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Free vibration of laminated composite stiffened saddle shell roofs with cutouts

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ABSTRACT : *In this paper, the finite element method has been applied to solve free vibration problems of laminated composite stiffened saddle shells with cutouts employing the eight-noded curved quadratic isoparametric element for shell with a three noded beam element for stiffener formulation. Specific numerical problems of earlier investigators are solved to compare their results. Moreover, free vibration problem of stiffened saddle shells with different size and position of the cutouts with respect to the shell centre for different edge constraints are examined to arrive at some conclusions useful to the designers. The results are presented in the form of figures and tables. The results are further analyzed to suggest guidelines to select optimum size and position of the cutout with respect to shell centre considering the different practical constraints.*

Keywords - *stiffened saddle shell, cutout, finite element, free vibration, laminated composites.*

I. INTRODUCTION

Among the different shell forms which are used as roofing units, saddle shells are one of them. Examples of such saddle roofs are: Warszawa Ochota railway station, Church Army Chapel, Blackheath, The Calgary, Saddledome, London Velopark. Quite often, to save weight and also to provide a facility for inspection, cutouts are provided in shell panels. In practice the margin of the cutouts must be stiffened to take account of stress concentration effects. Also, there can be some instruments directly fixed on these panels, and the safety of these instruments can be dependent on the vibration characteristics of the panels. Hence free vibration studies on saddle shell panels with cutouts are of interest to structural engineers.

Free vibration study of doubly curved shells was done by Qatu [1], Liew and Lim [2], Chakravorty et. al [3], Tan [4], and Kant et al [5]. Later, different researchers worked on doubly curved shells from time to time. Qatu et al. [7] reviewed the work done on the vibration aspects of composite shells during 2000 - 2009 and observed that most of the researchers dealt with closed cylindrical shells. Other shell geometries have also been investigated. But, saddle shells on rectangular planform with cutout (stiffened along the margin) are far from complete in the existing literature. Accordingly, the present endeavor focuses on the free vibration behavior of composite saddle shell with cutout (stiffened along the margin) with concentric and eccentric cutouts, and considers the shells to have various boundary conditions.

II. MATHEMATICAL FORMULATION

A laminated composite saddle shell of uniform thickness h (Fig.1) and radius of curvature R_x and R_y is considered. Keeping the total thickness the same, the thickness may consist of any number of thin lamina each of which may be arbitrarily oriented at an angle θ with reference to the X-axis of the co-ordinate system. An eight-noded curved quadratic isoparametric finite element is used. The five degrees of freedom taken into consideration at each node include two in-plane and one transverse displacement and two rotations about the X and Y axes. The detailed finite element formulation for doubly curved shells with cutout stiffened along the margin of the cutout is reported elsewhere [8]. The code developed can take the position and size of cutout as input.

III. VALIDATION STUDY

The present finite element approach is capable of modelling free vibration problem of composite stiffened saddle shells with cutout which is provided by solution of benchmark problems. The validation of stiffener formulation is reported elsewhere [8]. The close agreement of the present results (Table 1) with benchmark one [3] proves the correctness of present formulation.

IV. RESULTS AND DISCUSSION

In order to study the effect of cutout size and position on the free vibration response additional problems for saddle shells with different boundary conditions with 0/90/0/90 and +45/-45/+45/-45 laminations have been solved. The positions of the cutouts are varied along both of the plan directions of the shell to study the effect of eccentricity of cutout on the fundamental frequency.

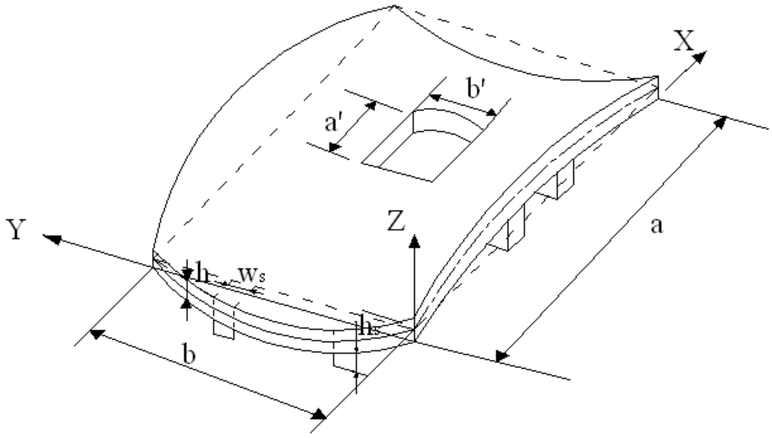

Fig 1 Saddle shell with a concentric cutout stiffened along the margins

Table 1: Non-dimensional fundamental frequencies ($\bar{\omega}$) for laminated composite saddle shell with cutout

a'/a	CS		SS		CL	
	Chakravorty et. al. [3]	Present model	Chakravorty et. al. [3]	Present model	Chakravorty et. al. [3]	Present model
0.0	13.485	13.249	14.721	14.686	113.567	112.926
0.1	13.060	13.084	14.350	14.695	97.753	98.041
0.2	12.530	12.449	13.544	13.507	97.599	97.032
0.3	12.016	12.038	12.908	12.882	111.489	111.033
0.4	11.721	11.733	12.560	12.559	110.210	110.20

a/b=1, a/h=100, $a^{/}/b^{/}$=1, h/R_{xx}= $-h/R_{yy}$=1/300, CS=Corner point supported, SS=Simply supported, CL=Clamped

4.1 Free vibration behaviour of shells with concentric cutouts

Tables 2 and 3 show the results of non-dimensional frequency ($\bar{\omega}$) of 0/90/0/90 and +45/-45/+45/-45 stiffened saddle shells with cutout respectively. The shells considered are of square plan form (a=b) and the cutouts are also taken to be square in plan ($a^{/}$=$b^{/}$). The cutout sizes (i.e. $a^{/}$/a) are varied from 0 to 0.4 and boundary conditions are varied along the four edges. Cutouts are concentric on shell surface. The boundary conditions are designated by describing the support clamped or simply supported as C or S taken in an anticlockwise order from the edge x=0. This means a shell with CSCS boundary is clamped along x=0, simply supported along y=0 and clamped along x=a and simply supported along y=b. The stiffeners are placed along the cutout periphery and extended up to the edge of the shell. The material and geometric properties of shells and cutouts are mentioned along with the Tables. From the tables it is seen that when a cutout is introduced to a stiffened shell the fundamental frequency increases in all the cases. This increasing trend is noticed for both cross ply and angle ply shells.

Table 2: Non-dimensional fundamental frequencies ($\bar{\omega}$) for laminated composite (0/90/0/90) stiffened saddle shell for different sizes of the central square cutout and different boundary conditions.

Boundary conditions	Cutout size (a'/a)				
	0	0.1	0.2	0.3	0.4
CCCC	93.08	107.18	129.7	135.35	138.56
CSCC	79.68	92.13	108.7	120.34	121.57
CSSC	61.39	72.13	81.88	87.4	91.17
CSCS	84.89	87.82	107.4	116.61	118.1
CSSS	57.39	69.92	76.06	80.13	83.51
SSSS	49.1	63.99	68.27	71.36	73.77

a/b=1, a/h=100, $a^{/}/b^{/}$=1, h/R_{xx}=- h/R_{yy}=1/300; E_{11}/E_{22} =0 25, G_{23} = 0.2E_{22}, G_{13} = G_{12} = 0.5E_{22}, ν_{12} =ν_{21} =0.25.

Table 3: Non-dimensional fundamental frequencies ($\bar{\omega}$) for laminated composite (+45/-45/+45/-45) stiffened saddle shell for different sizes of the central square cutout and different boundary conditions.

Boundary conditions	Cutout size (a'/a)				
	0	0.1	0.2	0.3	0.4
CCCC	81.34	98.71	103.2	107.23	112.06
CSCC	75.81	91.13	95.02	98.14	101.27
CSSC	68.36	81.17	85.72	89	92.23
CSCS	73	87.56	91.25	94.6	97.53
CSSS	64.37	77.65	81.45	84.33	87.34
SSSS	51.61	67	71.16	74.09	77.28

a/b=1, a/h=100, $a^{/}/b^{/}$=1, h/R_{xx}=- h/R_{yy}=1/300; E_{11}/E_{22} =0 25, G_{23} = 0.2E_{22}, G_{13} = G_{12} = 0.5E_{22}, ν_{12} =ν_{21} =0.25.

4.2 Free vibration behaviour of shells with eccentric cutouts

To study the effect of cutout positions on fundamental frequencies, results are obtained for different locations of a cutout with $a^{/}/a$ =0.2. Each of the non-dimensional coordinates of the cutout centre ($\bar{x} = x/a$, $\bar{y} = y/b$) is varied from 0.2 to 0.8 along both the plan directions. so that the distance of a cutout margin from the boundary is not less than one tenth of the plan dimension. The study is carried out for both 0/90/0/90 and +45/-45/+45/-45 saddle shells, but for the sake of brevity only the results of 0/90/0/90 saddle shell is presented here. The ratio of the fundamental frequency of a shell with an eccentric puncture to that of a shell with concentric puncture expressed in percentage is denoted by r. Table 4 contains the value of r for 0/90/0/90 saddle shells.

Table 4: Values of 'r' for 0/90/0/90 saddle shells

Edge condition	$\bar{y}$	$\bar{x}$						
		0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0.2	91.20	85.18	88.20	91.75	88.17	85.18	85.03
	0.3	84.39	85.22	89.00	93.85	89.00	85.22	84.39
	0.4	86.94	88.31	92.41	96.84	92.41	88.31	86.94
	0.5	90.16	91.96	96.15	100.00	96.15	91.96	90.16
	0.6	86.94	88.31	92.41	96.84	92.41	88.31	86.94
	0.7	84.39	85.22	89.00	93.64	89.00	85.22	84.39
	0.8	84.76	84.97	88.01	91.77	88.13	85.07	87.34
CSCC	0.2	91.12	90.60	91.90	93.11	91.90	90.60	91.10
	0.3	94.86	95.19	97.57	99.66	97.57	95.18	94.86

	0.4	95.93	99.74	105.62	109.00	106.03	99.76	95.88
	0.5	93.08	93.79	97.03	100.00	97.33	93.89	92.79
	0.6	86.93	85.83	87.77	89.93	87.92	85.92	86.50
	0.7	82.66	81.28	82.85	84.66	82.94	81.34	82.26
	0.8	80.75	79.37	80.75	82.34	80.82	79.43	80.40
CSSC	0.2	67.33	73.00	81.56	92.87	96.24	87.04	78.21
	0.3	72.96	79.10	88.04	99.06	101.48	92.50	83.52
	0.4	76.94	83.79	93.47	104.13	105.86	97.08	87.64
	0.5	73.28	78.60	87.24	100.00	104.47	94.43	83.29
	0.6	65.80	70.00	77.99	91.48	98.34	87.23	77.64
	0.7	60.39	64.56	72.67	86.24	92.93	81.74	72.47
	0.8	57.62	62.10	70.46	83.79	89.52	78.58	69.52
CSCS	0.2	78.77	78.00	78.58	79.36	78.58	78.00	78.77
	0.3	81.64	80.85	81.94	83.18	81.94	80.85	81.64
	0.4	86.58	86.04	87.77	89.54	87.77	86.04	86.58
	0.5	92.51	94.34	97.12	100.00	97.09	94.34	92.51
	0.6	86.58	86.04	87.77	89.54	87.77	86.04	86.58
	0.7	81.64	80.85	81.94	83.18	81.94	80.85	81.64
	0.8	78.76	78.00	78.58	79.35	78.58	78.00	78.77
CSSS	0.2	53.39	58.77	68.30	82.91	90.38	78.57	68.38
	0.3	58.51	63.66	72.90	87.51	94.99	83.39	73.30
	0.4	65.33	70.81	80.02	94.19	100.46	89.57	79.40
	0.5	70.35	77.20	87.33	100.00	103.67	93.82	83.51
	0.6	65.33	70.81	80.02	94.20	100.46	89.57	79.40
	0.7	58.51	63.66	72.89	87.51	94.99	83.39	73.30
	0.8	53.31	58.73	68.25	82.84	90.28	78.49	68.28
SSSS	0.2	50.81	60.23	73.00	87.40	73.00	60.25	50.81
	0.3	56.41	65.40	77.74	91.59	77.74	65.40	56.41
	0.4	63.78	72.87	84.90	96.78	84.90	72.87	63.78
	0.5	69.18	79.20	91.26	100.00	91.26	79.20	69.18
	0.6	63.78	72.87	84.90	96.78	84.90	72.87	63.78
	0.7	56.41	65.40	77.75	91.59	77.74	65.40	56.41
	0.8	50.71	60.19	72.95	87.33	72.96	60.19	50.73

$a/b=1$, $a/h=100$, $a'/b'=1$, $h/R_{xx}=- h/R_{yy}=1/300$; E_{11}/E_{22} =0 25, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} =\nu_{21} =0.25$.

Table 5 provides the maximum values of r together with the position of the cutout. These tables also show the rectangular zones within which r is always greater than or equal to 90. It is to be noted that at some other points r values may have similar values, but only the zone rectangular in plan has been identified. These tables indicate the maximum eccentricity of a cutout which can be permitted if the fundamental frequency of a concentrically punctured shell is not to reduce a drastic amount. So these tables will help practicing engineers.

Table 5: Maximum values of r with corresponding coordinates of cutout centres and zones where r≥90 for 0/90/0/90 saddle shells

Boundary	Maximum	Co-ordinate of cutout centre	Area in which the value of r≥90

Condition	values of r		
CCCC	100.00	(0.5,0.5)	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$.
CSCC	109.00	(0.5, 0.4)	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.5$.
CSSC	105.86	(0.6, 0.4)	$0.5 \leq \bar{x} \leq 0.6$, $0.2 \leq \bar{y} \leq 0.6$.
CSCS	100.00	(0.5, 0.5)	$0.2 \leq \bar{x} \leq 0.8$, $\bar{y} = 0.5$.
CSSS	103.67	(0.6, 0.5)	$0.5 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq .0.6$.
SSSS	100.00	(0.5, 0.5)	$\bar{x} = 0.5$, $0.3 \leq \bar{y} \leq 0.7$.

a/b=1, a/h=100, $a^{/}/b^{/}=1$, $h/R_{xx}=- h/R_{yy}=1/300$; E_{11}/E_{22} =0 25, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$.

V. CONCLUSION

From the present study the following conclusions are drawn.

1. The finite element code used here is suitable for analyzing free vibration problems of stiffened saddle shell roofs with cutouts as this approach produce results in close agreement with those of the benchmark problems.
2. Concentric cutouts may be provided safely on stiffened saddle shell surfaces for functional requirements upto $a^{/}/a=0.4$.
3. The information regarding behaviour of stiffened saddle shell with eccentric cutouts for wide range of eccentricity may be used as design aids by structural engineers.

Notations

a, b	length and width of the saddle shell
E_{11}, E_{22}	Young's moduli
G_{12}, G_{13}, G_{23}	shear moduli
h	shell thickness
ω	fundamental frequency
$\bar{\omega} = \omega a^2 \left(\rho / E_{11} h^2\right)^{1/2}$	non-dimensional fundamental frequency
ρ	density of material
ν_{12}, ν_{21}	Poisson's ratios

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