)-$ rgb –nhd of c ,since there exists a $(\tau_i, \tau_j)-$ rgb –open set G={c}such that c∈{c}⊆{a,c} However {a,c}is not $(\tau_i, \tau_j)-$ rgb- open set in X.

Proposition 4.22. Let (X, τ_i, τ_j) be bitopological space:

1) ∀ g ∈ X , $(\tau_i, \tau_j)-$ rgb –N(g)≠ φ

2) ∀ N∈ $(\tau_i, \tau_j)-$ rgb –N(g) ,then g ∈ N..

3) If N∈ $(\tau_i, \tau_j)-$ rgb –N(g) , N ⊆M ,then M∈ $(\tau_i, \tau_j)-$ rgb –N(g) .

4) If N ∈ $(\tau_i, \tau_j)-$ rgb –N(g) , then there exists M∈ $(\tau_i, \tau_j)-$ rgb –N(g) such that M⊆ N and exists M∈ $(\tau_i, \tau_j)-$ rgb –N(h) ∀, h∈ M.

Proof.1) Since X is an $(\tau_i, \tau_j)-$ rgb –open set,it is$(\tau_i, \tau_j)-$ rgb –nhd of every g ∈ X .Hence there exists at least one $(\tau_i, \tau_j)-$ rgb –nhd G for every g ∈ X.Therefore $(\tau_i, \tau_j)-$ rgb –N(g)≠ φ , ∀ g ∈ X

2)If N∈ $(\tau_i, \tau_j)-$ rgb –N(g) ,then N is $(\tau_i, \tau_j)-$ rgb –nhd G of g .Thus By Definition 4.13 g ∈ N.

3) If N∈ $(\tau_i, \tau_j)-$ rgb –N(g) ,then there is an$(\tau_i, \tau_j)-$ rgb –open set A such that g∈ A ⊆ N , since N ⊆M , g∈ A ⊆ M and M is an $(\tau_i, \tau_j)-$ rgb –nhd of g. Hence M∈ $\tau_i, \tau_j)-$ rgb –N(g) **.4)** If N∈ $(\tau_i, \tau_j)-$ rgb –N(g) ,then there exists is an$(\tau_i, \tau_j)-$ rgb –open set M such that g∈ M ⊆ N. Since M is an$(\tau_i, \tau_j)-$ rgb –open set , then it is $(\tau_i, \tau_j)-$ rgb –nhd of each of its points. Therefore M∈ $(\tau_i, \tau_j)-$ rgb –N(h) ∀, h∈ M.

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