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Unsteady state thermoelastic problem in a circular annular fin due to internal heat source

C.M. Jadhav[1]and B.R.Ahirrao[2]
[1](*Department of Mathematics, Dadasaheb Rawal College Dondaicha, NMU Jalgaon University, Jalgaon [M.S.], INDIA*
[2](*Departments of Mathematics, Z.B.Patil College, Dhule, NMU Jalgaon University, Jalgaon INDIA)*

Abstract : *This paper is concerned with the transient unsteady state thermoelastic problem of thin circular annular fin due to heat source .The transient heat conduction equation is solved by Marchi Zgrablich and Laplace transform with radiation boundary condition. The solution of the problem in the form of infinite series of Bessel function. To determine temperature distribution for heating process and their stresses .Numerical calculations are carried out by using mathematica software.*
Keywords - *Transient distribution, integral transform, heat source, circular annular fin, and transient heating*

I. INTRODUCTION

The circular annular fin is found in many field of thermal engineering such as air conditioning, heat exchangers microelectronics. Circular annular fin are used mostly in heat exchange devices to increase the heat transfer rate from a heat source for a given temperature difference or to decrease the temperature difference between the heat source and heat sink for a given heat flow rate. Several solution to the problem of one dimensional steady state condition within an annular fin of constant thickness have been presented [2],[5],[6]]and [8].The typical problem of transient thermal stresses in one dimensional steady state condition within an annular fin of constant annular fin have been investigated by Wu[9].

The present paper attempt to generalize the one dimension problem considered by Wu [9] and obtains the exact solution of two dimensional transient heat equation problem with radiation boundary condition subjected to internal heat source.

II. Formulation Of The Problem

We consider circular annular fin Fig. 1 occupying the space $D = \{(x, y, z)\epsilon R^3 : a \le r \le b, 0 \le z \le l$, where
$r = \sqrt{x^2 + y^2}$ The material of fin is isotropic homogeneous and all properties are assumed to be constant. The governing equations and boundary condition for the stress field [12] are

A nonzero stress strain –displacement equation $\varepsilon_r = \frac{\partial u}{\partial r}, \varepsilon_\varphi = \frac{u}{r}$ (2.1)

A single equilibrium equation $\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\varphi}{r} = 0$ (2.2)

Two equation of stress strain equation relation

$$\sigma_r = \frac{E}{1-\nu^2}\left[\varepsilon_r - \nu\varepsilon_\varphi - (1+\nu)\alpha T\right] \quad (2.3)$$

$$\sigma_\varphi = \frac{E}{1-\nu^2}\left[\varepsilon_r - \nu\varepsilon_\varphi + (1+\nu)\alpha T\right] \quad (2.4)$$

And boundary condition

$$\sigma_r = 0 \; at \; r = a \,, \sigma_\varphi = 0 \; at \; r = b \quad (2.5)$$

Combining equation (2.1)-(2.4), integrating twice the restive r and applying the boundary condition, one obtain the stress strain displacement relation as

$$\sigma_r = \frac{-\alpha E}{r^2}\int_a^r (T - T_\infty)\eta d\eta + \frac{\alpha E}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)\int_a^b (T - T_\infty)\eta d\eta \quad (2.6)$$

$$\sigma_\varphi = -\alpha E(T - T_\infty) + \frac{\alpha E}{r^2}\int_a^r (T - T_\infty)\eta d\eta + \frac{\alpha E}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)\int_a^b (T - T_\infty)\eta d\eta \quad (2.7)$$

Introduce dimensional quantities $\theta, \xi, \tau, R, S_r, S_\varphi$ in (2.6), (2.7)

$$S_r = \frac{-1}{\xi^2}\int_1^\xi \theta\xi d\xi + \frac{1}{\xi^2}\frac{\xi^2 - 1}{R^2 - 1}\int_1^R \theta\xi d\xi \quad (2.8)$$

$$S_\varphi = -\theta + \frac{1}{\xi^2}\int_1^\xi \theta\xi d\xi + \frac{1}{\xi^2}\frac{\xi^2 + 1}{R^2 - 1}\int_1^R \theta\xi d\xi \quad (2.9)$$

Unsteady –state conduction with an isotropic ,circular annular fin with internal heat source must satisfy two dimensional equation ,which in cylindrical coordinate can be obtained by substituting the radial and tangential stresses in to stress equilibrium equation ,lead as $a \le r \le b, 0 \le z \le l, t > 0$

$k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)-\frac{2h}{l}(T-T_\infty)+\psi(r,z,t,T)=\rho c\frac{\partial T}{\partial t}\quad , a\le r\le b, 0\le z\le l, t>0$ (2.10)

Substitute dimensional parameters

$\theta=\frac{k(T-T_\infty)}{\rho ba},\ \xi=\frac{r}{a},\zeta=\frac{z}{a},L=\frac{l}{a},\tau=\frac{kt}{\rho ca},R=\frac{b}{a},N^2=2ha^2/kl$

$\psi(r,z,t,T)=\Phi(r,z,t)+\varepsilon(t)T(r,z,t)$

$\chi(\xi,\zeta\ \tau)=\ \Phi(r,z,t)e^{-\int_0^t\varepsilon(y)dy}$

For sake of brevity, we consider $\chi(\mathrm{r},\mathrm{z},\mathrm{t})=\frac{\delta(\mathrm{r}-\mathrm{r}_0)\delta(\mathrm{z}-\mathrm{z}_0)}{2\pi \mathrm{r}_0}\mathrm{e}^{-\omega t}$ one obtain

$\frac{\partial^2\theta}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial\zeta^2}-N^2+\chi(\xi,\zeta\ \tau)=\frac{\partial\theta}{\partial t},1\le\xi\le R,0\le\zeta\le L,\tau>0$ (2.11)

Subject to initial and boundary conditions

$M_\tau(\theta,1,0,0)=0$, for $1\le\xi\le R,0\le\zeta\le L,\tau=0$ (2.12)

$M_\xi(\theta,1,\mathrm{k}_1,1)=0, M_\xi(\theta,1,\mathrm{k}_2,R)=\mathrm{F}_1(\zeta,\tau)$, for $0\le\zeta\le L,\tau>0$ (2.13)

$M_\zeta(\theta,1,0,0)=0$, $M_\zeta(\theta,1,0,L)=\mathrm{f}(\xi,\tau)$,for $1\le\xi\le R,\tau>0$ (2.14)

Being $M_\vartheta\left(f,\bar{k},\bar{\bar{k}},\$\right)=\left(\bar{k}f+\bar{\bar{k}}\dot{f}\right)_{\vartheta=\$}$ (2.15)

Where the dot denotes differentiation with respective to ϑ , $\mathrm{k}_1,\mathrm{k}_2$are radiation constant on the curved surface of the annular disc thus equations (2.1) to (2.15)constitute the mathematical formulation of the heating problem under consideration .

III. Solution Of The Problem

Applying March Zgrablich integral transform to the (2.11), (2.12) and (2.14) using (2.13) and taking Laplace transform, one obtain

$\bar{\theta}^*(n,\zeta,s)=\left[\bar{f}^*(n,s)-(PI)_{\zeta=L}\right]\frac{Sin\,hp\zeta}{Sin\,hpL}+(PI)_{\zeta=0}\frac{Sin\,hp(\zeta-L)}{Sin\,hpL}$ (3.1)

where $p^2=\mu_n{}^2+N^2+\frac{s}{k}$, $PI=\frac{k\,\Psi^*}{D^{2-}p^2}$

Applying inverse of March Zgrablich and Laplace integral transform to (3.1) we obtain

$\theta(\xi,\zeta\ \tau)=$
$-\frac{2k\pi}{L^2}\sum_{n,m=1}^\infty\frac{S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\xi)}{C_m}(-1)^n n\int_0^\tau Sinh(\lambda_n\zeta)\left[\bar{f}(n,s)-[PI]'_{\zeta=L}\right]+\ Sinh\lambda_n(\zeta-$
$L)[PI]'_{\zeta=0}e^{-(\mu_m{}^2+N^2+\lambda_n{}^2)(\tau-\tau')}d\tau'$ (3.2)

where μ_m are the positive roots of $J_0(\mathrm{k}_1,\mu)Y_0(\mathrm{k}_2,\mu R)-J_0(\mathrm{k}_2,\mu\mathrm{R})Y_0(\mathrm{k}_1,\mu)=0$ and dash indicate inverse Laplace transform , $\lambda_n=\frac{\pi n}{L}$

IV. Determination Thermal Stresses

Using (3.2)in (2.8),(2.9) we obtain the radical and tangential stresses are

$S_r=$
$\frac{2k\pi}{L^2}\frac{1}{\xi^2}\int_1^\xi\sum_{n,m=1}^\infty\frac{(-1)^n n\xi S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\xi)}{C_m}\int_0^\tau Sinh(\lambda_n\zeta)\left[\bar{f}(n,s)-[PI]'_{\zeta=L}\right]+\ Sinh\lambda_n(\zeta-$
*L)PI′ζ=0e−μm2+N2+λn2 τ−τ′dτ′dξ−2kπL21ξ2ξ2−1R2−11Rn,m=1∞ −1nnξS0*k1,k2,μm*ξCm0τSinh(λnζ) f(n,s)−PI′ζ=L+ Sinhλn(ζ−L)PI′ζ=0e−μm2+N2+λn2 τ−τ′dτ′dξ*
(4.1)

$S_\varphi=$
$\frac{2k\pi}{L^2}\sum_{n,m=1}^\infty\frac{(-1)^n n\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\xi)}{C_m}\int_0^\tau Sinh(\lambda_n\zeta)\left[\bar{f}(n,s)-[PI]'_{\zeta=L}\right]+\ Sinh\lambda_n(\zeta-$
$L)[PI]'_{\zeta=0}e^{-(\mu_m{}^2+N^2+\lambda_n{}^2)(\tau-\tau')}d\tau'-\frac{2k\pi}{L^2}\frac{1}{\xi^2}\int_1^\xi\sum_{n,m=1}^\infty\frac{(-1)^n n\xi\ S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\xi)}{C_m}\int_0^\tau Sinh(\lambda_n\zeta)\left[\bar{f}(n,s)-[PI]'_{\zeta=L}\right]+$
*Sinhλn(ζ−L)PI′ζ=0e−μm2+N2+λn2 τ−τ′dτ′dξ−2kπL21ξ2ξ2+1R2−11Rn,m=1∞ −1nnξS0*k1,k2,μm*ξCm 0τSinh(λnζ) f(n,s)−PI′ζ=L+ Sinhλn(ζ−L)PI′ζ=0e−μm2+N2+λn2 τ−τ′dτ′dξ*
(4.2)

V. Special Case

Setting $\mathrm{f}(\xi,\tau)=0$, $\chi(\xi,\zeta,\tau)=\frac{\delta(\xi-\xi_0)\mathrm{e}^{(\zeta-\zeta\,0)}}{2\pi\xi_0}\mathrm{e}^{-\omega\tau}$ (5.1)

Applying Marchi Zgrablich and Laplace transform to (4.1) one obtains

$$\bar{\Psi}^{*}(\xi,\zeta,\tau)=\frac{S_0(k_1,k_2,\mu_m\xi_0)}{2\pi C_n(s+\omega)}e^{(\zeta-\zeta_0)} \tag{5.2}$$

$$[PI]'_{\zeta=0}=\frac{S_0(k_1,k_2,\mu_m\xi_0)}{2\pi C_n(1-\lambda_n^2)}e^{\zeta}e^{-\omega\tau} \tag{5.3}$$

$$[PI]'_{\zeta=L}=\frac{S_0(k_1,k_2,\mu_m\xi_0)}{2\pi C_n(1-\lambda_n^2)}e^{(\zeta-L)}e^{-\omega\tau} \tag{5.4}$$

Substitute the value of (5.3),(5.4) in (3.2),(4.1)and (4.2)one obtains

$$\theta(\xi,\zeta\,\tau)=-\frac{k}{L^2}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m\xi_0)}{C_m(1-\lambda_n^2)}\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)e^{-(\mu_m^2+N^2+\lambda_n^2+\omega)\tau} \tag{5.5}$$

$$S_r=\frac{k}{L^2}\frac{1}{\xi^2}\int_1^{\xi}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m\xi_0)\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)}{C_m(1-\lambda_n^2)}e^{-(\mu_m^2+N^2+\lambda_n^2+\omega)\tau}d\xi$$
$$-\frac{k}{L^2}\frac{1}{\xi^2}\frac{\xi^2-1}{R^2-1}\int_1^{R}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m\xi_0)\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)}{C_m(1-\lambda_n^2)}e^{-(\mu_m^2+N^2+\lambda_n^2+\omega)\tau}d\xi \tag{5.6}$$

$$S_\varphi=\frac{k}{L^2}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m\xi_0)}{C_m(1-\lambda_n^2)}\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)e^{-(\mu_m^2+N^2+\lambda_n^2+\omega)\tau}$$
$$-\frac{k}{L^2}\frac{1}{\xi^2}\int_1^{\xi}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m\xi_0)\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)}{C_m(1-\lambda_n^2)}e^{-(\mu_m^2+N^2+\lambda_n^2+\omega)\tau}d\xi$$
$$-\frac{k}{L^2}\frac{1}{\xi^2}\frac{\xi^2+1}{R^2-1}\int_1^{R}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m\xi_0)\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)}{C_m(1-\lambda_n^2)}e^{-(\mu_m^2+N^2+\lambda_n^2+\omega)\tau}d\xi \tag{5.7}$$

The numerical calculation have been carried out for low carbon steel(AISI) with parameter $k_1=k_2=1$,Radius R=4, $\xi_0=1.2$, $\zeta_0=0.5$ k=1,h=1 L=1,N=2 , $\omega=1$ $\tau=1$ $\xi=1.5$ and μ_m =1.10821,2.13634,3.17041,4.21067,5.2536,6.29793.7.34305,8.38869,9.43467,21.9954 are the positive root of transcendental equation of $J_0(k_1,\mu)Y_0(k_2,\mu R)-J_0(k_2,\mu R)Y_0(k_1,\mu)=0$ and , $\lambda_n=3.142, 6.284, 9.426, 12.568, 15.71, 18.853, 21.994, 25.136, 28.278, 31.42$ are the positive root of transcendental equation of $\lambda_n=\frac{\pi n}{L}$

$$\theta(\xi,\zeta\,\tau)=-\sum_{n,m=1}^{\infty}\frac{(-1)^n n\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m 1.2)}{C_m(1-\pi n^2)}\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)e^{-(\mu_m^2+5+n\pi^2)} \tag{5.8}$$

$$S_r=\frac{1}{2.25}\int_1^{1.5}\sum_{n,m=1}^{\infty}\frac{(-1)^n\xi n S_0(k_1,k_2,\mu_m\xi)S_0(0.8,1,\mu_m 1.2)\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)}{C_m(1-\pi n^2)}e^{-(\mu_m^2+5+n\pi^2)}d\xi-$$
$$\frac{1}{2.7}\int_1^{2}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi\,S_0(k_1,k_2,\mu_m\xi)S_0(0.8,1,\mu_m 1.2)\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)}{C_m(1-\pi n^2)}e^{-(\mu_m^2+5+n\pi^2)}d\xi \tag{5.9}$$

$$S_\varphi=\sum_{n,m=1}^{\infty}\frac{(-1)^n n\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m 1.2)}{C_m(1-\pi n^2)}\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)e^{-(\mu_m^2+5+n\pi^2)}-$$
$$\frac{1}{2.25}\int_1^{1.5}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m 1.2)}{C_m(1-\pi n^2)}\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)e^{-(\mu_m^2+5+n\pi^2)}d\xi-$$
$$\frac{13}{13.5}\int_1^{2}\sum_{n,m=1}^{\infty}\frac{(-1)^n n\xi\,S_0(k_1,k_2,\mu_m\xi)S_0(k_1,k_2,\mu_m 1.2)}{C_m(1-\pi n^2)}\left(Sinh(\lambda_n\zeta)-Sinh\lambda_n(\zeta-\zeta_0)\right)e^{-(\mu_m^2+5+n\pi^2)}d\xi \tag{5.10}$$

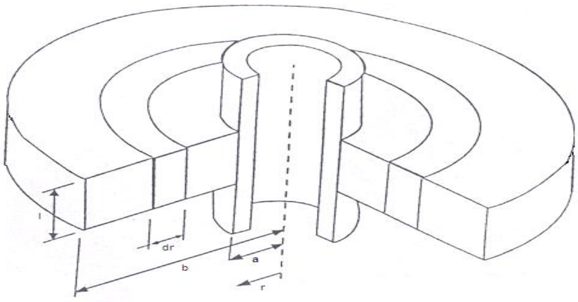

Figure 1: Characteristics of the annular fin

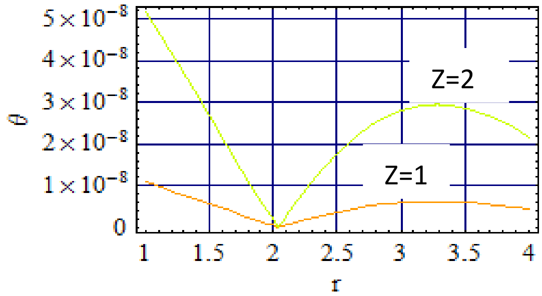

Fig.2Temperature verses r with different value of z at t=1

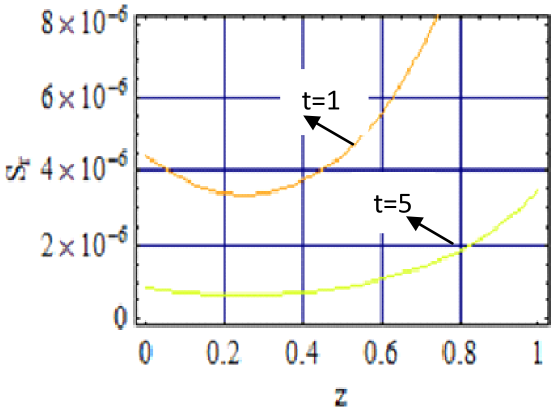

Fig.3 Thermal Stress S_r verses r with different time

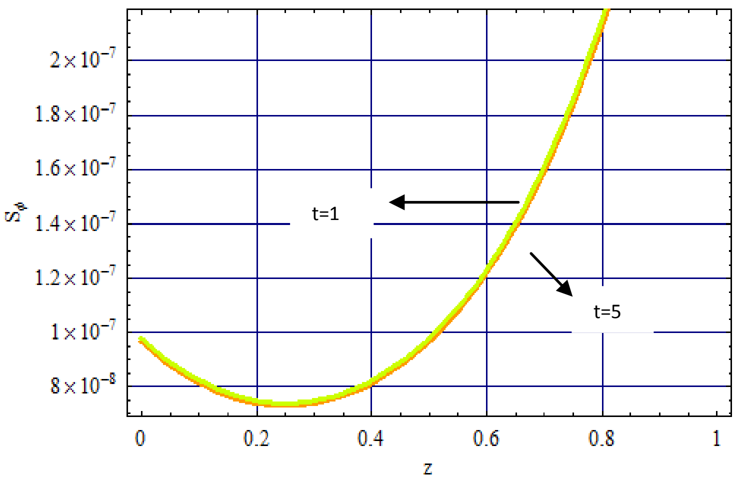

Fig.4Thermal stress S_φ verses r with different time

V. Conclusion

In this paper, we generalized the idea proposed by Wu et.al(1997) for two dimensional non homogeneous radiation boundary value problem of circular annular fin with heat source and temperature distribution Displacement and stress function for annular fin have been obtained we developed the analysis for temperature field for heating processes by using March Zgrablich and Laplace transform technique with boundary condition of radiation type. the series solution is converges since the thickness of annular fin is very small also any particular case of special interest may be derived by assigning suitable value of the parameter and function in the series expansion. The result can be applied to the design of useful structures or machines in engineering applications.

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Appendix

The finite Marchi-Zgrablich integral transform of order p is defined as

$$\bar{f}_p(n) = \int_a^b r f(r) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n r) dr$$

And inverse Marchi-Zgrablich integral transform as

$$f(r) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n r)}{C_n}$$

Where

$$S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n r) = J_p(\mu_n r)\{Y_p(\mathrm{k}_1, \mu_n a) + Y_p(\mathrm{k}_2, \mu_n b)\} - Y_p(\mu_n r)\{J_p(\mathrm{k}_1, \mu_n a) + J_p(\mathrm{k}_2, \mu_n b)\}$$

$$C_n = \int_a^b [r S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n r)]^2 dr$$

The eigenvalue μ_n are the positive roots of the equation $J_p(\mathrm{k}_1, \mu_n a) Y_p(\mathrm{k}_2, \mu_n b) - J_p(\mathrm{k}_2, \mu_n b) Y_p(\mathrm{k}_1, \mu_n a) = 0$

An operational property is given by

$$\int_a^b \left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} + \frac{p^2 f}{x^2}\right] S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n r x)$$

$$= \frac{b}{\mathrm{k}_2} S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n b)\left[f + \mathrm{k}_2 \frac{\partial f}{\partial r}\right]_{r=b}$$

$$-\frac{a}{\mathrm{k}_1} S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n a)\left[f + \mathrm{k}_1 \frac{\partial f}{\partial r}\right]_{r=a} - \mu_n^2 \bar{f}_p(n)$$

and $J_p(\mu_n r)$ and $Y_p(\mu_n r)$ are the Bessel function of first and second kind respectively.

Nomenclature

a, b: Inner and outer radii of the fin
c: Specific heat of material of the fin
c_1, c_2: Constants
E: Young's modulus of material of the fin
h: Heat transfer coefficient
k: Thermal conductivity of material of the fin
N: Dimensionless parameter
t: Time
q_b: Heat flux from the base of the fin
R: Dimensionless outer radius,
S_r,: Dimensionless radial
S_φ: Dimensionless tangential stresses
T: Temperature of the fin
T_∞ : Ambient temperature
U: Radial displacement
l: Thickness of the fin
ε_r, ε_φ: Radial and tangential strains
θ: Dimensionless temperature of the fin
σ_r, σ_θ: Radial and tangential stresses
τ: Dimensionless time
r, φ :Polar coordinates
L: Dimensionless thickness