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A new argument for the non-existence of Closed Timelike Curves

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Abstract: *In this paper, we attempt to present a short argument, different from that of the original proofs by that of Hawking, for a theorem stated that no closed timelike curves can exist. In a later paper, we apply this to quantum gravity and relate the curvature of spacetime to this theorem. Also, we present this paper as a preliminary introduction to the complete argument of this, and we also provide a preliminary notion of the concepts which will be narrated in the later papers. We also use this as a starting basis for a true theory of everything for a theory of everything. We use the notation of [1] and of [2].*

Keywords: *Theoretical Physics, Quantum Gravity, Mathematical Physics, General Relativity, Quantum Mechanics*

I. The elimination of Closed Timelike Curves in Loop Quantum Gravity

Theorem : *There exists no closed timelike curve in the physical world.*

To prove this, we introduce spacetime as $(\mathcal{M}, {}^{(4)}g)$ where we can define $\mathcal{M}$ as a four dimensional manifold and ${}^{(4)}g$ as a Lorentzian metric on it. In the third part, we introduce a ADM $3 + 1$ split of the four dimensional $(\mathcal{M}, {}^{(4)}g)$.

II. ADM 3+1 split of classical four dimensional $(\boldsymbol{\mathcal{M}}, {}^{(4)}\boldsymbol{g})$

In this section, we mainly reconstruct the topics in [1].

To do a 3+1 split of classical four dimensional $(\mathcal{M}, {}^{(4)}g)$, we need $(\mathcal{M}, {}^{(4)}g)$ to be homeomorphic to the direct product space formed by $\mathbb{R} \times \Sigma$, where Σ is a three-manifold representing space and $t \in \mathbb{R}$ represents time. $(\mathcal{M}, {}^{(4)}g)$ needs to be globally hyperbolic, and we assume causality, that is, no closed timelike curves (CTC) exist. We define closed timelike curves in the following manner.

A curve π such that, we have, in a particular coordinate system $\mathfrak{K}$ on $(\mathcal{M}, {}^{(4)}g)$, the following equations satisfied:

$$\pi: S^1 \to (\mathcal{M}, {}^{(4)}g) \quad 1)$$

And in a shifted coordinate system, $\mathfrak{K}1$, we have

$$g(\pi^{\mathfrak{K}1}, \pi^{\mathfrak{K}1}) < 0 \quad 2)$$

This allows us to state that the time function is regular. A particular slicing of spacetime, though, would be a matter of choice. A choice of slicing is equivalent to the choice of a regular function f (i.e., a scalar field on $(\mathcal{M}, {}^{(4)}g)$) for which $\partial^\mu t$ is timelike.

Suppose there exist two spaces, so that there is a mapping of some sort between them, defined as

Proposition 1: *f on $\mathcal{A}$ maps to $\mathcal{B}$. Then, it could be said that $\{f\}$ is a space itself, multiplied (i.e., it forms a product space with) by $\mathcal{A}$ to yield $\mathcal{B}$. Then we could say that $\{f\} = \mathcal{B}/\mathcal{A}$.*

I.e., if

$$f: \mathcal{A} \to \mathcal{B} \quad 3)$$

Then we have

$$\{f\} = \mathcal{B}/\mathcal{A} \quad 4)$$

Proof: If f is defined as

$$f: \mathcal{A} \to \mathcal{B} \quad 5)$$

Then we need $\mathcal{B}$ and $\mathcal{A}$ to have their points as discrete eigenvalues of some operator corresponding to each space (we wish to call this the *soperator*). Thus, as f should have eigenvalues corresponding to each soperator, we need f to eigenvalues which exist in both the spaces, which is obviously the union of the two, which is given by

$$\{f\} = \mathcal{B}/\mathcal{A} \tag{6)}$$

■

The regular values of f then form 3-manifolds, our Σ, defined by $\Sigma(t_0) = f^{-1}(t_0)$. Using the submersion theorem, we can find a local coordinate system $\{x^{\bar{\mu}}\}$ over the open set U, where $\forall\, p \in U$, $f(p) = f(x^0(p), x^1(p), x^2(p), x^3(p)) = x^0(p)$.

The 1-form df is then dx^0 and the intrinsic coordinates of each hypersurface are given by x^1, x^2, x^3. Thus the vectors $\partial_\alpha := \frac{\partial}{\partial x^\alpha}$ span the target space to each hypersurface. We can express the components of each vector field in terms of a general basis $\{y^\alpha\}$ as $e_{\bar{v}}$:

$$\frac{\partial}{\partial x^{\bar{v}}} = \frac{\partial x^\alpha}{\partial x^{\bar{v}}} \frac{\partial}{y^\alpha} =: e^\alpha_{\bar{v}} \partial_\alpha \tag{7)}$$

Similar to [1], $\sqrt{4}$ means the metric dual to dx^0. We have a vector field

$$(dx^0)^{\sqrt{4}} := {}^{(4)}g(dx^0, \cdot) = ({}^{(4)}g)^{0v} \partial_v \tag{8)}$$

Thus the vector field with components $\partial_v f$ or $({}^{(4)}g)^{0v}$ is a normal to the hypersurface. If n^μ is the unit normal to Σ, then on decomposing ∂_0 into its components parallel to the hypersurface N^μ and orthogonal to is Nn^μ, we obtain

$$\partial_0 = Nn + \vec{N} \tag{9)}$$

Since we have $dx^0(\partial_0) = 1$, we see that we must have

$$-N^2 = ({}^{(4)}g)^{00} = -||dx^0||^2 \tag{10)}$$

III. Loop Part

We first look at Definition 1.0.2. So, we can define *elements of a curve* as

Definition : If there exists a curve γ, then the elements of it are its x, y, z and/or t components, given by γ^a, or by γ_a.

So, using this convention, we have a new definition of a closed timelike curve:

Definition : A curve π such that, we have, in a particular coordinate system $\mathfrak{K}$ on $(\mathcal{M}, {}^{(4)}g)$, the following equations satisfied:

$$\pi: S^1 \to (\mathcal{M}, {}^{(4)}g) \tag{11)}$$

And in a shifted coordinate system, $\mathfrak{K}1$, we have

$$({}^{(4)}g)_{ab} (\pi^{\mathfrak{K}1})^a, (\pi^{\mathfrak{K}1})^b < 0 \tag{12)}$$

We then define a set of all possible timelike curves defined by $\pi: S^1 \to (\mathcal{M}, {}^{(4)}g)$. This set we call the timelike loop space of $(\mathcal{M}, {}^{(4)}g)$ and call it $\Omega(\mathcal{M}, {}^{(4)}g)$. Now, if we consider our coordinate system change, to the system $\mathfrak{K}1$, we would have the following:

$$\Omega(\mathcal{M}, {}^{(4)}g) = \emptyset \tag{13)}$$

Therefore, we may define the coordinate shift such that the causal structure makes the timelike loops (that is, curves) to vanish and therefore make $\Omega(\mathcal{M}, {}^{(4)}g) = \emptyset$. The $\pi^{\mathfrak{K}1}$ may lie on $\mathbb{R}$ or Σ. If $\pi^{\mathfrak{K}1}$ lies on Σ, then we need $\pi^{\mathfrak{K}1}$ to lie on the regular values of f.

But the introduction of a metric in $(\mathcal{M}, {}^{(4)}g)$ induces a metric on Σ, which in turn causes a causal structure to be formed. Since spacetime is not necessarily Ricci flat, $\pi^{\mathfrak{K}1}$ must, to introduce a causal structure on $(\mathcal{M}, {}^{(4)}g)$, exist on $\mathbb{R}$. Then we may define $\mathbb{R}$ as having no causal structure. The coordinate shift creates the closed timelike curves on the $\mathbb{R}$, and therefore a restriction of vectors tangential to $\mathbb{R}$ must not exist. This is wrong.

But $\mathbb{R}$ is Riemannian, therefore $\pi^{\mathfrak{K}1}$ must not exist. That is, slicing causing closed timelike curves do not exist. This means that all slicings of $\mathbb{R} \times \Sigma$ have no closed timelike curves in them, even on coordinate shifts, and therefore $(\mathcal{M}, {}^{(4)}g)$ is globally hyperbolic, $\forall (\mathcal{M}, {}^{(4)}g)$ and ${}^{(4)}g$. This is the proof of the proposition.

■

IV. Introduction to the preliminaries of Future Papers

Define $t^{\alpha} = Nn^{\alpha} + N^{a} e_{a}^{\alpha}$ as the components of ∂_0. We then obtain from [1], the following equation:

$$D_{(a} t_{b)} = n_{(\alpha} N_{;\beta)} + N D_{(\beta}\, n_{\alpha)} + D_{(\alpha} N_{\beta)} \qquad 14)$$

We then obtain the extrinsic curvature as

$$K_{ab} = \frac{1}{2N}(\dot{g}^{ab} - 2N_{(a;b)}) \qquad 15)$$

Now we use the Ashtekar approach. The canonical variables (these are x^{μ}) are components of the inverse 3-metric g_{ab} intrinsic to Σ and the components of the spin connection ∇ on Σ. The spin connection ∇ defines a bundle connection with base space Σ and the spin space $\mathbb{S}$. ∇ is then given by

$$\nabla = \nabla_{\text{intrinsic}} + i\boldsymbol{K} \qquad 16)$$

Defining a particular $\mu = 0$ would lead to f being a component of g_{ab} intrinsic to Σ and the components of the spin connection ∇ on Σ. For the $3+1$ split, we say that $(\mathcal{M}, {}^{(4)}g) = \Sigma \times \mathbb{R}$, and in the spin connections's space, $\Sigma \times \mathbb{S}$. We now define $\Sigma \times \mathbb{S}$ as a five-dimensional space $(\mathcal{D}, {}^{(5)}g)$, with a 5-metric.

We may perform a $4+1$ split of $(\mathcal{D}, {}^{(5)}g)$, by stating that $\mathbb{S}$ would have a $1+1$ split on it, so we can define $\mathbb{S} = \mathbb{R} \times Q$. What is Q? We can define the points of Q as below:

Definition : *Q is defined as $\forall p \in Q$, $p \times \mathbb{R}$ is a vector in $\mathbb{S}$.*

Here $\mathbb{R}$ denotes time, and Q denotes space. (We can see the reasons for this split later.) Consider the following equation:

$$(\mathcal{D}, {}^{(5)}g) = \Sigma \times \mathbb{R} \times Q \qquad 17)$$

Look at the first part of the right hand side. It has $\Sigma \times \mathbb{R}$, which is the $3+1$ split of gravity! So, this can be reformulated as

$$(\mathcal{D}, {}^{(5)}g) = (\mathcal{M}, {}^{(4)}g) \times Q \qquad 18)$$

The spin connection is basically a functional derivative's component, that is,

$$g^{\mu v} = i\hbar \frac{\delta}{\delta \nabla_{\mu v}} \qquad 19)$$

The metric made is $g^{\mu v} = {}^{(4)}g^{\mu v} + n^{\mu} n^{v}$. Therefore, we have a refined equation, that is

$${}^{(4)}g^{\mu v} + n^{\mu} n^{v} = i\hbar \frac{\delta}{\delta \nabla_{\mu v}} \qquad 20)$$

For uncontrollable infinities to vanish, we need $\nabla_{\mu v} \neq 0$, else the 3-dimensional metric would yield infinite distances between any two (even infinitesimally close) points as ∞.

Now we ask ourselves a question. In the above statement, why was "3-dimensional metric" mentioned? Why not "*the* metric" or "4-dimensional metric"?

To answer this, look at the fact that $g^{\mu v} = {}^{(4)}g^{\mu v} + n^{\mu} n^{v}$. We need the 4-metric ${}^{(4)}g$ to be finite, so we can introduce $n^{\mu} n^{v} = -\infty + \delta^{\mu}_{v}$. This then yields a finite answer for the 4-metric ${}^{(4)}g$, but what about the effect on Σ? Look at how we had defined n^{μ} – "n^{μ} is the unit normal to Σ…". The unit normal cannot be infinity, so saying that the 3-dimensional metric would yield infinite distances between any two (even infinitesimally close) points as ∞ is equivalent to saying that the 4-dimensional metric would yield infinite distances between any two (even infinitesimally close) points as ∞.

In the classical limit, ${}^{(4)}g^{\mu v} + n^{\mu} n^{v} = i\hbar \frac{\delta}{\delta \nabla_{\mu v}}$ becomes ${}^{(4)}g^{\mu v} = -n^{\mu} n^{v}$ as we estimate $\lim_{\hbar \to 0} \hbar$, and therefore we need $-1 = {}^{(4)}g_{\mu v} n^{\mu} n^{v}$, so that n^{μ} and n^{v} are negatively normalized to each other.

The space of the spin connection is $(\mathcal{M}, {}^{(4)}g) \times Q$, and we have a Dirac spinorial wavefunction, defined as $\Psi = (\alpha_A, \beta_{A'})$. ∇ tells us how to carry the (two) spinor α_A, (and also $\beta_{A'}$), parallel to itself with respect to the $(\mathcal{M}, {}^{(4)}g)$'s metric connection along some curve δ that lies on Σ, defined as

Definition : *A curve δ such that, we have, in a particular coordinate system $\mathfrak{K}$ on Σ, the following equations satisfied: $\delta: S^1 \to \Sigma$.*

Since $(\mathcal{M}, {}^{(4)}g)$ is globally hyperbolic, from the above discussion, and $\mathbb{R}$ is flat, Σ may be considered to be the global hyperbolicity "generator". We are using the δ on a globally hyperbolic surface, and the curve then becomes a timelike loop, but with restriction that if

Proposition 2: *If $\delta: S^1 \to \Sigma$ then we should have $\delta: S^1 \times \mathbb{R} \to \Sigma \times \mathbb{R}$ so that $\delta \neq \pi^{\mathfrak{K}1}$, i.e., δ is not a closed timelike curve.*

Define the connection now as ∇ tells us how to carry the two spinor Ψ parallel to itself with respect to the $(\mathcal{M}, {}^{(4)}g)$'s metric connection along some curve δ that lies on Σ, defined as

Definition : A curve δ such that, we have, in a particular coordinate system $\mathfrak{K}$ on Σ, the following equation satisfied $\delta: S^1 \to \Sigma$.

This new definition allows us to say that ∇ causes a linear transformation of the spin space $\mathbb{R} \times \mathcal{Q}$, by a matrix $\mathfrak{T}_A^B$ and $\mathfrak{T}_A^B$, when acted on $\mathbb{R}$ may/may not yield $\mathbb{R}$, but the space $\mathcal{Q}$ is for sure transformed.

As we split $(\mathcal{D}, {}^{(5)}g)$, we can say that $\mathfrak{T}_A^B$ on $\mathbb{R}$ would transform, but $\mathfrak{T}_A^B$ on $\mathcal{Q}$ would yield $\mathcal{Q}$. $\mathfrak{T}_A^B$ then operates only on $(\mathcal{M}, {}^{(4)}g)$'s $\mathbb{R}$ components. The elements of $\mathfrak{T}_A^B$ are determined by some sort of basis β in Σ. We need the basis β to be extended to $(\mathcal{M}, {}^{(4)}g)$, so we do a direct product by $\mathbb{R}$ on $(\mathcal{M}, {}^{(4)}g) \times \mathcal{Q}$ and $(\mathcal{M}, {}^{(4)}g)$, and retrieve back the full spin space (with the $1+1$ split). This ensures that the $\mathfrak{T}_A^B$ are determined by some sort of basis $\mathcal{B}$ in $\left(\mathcal{M}, {}^{(4)}g\right)$. This allows wavefunctions to be existent as functions of time, so that $i\,\hbar \frac{\partial}{\partial t} |\Psi\rangle$ is not necessarily *0*.

See that if the direct product by $\mathbb{R}$ had not been done, we would have had a vanishing Hamiltonian and therefore a Ricci flat spacetime, that is Minkowski space. It must be noted that after the direct product, we end up with a 6-dimensional spin connection space and a 5-dimensional spacetime, **not necessarily Ricci flat**. How are we to explain the sudden introduction of mass into the spacetime picture by doing a direct product? This we will try examining later. We have already split our 5-dimensional spacetime $\left(\mathcal{M}, {}^{(5)}g\right)$ as a $(3+1)+1$ split, as we have $\left(\mathcal{M}, {}^{(5)}g\right) = (\Sigma \times \mathbb{R}) \times \mathbb{R}$. $\mathbb{R} \times \mathbb{R}$, the direct product, is the 2-dimensional $\mathbb{R}^2$. We then have the two main equations $\left(\mathcal{D}, {}^{(6)}g\right) = \Sigma \times \mathbb{R}^2 \times \mathcal{Q}$ and $\left(\mathcal{M}, {}^{(5)}g\right) = (\Sigma \times \mathbb{R}) \times \mathbb{R}$. The 6-metric ${}^{(6)}g^{\mu\nu}$ is defined as ${}^{(6)}g^{\mu\nu} + n^\mu n^\nu = i\,\hbar \frac{\delta}{\delta \nabla_{\mu\nu}}$. We need μ, ν to run over $0, \dots, 5$. A problem here is that $\nabla_{\mu\nu}$ wanders away from its original background space – if μ, ν is to run over $0, \dots, 5$, then $\nabla_{\mu\nu}$ would exist on $\left(\mathcal{D}, {}^{(6)}g\right)$, and it has obviously deviated from its original background space Σ. So, for $\nabla_{\mu\nu}$, we define different indices h, k which run over $0, \dots, 2$, so the above equation becomes ${}^{(6)}g^{\mu\nu} + n^\mu n^\nu = i\,\hbar \frac{\delta}{\delta \nabla_{hk}}$.

Our $-+++$ metric is formed by restricting the Killing form of the group $\mathcal{G}$'s (of $\left(\mathcal{M}, {}^{(6)}g\right)$) Lie algebra g to its Cartan subalgebra, say g_0. If the (Killing form) field equations vanish, a topological field theory (TFT) can be created on $(\mathcal{M}, \cdot)$, and so even the ∇_{hk} vanishes. If we do the Chas-Sullivan timelike loop product of any two timelike loops $\nabla_{hk}{}^a$ and $\nabla_{hk}{}^b$, we need to have the Killing form of the Lie group to be restrictable to its Cartan subalgebra. If the Killing form vanishes, we have a TFT and the timelike loops vanish. Now we ask ourselves, is it a local or global TFT? The group of $(\mathcal{M}, \cdot)$ can be said to be $\mathcal{G}_u$. \emph{If $\mathcal{G}_u$ has a locally different structure from the global structure of $\mathcal{G}_u$, and if the local structure of $\mathcal{G}_u$ has a non-restrictable Killing form, then the TFT is local. For an Abelian $\mathcal{G}_u$, the TFT vanishes, but for a non-Abelian G, the TFT may vanish.}

We see that we can define the proposition in section 1 as a *theorem*:

Theorem : *There exists no closed timelike curve in the physical world.*

It seems that the above can also be stated alternatively, as a corollary, as below:

Corollary : *There exists no map between any two charts on \$(\mathcal{M}, ^{(4)}g)\$ such that causality is violated in any of them, i.e.,t here exists no map between any two charts on \$(\mathcal{M}, ^{(4)}g)\$ such that closed timelike structures (here proved only for curves) is violated in any of them.*

Proof : Basically, the *main concept we assume is that causality exists*. If that exists, then it should hold in all coordinate systems. According to the definition of closed timelike curves, if there exists a map between the two coordinate systems, say $\mho: \mathfrak{K} \to \mathfrak{K}1$, and the above theorem is false, then $\Omega\left(\mathcal{M}, {}^{(4)}g\right) = \emptyset$. ■

If we look at proposition 1, then if we look at the transition map between two charts, $\mathfrak{K}$ and $\mathfrak{K}1$, and compare it with the above then we see that we can propose:

Proposition 3: *The transition map between two charts, $\mathfrak{K}$ and $\mathfrak{K}1$, does not exist.*

Proof : We have, if $\mathfrak{K}/\mathfrak{K}1$ exists (as it should), a map $\mho: \mathfrak{K} \to \mathfrak{K}1$. Then, from proposition 1, $\mathfrak{K}/\mathfrak{K}1 = \{\mho\}$. As $\{\mho\}$ does not exist, by the definition of a transition map, $\tau: \{\mho\} \to \{\mho\}$ also *cannot exist*. ■

V. Conclusion

In this paper, we have proved the theorem which states that no CTCs can exist. Also, we have a concept simply stated as "functions as spaces". This has many great implications, as can be exhibited: In future papers, we see that the symmetry group of the quantum universe during the so-called 'quantum super-bounce' of super-LQC must have been $\mathcal{SU}$ (14) (due to complex reasons explained in those papers). We can easily perform a breaking as $\mathcal{SU}$ $(14) \rightarrow \mathcal{SU}$ $(5) \times \mathcal{SU}$ (9), where $\mathcal{SU}$ (9) is the symmetry group of loop quantum supergravity (due to complex reasons explained in those papers) and $\mathcal{SU}$ (5) is the Georgi-Glashow model. We obtain gravity, QCD, and the Electroweak force. By this method, we can obtain quantum electrodynamics as a unitary representation of the $\mathcal{SU}$ (14) group through the concept "functions as spaces".

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