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On Fuzzy Supra Semi $\tilde{T}_{i=0,1,2}$ Space In Fuzzy Topological Space On Fuzzy Set

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Abstract: *This paper is devoted to introduce the notion of fuzzy supra semi $\tilde{T}_{i=0,1,2}$ space, fuzzy supra semi $D_{i=0,1,2}$ space, and use the notion of fuzzy quasi coincident in their definitions, study some properties and theorems related to these subjects.*
Keywords: *fuzzy supra semi open set, fuzzy supra semi D set, fuzzy supra semi $\tilde{T}_{i=0,1,2}$ space, fuzzy supra semi $D_{i=0,1,2}$ space.*

I. Introduction

The concept of fuzzy set and fuzzy set operation was first introduced by Zadeh[13] .Chakrabarty and Ahsanullah[2] introduced the notion of fuzzy topological space on fuzzy set.In 1986 Abd EL-Monsef and Ramadan [1] introduced fuzzy supra topological space. In this paper we introduced and study the concept of fuzzy supra semi $\tilde{T}_{i=0,1,2}$space,fuzzy supra semi $D_{i=0,1,2}$space in fuzzy topological space on fuzzy set.

1. Basic Definitions

Definition 1.1 [13]: Let X be anon-empty set ,a fuzzy set $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x) : X \to I$, where I is the closed unite interval [0,1] which is written as $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)) : x \in X, 0 \le \mu_{\tilde{A}}(x) \le 1\}$,the collection of all fuzzy subsets in X will be denoted by I^x,that is $I^x=\{\tilde{A}:\tilde{A}$ is fuzzy subset of X$\}$ and $\mu_{\tilde{A}}(x)$ is called the membership function.

Proposition 1.2 [9, 12, 13]: Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets in X with membership function $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ respectively then for all $x \in X$.

- $\tilde{A} \subseteq \tilde{B}$ iff $\mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$
- $\tilde{A} = \tilde{B}$ iff $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$
- $\tilde{A}^c$ is the complement of $\tilde{A}$ with membership function $\mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x)$
- $\tilde{C} = \tilde{A} \cap \tilde{B}$ if $\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$
- $\tilde{D} = \tilde{A} \cup \tilde{B}$ if $\mu_{\tilde{E}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$

Remark 1.3 [2, 8]: Let $\tilde{A} \in I^x$ then $p(\tilde{A}) = \{\tilde{B} : \tilde{B} \in I^x$ and $\mu_{\tilde{B}}(x) \le \mu_{\tilde{A}}(x)\ \forall x \in X\}$.

Definition 1.4 [2]: A collection$\tilde{\tau}$ of fuzzy subset of $\tilde{A}$, that is $\tilde{\tau} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if satisfied the following conditions:

- $\tilde{\varphi}, \tilde{A} \in \tilde{\tau}$.
- If $\tilde{B},\tilde{C} \in \tilde{\tau}$, then $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\} \in \tilde{\tau}$.
- If $\tilde{B}_i \in \tilde{\tau}$, then $\max\{\mu_{\tilde{B}_i}(x)\ i \in J\} \in \tilde{\tau}$.

The pair $(\tilde{A},\tilde{\tau})$ is said to be fuzzy topological space and every member of $\tilde{\tau}$ is said to be fuzzy open set in $\tilde{A}$,and a fuzzy set is called fuzzy closed set in iff its complement is fuzzy open set in $\tilde{A}$.

Remark 1.5 [2, 8]: If $(\tilde{A},\tilde{\tau})$ is a fuzzy topological space and $\tilde{B} \in p(\tilde{A})$, the complement of $\tilde{B}$ revered to $\tilde{A}$, denoted by $\tilde{B}^c$ is defined by $\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x) \forall x \in X$.

Definition 1.6 [1]: A subfamily of $\tilde{\tau}^*$ of $\tilde{A}$ is said to be fuzzy supra topology on $\tilde{A}$ if satisfied the following conditions**:**

- $\tilde{\varphi}, \tilde{A} \in \tilde{\tau}^*$.
- If $\tilde{B}_i \in \tilde{\tau}^*$, then $\max\{\mu_{\tilde{B}_i}(x), i \in J\} \in \tilde{\tau}^*$.

The pair $(\tilde{A},\tilde{\tau}^*)$ is said to be fuzzy supra topological space, the element of $\tilde{\tau}^*$ is said to be fuzzy supra open set in $\tilde{A}$, and the complement of fuzzy supra open set is called fuzzy supra closed set.
Remark 1.7 [4]: Every fuzzy topological space is a fuzzy supra topological space.

Definition 1.8 [4, 6, 7]: The support of a fuzzy set B̃ in Ã will be denoted by Supp(B̃) and defined by Supp(B̃) = $\{x \in X : \mu_{\tilde{B}}(x) > 0\}$.

Definition 1.9 [4, 6]: A fuzzy point x_r in X is a fuzzy set with membership function $\mu_{x_r}(x) = r$, if x= v where $0 < r \leq 1$ and $\mu_{x_r}(x) = 0$, if $x \neq$ v, such that v is called the support of x_r and r the value of x_r

Definition 1.10 [2, 5, 10]: Let B̃ , C̃ be a fuzzy sets in (Ã, $\tilde{\tau}$), then:

- A fuzzy Point x_r is said to be quasi coincident with a fuzzy set B̃ , if there exists $x \in X$ such that $\mu_{x_r} + \mu_{\tilde{B}}(x) > \mu_{\tilde{A}}(x)$ and denoted by $x_r\ q\ \tilde{B}$, if $\mu_{x_r}(x) + \mu_{\tilde{B}}(x) \leq \mu_{\tilde{A}}(x) \forall x \in X$, then x_r is not quasi coincident with a fuzzy set B̃ and denoted by $x_r\ \tilde{\bar{q}}\tilde{B}$.
- A fuzzy set B̃ is said to be quasi coincident with a fuzzy set C̃ , if there exists $x \in X$ such that $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) > \mu_{\tilde{A}}(x)$ and denoted by $\tilde{B}\ q\ \tilde{C}$, if $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) \leq \mu_{\tilde{A}}(x) \forall x \in X$, then B̃ is not quasi coincident with a fuzzy set C̃ and denoted by $\tilde{B}\ \tilde{\bar{q}}\tilde{C}$.

Proposition 1.11 [6, 11]: Let B̃, C̃, D̃ be any fuzzy sets in (Ã, $\tilde{\tau}$), then

- If $\min\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\varphi}}(x)\}$, then $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) \leq \mu_{\tilde{A}}(x)$
- $\mu_{\tilde{B}}(x) + \mu_{\tilde{B}}{}^{c}(x) \leq \mu_{\tilde{A}}(x)$
- $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) \leq \mu_{\tilde{A}}(x)$, $\mu_{\tilde{D}}(x) \leq \mu_{\tilde{C}}(x) \leq \mu_{\tilde{A}}(x)$ then $\mu_{\tilde{B}}(x) + \mu_{\tilde{D}}(x) \leq \mu_{\tilde{A}}(x)$.

2. Fuzzy supra semi open set

Definition 2.1: A fuzzy set B̃ of a fuzzy topological space (Ã,$\tilde{\tau}$) is said to be fuzzy supra s-open (fuzzy supra s-closed) sets if $\mu_{\tilde{B}}(x) \leq \mu_{\text{supracl (supraint } (\tilde{B}))}(x)$ $(\mu_{\tilde{B}}(x) \geq \mu_{\text{supraint (supracl } (\tilde{B}))}(x))$ $\forall$ x $\in$ X

The family of fuzzy supra s-open [fuzzy supra s-closed] sets is denoted by FSSO(Ã) [FSSC(Ã)] sets.

Definition 2.2: If B̃ is a fuzzy set in (Ã,$\tilde{\tau}^*$), then:

- supra s-closure of B̃ is denoted by (suprascl(B̃)) and defined by:$\mu_{\text{suprascl } (\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x)$: F̃ is a fuzzy supra s-closed set in Ã,$\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$.
- supra s-interior of B̃ is denoted by (suprasint(B̃)) and defined by $\mu_{\text{suprasint } (\tilde{B})}(x) = \max\{\mu_{\tilde{G}}(x)$: G̃ is a fuzzy supra s-open set in Ã, $\mu_{\tilde{G}}(x) \leq \mu_{\tilde{B}}(x)\}$.

Proposition 2.3: Every fuzzy supra open set (resp. fuzzy supra closed set) in (Ã,$\tilde{\tau}$) is a fuzzy supra s-open set (resp. fuzzy supra s-closed set) in (Ã,$\tilde{\tau}$).

Proof: Obvious

Remark 2.4: The converse of ***proposition 2.3*** is not true in general as shown in the following example.

Example 2.5: Let X={a, b, c} , Ã={(a,0.6), (b,0.4), (c,0.4)}
B̃={(a, 0.3), (b, 0.2), (c,0.2)}, C̃={(a, 0.5), (b, 0.4), (c,0.3)},
D̃={(a, 0.1), (b, 0.0), (c,0.1)}, be fuzzy sets in Ã,
$\tilde{\tau}$={$\tilde{\varphi}$,Ã,B̃}, be a fuzzy topology on Ã,
Then C̃ is a fuzzy supra s-open set but not fuzzy supra open set and D̃ is a fuzzy supra s-closed set but not fuzzy supra closed set.

Definition 2.6: A fuzzy set B̃ of a fuzzy topological space (Ã,$\tilde{\tau}$) is said to be fuzzy supra s-difference set (s-D set) if $\mu_{\tilde{B}}(x) = \mu_{\tilde{G}}(x) - \mu_{\tilde{H}}(x)$, where G̃ , H̃ are fuzzy supra s-open sets and $\mu_{\tilde{G}}(x) \neq \mu_{\tilde{A}}(x)$

Proposition 2.7: Every fuzzy supra s-open set is a fuzzy supra s-D-set
Proof: Obvious.

Remark 2.8: The converse of ***proposition 2.7*** is not true in general as shown in the following example.
Example 2.9: Let X={a, b, c} , Ã={(a,0.4), (b,0.5), (c,0.7)}
B̃={(a, 0.3), (b, 0.3), (c,0.4)}, C̃={(a, 0.3), (b, 0.2), (c,0.2)},
D̃={(a, 0.0), (b, 0.1), (c,0.2)}, be fuzzy sets in Ã,
$\tilde{\tau}$={$\tilde{\varphi}$,Ã,B̃,C̃}, be a fuzzy topology on Ã,
Then D̃ is a fuzzy supra s-D set but not fuzzy supra s- open set.

3. Fuzzy supra semi $\tilde{T}_{i=0,1,2}$ space

Definitions 3.1: A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is said to be:

- ***Fuzzy supra s-$\tilde{T}_0$ space*** if for every pair of distinct fuzzy points x_r, y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x), \mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, there exists fuzzy supra s-open set $\tilde{G}$ in $\tilde{A}$ such that either $\mu_{x_r}(x) < \mu_{\tilde{G}}(x), y_s \tilde{\bar{q}} \tilde{G}$ or $\mu_{y_s}(x) < \mu_{\tilde{G}}(x), x_r \tilde{\bar{q}} \tilde{G}$.

- ***Fuzzy supra s -$\tilde{T}_1$ space*** if for every pair of distinct fuzzy points x_r, y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, there exists two fuzzy supra s -open sets $\tilde{G}$, $\tilde{U}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x), y_s \tilde{\bar{q}} \tilde{G}$ and $\mu_{y_s}(x) < \mu_{\tilde{U}}(x), x_r \tilde{\bar{q}} \tilde{U}$.

- ***Fuzzy supra s -$\tilde{T}_2$ space*** if for every pair of distinct fuzzy points x_r, y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, there exists two fuzzy supra s -open sets $\tilde{G}$,$\tilde{U}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)$, $\mu_{y_s}(x) < \mu_{\tilde{U}}(x)$ and $\tilde{G} \tilde{\bar{q}} \tilde{U}$.

Propositions 3.2:

1. Every fuzzy supra s-$\tilde{T}_1$ space is a fuzzy supra s-$\tilde{T}_0$ space
2. Every fuzzy supra s-$\tilde{T}_2$ space is a fuzzy supra s-$\tilde{T}_1$ space

Proof: Obvious

Remark 3.3: The converse of ***propositions 3.2*** is not true in general as shown in the following examples.

Examples 3.4:

1. Let X={a, b} , $\tilde{A}$={(a,0.8), (b,0.7)},
   $\tilde{B}$={(a, 0.0), (b, 0.6)},$\tilde{C}$={(a, 0.0), (b, 0.7)}, be a fuzzy sets in $\tilde{A}$,
   $\tilde{\tau}$= {$\tilde{\varphi}$,$\tilde{A}$,$\tilde{B}$,$\tilde{C}$} be a fuzzy topology on $\tilde{A}$
   Then $(\tilde{A},\tilde{\tau})$ is fuzzy supra s-$\tilde{T}_0$ space but not fuzzy supra s-$\tilde{T}_1$space.
2. Let X={a, b} , $\tilde{A}$={(a,0.5), (b,0.4)},
   $\tilde{B}$={(a, 0.4), (b, 0.0)},$\tilde{C}$={(a, 0.0), (b, 0.1)},$\tilde{D}$={(a, 0.4), (b, 0.1)},
   $\tilde{E}$={(a, 0.4), (b, 0.4)}, $\tilde{F}$={(a, 0.0), (b, 0.4)}, $\tilde{G}$={(a, 0.5), (b, 0.1)}, be a fuzzy sets in $\tilde{A}$,
   $\tilde{\tau}$= {$\tilde{\varphi}$,$\tilde{A}$,$\tilde{B}$,$\tilde{C}$,$\tilde{D}$,$\tilde{E}$,$\tilde{F}$} be a fuzzy topology on $\tilde{A}$
   The FSSO($\tilde{A}$)={$\tilde{\varphi}$,$\tilde{A}$,$\tilde{B}$,$\tilde{C}$,$\tilde{D}$,$\tilde{E}$,$\tilde{F}$,$\tilde{G}$}
   Then $(\tilde{A},\tilde{\tau})$ is fuzzy supra s-$\tilde{T}_1$ space but not fuzzy supra s-$\tilde{T}_2$ space.

Theorem 3.5: A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is fuzzy supra s -$\tilde{T}_0$ space if and only if for each pair of distinct fuzzy points x_r, y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x), \mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, then $x_r \tilde{\bar{q}}$ suprascl(y_s) or $y_s \tilde{\bar{q}}$ suprascl(x_r)

Proof: Let $\mu_{x_r}(x) < \mu_{\tilde{A}}(x), \mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, then there exist a fuzzy supra s-open set $\tilde{G}$ in $\tilde{A}$ such that either $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)$, $\tilde{G} \tilde{\bar{q}} y_s$ or $\mu_{y_s}(x) < \mu_{\tilde{G}}(x)$,$\tilde{G} \tilde{\bar{q}} x_r$

If $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)$,$\tilde{G} \tilde{\bar{q}} y_s$ then $\tilde{G}^c \tilde{q} x_r$, $\mu_{y_s}(x) \leq \mu_{\tilde{G}^c}(x)$

Since $\tilde{G}^c$ is a fuzzy supra s-closed set

Therefore $\mu_{\text{suprascl }(y_s)}(x) \leq \mu_{\tilde{G}}{}^c(x)$

Hence $x_r \tilde{\bar{q}}$ suprascl(y_s)

Similarly if $\mu_{y_s}(x) < \mu_{\tilde{G}}(x)$,$\tilde{G} \tilde{\bar{q}} x_r$

Conversely, let $x_r \tilde{\bar{q}}$suprascl(y_s) or $y_s \tilde{\bar{q}}$ suprascl(x_r)

Then $\mu_{x_r}(x) \leq \mu_{(\text{suprascl }(y_s))^c}$ or $\mu_{y_s}(x) \leq \mu_{(\text{suprascl }(x_r))^c}$

If $\mu_{x_r}(x) \leq \mu_{(\text{suprascl }(y_s))^c}$ and since $y_s \tilde{\bar{q}}$ [suprascl(y_s)]^c

Then $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_0$ space

And if $\mu_{y_s}(x) \leq \mu_{(\text{suprascl }(x_r))^c}$ and since $x_r \tilde{\bar{q}}$ [suprascl(x_r)]^c

Then $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_0$ space.

Theorem 3.6: If $(\tilde{A},\tilde{\tau})$ is a fuzzy topological space then the following statements are equivalents:

1. $(\tilde{A},\tilde{\tau})$ is fuzzy supra s-$\tilde{T}_1$ space
2. For each two distinct fuzzy points, x_r , y_s then $x_r \tilde{\bar{q}}$ suprascl(y_s) and $y_s \tilde{\bar{q}}$ suprascl(x_r)

Proof: Obvious

Theorem 3.7: If $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_2$ space then for each two fuzzy points x_r, y_s in $\tilde{A}$ there exists two fuzzy supra s-closed sets $\tilde{F}_1$ and $\tilde{F}_2$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{F}_1}(x)$, $y_s \tilde{\bar{q}} \tilde{F}_1$, $\mu_{y_s}(x) < \mu_{\tilde{F}_2}(x)$,$x_r \tilde{\bar{q}} \tilde{F}_2$ and max{$\mu_{\tilde{F}_1}(x)$,$\mu_{\tilde{F}_2}(x)$}= $\mu_{\tilde{A}}(x)$

Proof: Obvious

Theorem 3.8: If $(\tilde{A},\tilde{\tau})$ is a fuzzy topological space then the following statements are equivalents:

1. $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_2$ space
2. For each two distinct fuzzy points x_r , y_s in $\tilde{A}$ there exist fuzzy supra s-open set $\tilde{G}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x) < \mu_{\mathrm{suprascl}\ (\tilde{G})}(x) < \mu_{y_s{}^c}(x)$

Proof:$(1) \rightarrow (2)$Let $(\tilde{A},\tilde{\tau})$ be a fuzzy supra s-$\tilde{T}_2$ space , x_r , y_s be two distinct fuzzy point such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$,$\mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, and $\tilde{G}$,$\tilde{U}$ are fuzzy s-open set in $\tilde{A}$

Such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)\mu_{y_s}(x) < \mu_{\tilde{U}}(x)$, and $\tilde{G}\tilde{\bar{q}}\tilde{U}$

Since $\mu_{\mathrm{suprascl}\ (\tilde{G})}(x)$= $\min\{\mu_{\tilde{U}^c}(x)$: $\tilde{U}^c$ is fuzzy supra s-closed set,$\mu_{\tilde{G}}(x) \leq \mu_{\tilde{U}^c}(x)\}$

Therefore $\mu_{x_r}(x) < \mu_{\tilde{G}}(x) < \mu_{\mathrm{suprascl}\ (\tilde{G})}(x) < \mu_{y_s{}^c}(x)$

$(2) \rightarrow (1)$Let x_r , y_s be a distinct fuzzy points in $\tilde{A}$ and $\tilde{G}$ be a fuzzy supra s-open set in $\tilde{A}$ such that $\mu_{y_s}(x) < \mu_{(\mathrm{suprascl}\ (\tilde{G}))^c}(x) < \mu_{x_r{}^c}(x)$

since suprasint($\tilde{G}^c$) is a fuzzy supra s-open set and $\mu_{\mathrm{suprasint}\ (\tilde{G}^c)}(x) \leq \mu_{\tilde{G}^c}(x)$,

Then there exists two fuzzy supra s-open sets $\tilde{G}$, suprasint($\tilde{G}^c$)

such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)$, $\mu_{y_s}(x) < \mu_{\mathrm{suprasint}\ (\tilde{G})^c}(x)$ and $\tilde{G}\,\tilde{\bar{q}}$suprasint($\tilde{G}^c$)

Hence the space $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_2$ space.

Theorem 3.9: A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_1$ space if for every fuzzy point is a fuzzy supra s-closed set.

Proof: Let x_r , $y_s$be two distinct fuzzy points in $\tilde{A}$ which are fuzzy supra s-closed set,
then x_r^c, y_s^c are fuzzy supra s-open sets
since $\mu_{x_r}(x) \leq \mu_{\mathrm{suprascl}\ (x_r)}(x)$, and $\mu_{y_s}(x) \leq \mu_{\mathrm{suprascl}\ (y_s)}(x)$,
Then $x_r\ \tilde{\bar{q}}$ [suprascl(x_r)]^c and $y_s\ \tilde{\bar{q}}$ [suprascl(y_s)]^c
Let $\mu_{\tilde{G}}(x)$=$\mu_{(\mathrm{suprascl}\ (x_r))^c}(x)$ and $\mu_{\tilde{U}}(x)$=$\mu_{(\mathrm{suprascl}\ (y_s))^c}(x)$
Hence $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_1$ space.

Remark 3.10: The converse of ***theorem 3.9*** is not true in general as shown in the following example.

Example 3.11: The space $(\tilde{A},\tilde{\tau})$ in the ***examples 3.4(2)*** is a fuzzy supra s-$\tilde{T}_1$ space but $a_{0.2}$ is not fuzzy supra s-closed set.

Theorem 3.12: If $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_2$ space then for each two fuzzy points x_r ,y_s in $\tilde{A}$ there exists two fuzzy supra s- closed sets $\tilde{F}_1$ and $\tilde{F}_2$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{F}_1}(x)$, $y_s\ \tilde{\bar{q}}\ \tilde{F}_1$, $\mu_{y_s}(x) < \mu_{\tilde{F}_2}(x)$,$x_r\ \tilde{\bar{q}}\ \tilde{F}_2$ and $\max\{\mu_{\tilde{F}_1}(x)$,$\mu_{\tilde{F}_2}(x)\}$= $\mu_{\tilde{A}}(x)$

Proof: Obvious

4. Fuzzy supra semi $\mathbf{D_{i=0,1,2}}$ space

Definition 4.1:A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is said to be fuzzy supra s-D_0 space if for every pair of distinct fuzzy points x_r,y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$,$\mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, there exists fuzzy supra s-D set $\tilde{B}$ in $\tilde{A}$ such that either $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s\ \tilde{\bar{q}}\ \tilde{B}$ or $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$, $x_r\ \tilde{\bar{q}}\ \tilde{B}$.

Example 4.2: The space $(\tilde{A},\tilde{\tau})$ in the ***examples 3.4(1)*** is a fuzzy supra s-D_0 space.

Theorem 4.3: If $(\tilde{A},\tilde{\tau})$ is a fuzzy topological space then the following statements are equivalents:

1. $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-D_0 space
2. $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_0$ space

Proof:

$(1) \rightarrow (2)$ Let $(\tilde{A},\tilde{\tau})$ be a fuzzy supra s-D_0 space,
Then for each distinct fuzzy points x_r,y_s $\tilde{\in}$ $\tilde{A}$, there exist fuzzy supra s-D set $\tilde{B}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s\ \tilde{\bar{q}}\ \tilde{B}$or $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$, $x_r\ \tilde{\bar{q}}\ \tilde{B}$
Since $\tilde{B}$ is a fuzzy supra s-D set, then $\mu_{\tilde{B}}(x)$=$\mu_{\tilde{G}}(x) - \mu_{\tilde{H}}(x)$,where $\tilde{G}$,$\tilde{H}$ are fuzzy supra s-open set
If $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s\ \tilde{\bar{q}}\ \tilde{B}$
Then $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)$, $x_r\ \tilde{\bar{q}}\ \tilde{H}$ ……..(*)
Since $y_s\ \tilde{\bar{q}}\ \tilde{B}$ then $y_s\ \tilde{\bar{q}}\ \tilde{G}$ or $\mu_{y_s}(x) < \mu_{\tilde{G}}(x)$ and $\mu_{y_s}(x) < \mu_{\tilde{H}}(x)$,
If $y_s\ \tilde{\bar{q}}\ \tilde{G}$ and by (*) we get $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_0$ space
and if $\mu_{y_s}(x) < \mu_{\tilde{H}}(x)$, and by (*) we get $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-$\tilde{T}_0$ space
Similarly if $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$, $x_r\ \tilde{\bar{q}}\ \tilde{B}$
$(2) \rightarrow (1)$Obvious.

Definition 4.4: A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is said to be fuzzy supra s-D_1 space if for every pair of distinct fuzzy points x_r , y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, there exists two fuzzy supra s-D sets $\tilde{B}$,$\tilde{C}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s \tilde{\bar{q}}\, \tilde{B}$ and $\mu_{y_s}(x) < \mu_{\tilde{C}}(x)$, $x_r \tilde{\bar{q}}\, \tilde{C}$.
Proposition 4.5: Every fuzzy supra s- $\tilde{T}_1$ space is a fuzzy supra s-D_1 space.

Proof: Obvious

Remark 4.6: The converse of ***proposition 4.5*** is not true in general as shown in the following example.

Example 4.7: Let X={a, b, c} , $\tilde{A}$={(a,0.6), (b,0.5), (c, 0.4)},$\tilde{B}$={(a, 0.6), (b, 0.1), (c, 0.0)}, $\tilde{C}$={(a, 0.1), (b, 0.5), (c, 0.0)},$\tilde{D}$={(a, 0.6), (b, 0.5), (c, 0.0)},$\tilde{E}$={(a, 0.1), (b, 0.1), (c, 0.0)}, $\tilde{F}$={(a, 0.6), (b, 0.4), (c, 0.4)},$\tilde{G}$={(a, 0.1), (b, 0.4), (c, 0.0)},$\tilde{H}$={(a, 0.6), (b, 0.4), (c, 0.0)}, $\tilde{I}$={(a, 0.0), (b, 0.0), (c, 0.4)},be a fuzzy sets in $\tilde{A}$, $\tilde{\tau}$={$\tilde{\varphi}$,$\tilde{A}$,$\tilde{B}$,$\tilde{C}$, $\tilde{D}$,$\tilde{E}$,$\tilde{F}$,$\tilde{G}$,$\tilde{H}$}, be a fuzzy topology on $\tilde{A}$
Then $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-D_1 space but not fuzzy supra s-$\tilde{T}_1$ space.
Proposition 4.8: Every fuzzy supra s-D_1 space is a fuzzy supra s-D_0 space.

Proof: Obvious

Remark 4.9: The converse of ***proposition 4.8*** is not true in general as shown in the following example.

Example 4.10: The space $(\tilde{A},\tilde{\tau})$ in the ***examples 3.4(1)*** is a fuzzy supra s-D_0 space but not fuzzy supra s-D_1 space.

Definition 4.11: A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is said to befuzzy supra s-D_2space if for every pair of distinct fuzzy points x_r,y_s such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_s}(x) < \mu_{\tilde{A}}(x)$, there exists two fuzzy supra s-D sets $\tilde{B}$,$\tilde{C}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $\mu_{y_s}(x) < \mu_{\tilde{C}}(x)$, and $\tilde{B}\, \tilde{\bar{q}}\, \tilde{C}$.
Proposition 4.12: Every fuzzy supra s-$\tilde{T}_2$ space is a fuzzy supra s-D_2 space.
Proof: Obvious
Remark 4.13: The converse of ***proposition 4.12*** is not true in general as shown in the following example.
Example 4.14: Let X={a, b, c, d},$\tilde{A}$ = {(a,0.4), (b,0.4), (c,0.4), (c,0.4)},$\tilde{B}$={(a, 0.4), (b, 0.0), (c, 0.0), (d, 0.0)}, $\tilde{C}$={(a, 0.4), (b, 0.4), (c, 0.0), (d, 0.0)},$\tilde{D}$={(a, 0.4), (b, 0.4), (c, 0.4), (d, 0.0)}, $\tilde{E}$={(a, 0.4), (b, 0.0), (c, 0.0), (d, 0.4)},$\tilde{F}$={(a, 0.4), (b, 0.4), (c, 0.0), (d, 0.4)}, $\tilde{G}$={(a, 0.0), (b, 0.4), (c, 0.0), (d, 0.0)},$\tilde{H}$={(a, 0.0), (b, 0.0), (c, 0.4), (d, 0.0)}, $\tilde{I}$={(a, 0.0), (b, 0.0), (c, 0.0), (d, 0.4)}, be a fuzzy sets in $\tilde{A}$,$\tilde{\tau}$={$\tilde{\varphi}$,$\tilde{A}$,$\tilde{B}$, $\tilde{C}$,$\tilde{D}$,$\tilde{E}$,$\tilde{F}$}, be a fuzzy topology on $\tilde{A}$,
Then $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-D_2 space but not fuzzy supra s-$\tilde{T}_2$ space.
Proposition 4.15: Every fuzzy supra s-D_2 space is a fuzzy supra s-D_1 space.

Proof: Obvious

Remark 4.16: The converse of ***proposition 4.15*** is not true in general as shown in the following example.

Example 4.17: The space $(\tilde{A},\tilde{\tau})$ in the ***example 4.7*** is a fuzzy supra s-D_1 space but not fuzzy supra s-D_2 space.

Theorem 4.18: A fuzzy topological space $(\tilde{A},\tilde{\tau})$ is a fuzzy supra s-D_2 space if for each two distinct fuzzy points x_r , $y_s$in $\tilde{A}$ there exist fuzzy supra s-open set $\tilde{G}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x) < \mu_{\text{suprascl }(\tilde{G})}(x) < \mu_{y_s}{}^{c}(x)$
Proof: Obvious
Propositions 4.19:

1. Every fuzzy supra s-D_2 space is a fuzzy supra s-$\tilde{T}_0$ space.
2. Every fuzzy supra s-D_1 space is a fuzzy supra s-$\tilde{T}_0$ space.

Proof: Obvious

Remark 4.20: The converse of ***propositions 4.19*** is not true in general as shown in the following example.

Example 4.21: The space $(\tilde{A},\tilde{\tau})$ in the ***examples 3.4(1)*** is a fuzzy supra s-$\tilde{T}_0$ space but not fuzzy supra s-D_2 space, fuzzy supra s-D_1 space.

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