

Study on Effect of Soil Inertia Contribution on Dynamic Response of a Single Pile

R. Ravikumar¹, Dr. M.V. Ramarao²

¹*Ph.D Scholar, Osmania collage of engineering, Hyderabad – 500033, India*

²*Professor, Department of Civil engineering, Vasavi Collage of Engineering, Hyderabad – 500031, India*

ABSTRACT: *To adequately address pile response under earthquake actions, or to analyze heavy vibratory machine foundations, it is often required to perform a dynamic analysis of the pile for transverse (lateral) vibrations. For a dynamic analysis it is critical to have an adequate representation of the system stiffness (force-deflection relationships), and adequate representation of the system mass involved in the vibration phenomena. The most widely used model to perform the analysis of piles under lateral loads consists in modeling the pile as a series of beam elements, and considering the pile-soil interaction by representing the soil as a group of unconnected, concentrated springs perpendicular to the pile (Discrete Winkler Model). The literature review shows that the soil around the pile deforms in pure shear and that soil deformations are negligible beyond a radial distance of 10times radius of the pile.*

The main objective of this investigation is to study the importance of the soil mass in the soil-pile system response and to study the soil contribution from zero to 3-times the pile radius to the inertial properties of a soil-pile system under dynamic lateral loads, through lumped masses with the discrete Winkler model and to evaluate the importance of such lumped mass in the system response.

Keywords: *Dynamic Lateral Response, Soil Inertia, Added Mass Co-efficient, Single Pile*

I. INTRODUCTION

Pile foundations are generally used to support structures on soft soil with low bearing capacity. The pile foundations have to be designed to support lateral loads due to earthquakes, wind, and vehicle impact loads, among others. The most widely used model to perform the analysis of piles under lateral loads consists the pile as a series of beam elements, and considering the pile-soil interaction by representing the soil as a group of unconnected, concentrated springs perpendicular to the pile (Discrete Winkler Model). For a dynamic analysis it is critical to have an adequate representation of the system stiffness (force-deflection relationships), and adequate representation of the system mass involved in the vibration phenomena. The literature review shows that the soil deformations are negligible beyond a radial distance of 10times radius of the pile also recommend that for lateral analysis the added mass co-efficient (C_m) can be consider 3.0 times dia. of pile and for axial it is recommended 4.0 times the dia of the pile. Stiffness and damping properties are included in a dynamic analysis through lumped springs and dashpots.

This investigation is to study the importance of the soil mass in the soil-pile system response and to study the effect of soil contribution in the inertial properties of a soil-pile system under dynamic lateral loads through lumped masses with the discrete Winkler mode.

II. DYNAMIC ANALYSIS OF PILES UNDER LATERAL LOADS

Similar to the static lateral analysis approaches the following are the three major approaches for the dynamic analysis of laterally loaded piles. Namely Finite Element Method (FEM) and the Boundary Element Method (BEM) that treat the soil as a continuous medium, the Beam on Nonlinear Winkler Foundation (BNWF), that treats the soil as a series of disconnected springs, and the Continuum Approach, that provides closed form solutions by considering the soil as an infinite semi-space. A brief description of these approaches is presented in this section.

- a) Winkler Approach or Beam on Elastic Foundation Approach
- b) Continuum Approach.
- c) Finite Element Approach.

2.1. Winkler Approach or Beam on Elastic Foundation Approach

This approach was originally proposed by Winkler in 1867. The model, also known as Beam on Elastic Foundation (BEF) and Beam on Winkler Foundation (BWF), Considering that both piles and soil can behave in a nonlinear manner during extreme events, the use of p-y methods for defining the lateral stiffness of pile-soil model for Lateral analysis has been used since the seventies (Matlock et al., 1978).

2.1.1 Mass

The mass to be assigned to nodes along a pile should include the mass of the pile, the mass of soil inside the pile in case of group piles and some added (or virtual) mass representing the portion of surrounding soils accelerated by the pile. The added mass is not easy to compute. Moreover, it is not a constant quantity, but varies with the induced deformations. Rough estimates may be obtained by assuming a strain field around the pile and integrating the squares of the corresponding displacements over the volume of the assumed strain field. In this manner, the added mass M_a per unit length of pile may be expressed as

$$M_a = C_m \rho \pi r_o^2 = \iint_A u^2 dA \tag{1}$$

where C_m is the added mass coefficient, ρ the soil density, r_o the pile radius, u the displacement field in the soil (normalized to unity at the pile surface) and A the near field area contributing to the added mass. In this paper the study is carried out by varying the added soil mass coefficient from $0.50r_o$ to $3r_o$ and the results are compared.

2.1.2 Damping

There are two main sources of damping in the soil, material and radiation damping. Material damping is predominantly of a hysteretic nature and is explicitly accounted for by the nonlinear force-deformation relationships of the near field elements (except at low strains at which the nonlinear soil reactions have been linearized). Radiation damping is due to energy dissipation by waves propagating in the half-space. This type of damping is simulated by the viscous dashpots associated with the lateral and axial nearfield soil elements.

In this paper main focus is given on the soil inertia contribution on dynamic effect of the soil, so the material damping is considered in default concrete nonlinear damping ratio.

2.1.3 Lateral motion

For lateral motion, the displacement field resulting from the Mindlin solution for a horizontal force P acting parallel to the horizontal axis (X-axis) within a linear, isotropic half-space was assumed. To further simplify the computations, displacement components perpendicular to the direction of loading were neglected. For Poisson's ratio equal to 0.5, Mindlin's equation takes the form (in cylindrical coordinates)

$$u_x(r, \theta, z) = \frac{3P}{8\pi E} \left\{ \left(\frac{1}{S_1^{0.5}} + \frac{1}{S_2^{0.5}} + \frac{2bz}{S_2^{1.5}} \right) + r^2 \cos^2 \theta \left(\frac{1}{S_1^{1.5}} + \frac{1}{S_2^{1.5}} - \frac{6bz}{S_2^{2.5}} \right) \right\} \tag{2}$$

where u_x is the horizontal displacement of the half-space, E is Young's modulus for the half-space, b is the depth of the force P from the free surface, and $S_1 = r^2 + (z - b)^2$ and $S_2 = r^2 + (z + b)^2$. Added mass coefficients have been obtained from “equations (1) and (2)” for various depths ($b = z = kr_o$) and circular areas of integration ($r = \lambda r_o$), and these are summarized in *Table 1* below. The rather wide range of values should not cause any great concern, because the dynamic response is quite insensitive to changes in the mass associated with nodes along the piles. For practical applications, a constant value of $C_m \approx 3.0$ appears to be a reasonable choice for lateral motion and $C_m \approx 4.0$ for an axial motion.

Table1: Values of added mass coefficient for Horizontal & Axial pile motion

Λ	k (Horizontal Pile Motion)						μ (Axial Pile Motion)				
	2	5	10	20	50	100	0	0.50	0.67	0.80	0.95
6	3.25	2.86	2.53	2.34	2.22	2.17	3.89	3.08	2.65	2.21	1.35
8	3.94	3.51	3.07	2.79	2.60	2.54	5.80	4.52	3.87	3.21	1.96

10	4.49	4.07	3.54	3.16	2.91	2.83	7.90	6.09	5.21	4.32	2.65
20	6.30	5.98	5.23	4.51	3.95	3.76	---	---	---	---	---

λ = radius of integration/pile radius, k = depth of pile segment/pile radius, $\mu = \tau_{max}/c$

3.0 Parametric Study

This analysis is carried out based on the assumptions that the soil around the pile deforms in pure shear and that soil deformations are negligible beyond a radial distance of $10r_o$ from the pile axis, where r_o is the pile radius.

3.1 Analytical Model

Pile is modeled as a frame element with 3-degrees of freedom at each node divided into 12 elements based on the soil layer as shown in Fig.1. Each soil layer is divided separately with a node. The soil stiffness is modeled as an equivalent series of disconnected nodal springs. The soil stiffness is considered by modifying the element stiffness matrix by adding equivalent soil spring stiffness. A computer program is developed in MAT-LAB computer software version R2010a and the analysis results like Translations, Rotations and Bending moments are validated with standard text book example given in Foundation Analysis and Design by Joseph E Bowels 5th edition. Also all the results are compared and validated with the standard finite element software STAAD Pro.V8i and presented in Table-2.

3.2 Soil Properties

Study is carried out by considering soil profiles from one of the India’s prestigious project. All the soil properties like type of the soil layer, standard penetration number (N), poisson ratio are (μ) are tabulated as shown in Table-3 below. And the empirical formulae used to calculate the engineering properties of the soil like Soil elastic modulus (Es), Subgrade soil modulus (Ks) and spring stiffness (Ki) are also shown in below table.

Table 2: Comparison of Results for Computer Program with Text book results

Node No	Translations (m)		Rotations (Rads)		Bending Moment (Kn-m)	
	Bowels	Computer Program	Bowels	Computer Program	Bowels	Computer Program
1	0.00544	0.0054330	0.00299	0.002978	0.000	0.000
2	0.00445	0.0044300	0.00292	0.002906	29.855	29.857
3	0.00360	0.0035950	0.00274	0.00273	52.463	52.470
4	0.00212	0.0021140	0.00217	0.00217	78.643	78.691
5	0.00102	0.0010160	0.00149	0.001487	80.827	80.931
6	0.00032	0.0003193	0.00086	0.000856	66.255	66.404
7	0.00004	0.0000400	0.00038	0.00038	44.799	44.962
8	0.00018	0.0001800	0.0000252	0.000025	11.754	11.865
9	0.00008	0.0000800	0.00009	0.00009	4.037	4.020
10	0.00000	0.0000000	0.00003	0.00003	2.094	2.110
11	0.00000	0.0000000	0.00001	0.00001	0.523	0.527
12	0.00000	0.0000000	0.00000	0.00000	0.101	0.102
13	0.00000	0.0000000	0.00000	0.00000	0.000	0.000

Table 3. Soil Properties from Bore Log

SOIL TYPE	Depth - Z (m)	N Value	Formulae Used to Calculate Engineering Properties of Soil		
	0.00	19.00			
Clay Silt	0.5 to 3.10	19.00	Soil Modulus	Clay	300 (N+6) kPa
Silt with Fine	4.10	49.00		Clayey/silty	320

Sand				sand	(N+15) kPa
Silt with Fine Sand	5.10	49.00			
Silt with Fine Sand	6.50	49.00	Subgrade Modulus of Soil	$K_s' = 1.3 \left[\frac{(E_s \times B^4)}{(E_p \times I_p)} \right]^{1/12} \times (E_s / (1-m^2))$	
Silt with Fine Sand	8.00	49.00			
Silt with Kankar	10.00	16.00		$K_s = K_s' * Z^n / B$	
Silt with Kankar	12.50	16.00			
Silt with Kankar	14.90	16.00			
Silt with Kankar	17.40	16.00	Soil Spring Stiffness Calculations	$K_{s1} = BL/6 \times (2 \times K_{si} + K_{si+1})$	
Silt with Kankar	20.2 to 23.30	29.00		$K_{s2} = BL/6 \times (2 \times K_{si} + K_{si-1})$	
Silt with Kankar	26.10	34.00			
Clay & Gravel	28.50	58.00			

3.3 Structural properties

Single reinforced concrete pile with constant diameter is considered and analyzed. The structural properties of the pile used for the analysis are Dia. of the pile (D_p) = 0.40m, Total length of the pile (L_p) = 28.50m, Grade of Concrete = M30 and Modulus of Elasticity of Pile (E_p) is obtained by using = 5000 x sqrt (F_{ck}).

III. RESULTS AND DISCUSSIONS

Based on the geotechnical data, analysis is carried out by considering the soil added mass co-efficient values vary from $0.5r_0$ to $3.0r_0$. The results are compared below for without soil mass condition with the varied added soil mass co-efficient conditions.

Table 4. Comparisons of Frequencies (Hz)

Added Mass Index	First Ten Modes									
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
0% - without soil mass	34.802	48.275	66.798	89.473	104.71	117.32	129.89	134.73	142.56	154.28
0.5 C_m	29.840	41.390	57.270	76.720	89.780	100.59	111.38	115.53	122.23	132.29
1.0 C_m	26.534	36.807	50.929	68.218	79.833	89.448	99.037	102.72	108.69	117.63
1.5 C_m	24.128	33.470	46.312	62.033	72.595	81.338	90.058	93.414	98.836	100.96
2.0 C_m	22.277	30.902	42.758	57.278	67.025	75.097	83.149	86.246	91.253	98.762
2.5 C_m	20.796	28.847	39.915	53.465	62.568	70.103	77.619	80.511	85.184	92.194
3.0 C_m	19.575	27.154	37.572	50.327	58.896	65.989	73.060	75.780	80.180	86.780

Table-4 shows the comparison of first 10 Natural frequencies of a pile without soil mass and with added soil mass co-efficient varies from 0.5 to 3.0. The natural frequencies of a pile without soil mass condition give higher than the added soil mass. As the soil mass increases the natural frequencies will reduce.

Table 5. Comparisons of Shear Force & Bending Moments (sec)

Added Mass Index	Shear Force (kN)					Bending Moment (kN-m)				
	Node 2	Node 3	Node 4	Node 9	Node 10	Node 2	Node 3	Node 4	Node 9	Node 10
0% - without soil mass	6.543	4.978	4.72	1.20	1.41	3.62	2.27	1.90	0.58	0.57
0.5 C_m	7.20	6.77	4.57	1.33	1.92	4.92	3.09	2.58	0.79	0.78
1.0 C_m	11.25	8.56	8.13	2.05	2.43	6.22	3.91	3.26	1.00	0.98

1.5 C _m	13.61	11.02	9.83	2.94	3.50	7.52	4.75	3.95	1.21	1.19
2.0 C _m	15.96	12.93	12.14	2.92	3.65	8.82	5.54	4.62	1.42	1.40
2.5 C _m	18.32	14.84	13.94	3.35	3.95	10.12	6.36	5.31	1.63	1.60
3.0 C _m	20.68	16.75	14.94	3.78	4.46	11.43	7.18	6.00	1.84	1.81

Table-5 shows the comparison of shear force & bending moment for the selected nodes. As the soil added mass index increases both the design shear and design moment will also increase.

Table 6. Comparisons of Amplitudes (mm)

Added Mass Index	First Ten Modes									
	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Node 9	Node 10
0% - without soil mass	0.268	0.226	0.190	0.125	0.079	0.052	0.040	0.032	0.019	0.017
0.5 C _m	0.364	0.308	0.258	0.171	0.108	0.071	0.054	0.044	0.026	0.024
1.0 C _m	0.461	0.389	0.326	0.216	0.136	0.090	0.068	0.055	0.033	0.032
1.5 C _m	0.557	0.470	0.395	0.261	0.165	0.108	0.083	0.067	0.039	0.033
2.0 C _m	0.654	0.552	0.463	0.306	0.193	0.127	0.097	0.078	0.046	0.044
2.5 C _m	0.750	0.633	0.531	0.351	0.222	0.146	0.111	0.090	0.053	0.050
3.0 C _m	0.847	0.715	0.600	0.396	0.250	0.165	0.125	0.101	0.060	0.058

Table-6 shows the comparison of amplitudes/Deformations for first 10Modes of a pile. Amplitudes of a pile without soil mass condition gives lower values compared than the added soil mass. As the soil mass increases the maximum deformations will increase as the design shear force will get increase.

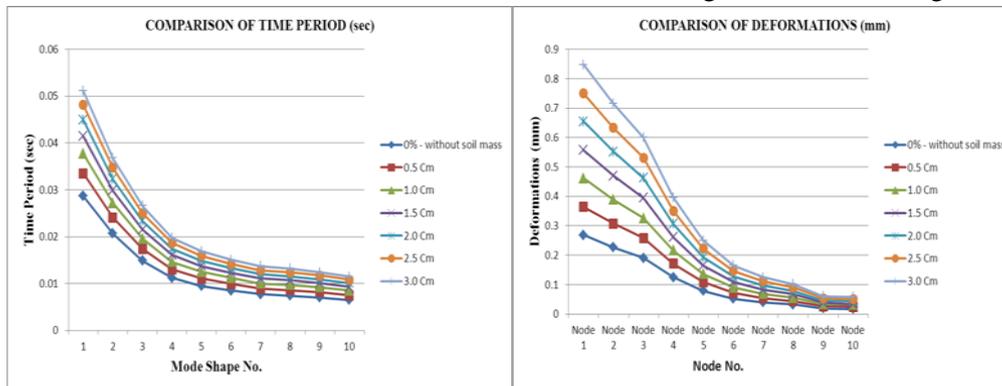


Figure 2: Comparison of Time periods

Figure 3: Comparison of Amplitudes

Fig. 2 shows the graphical representation of the time period of the pile for first 10 Modes with and with added soil mass index. Fig. 3 shows the graphical representation of the Deformations of the pile for first 10 Modes with and with added soil mass index.

IV. CONCLUSIONS

The results of the present study are strictly applicable to the structure and assumptions considered herein. In a qualitative sense, however, they indicate certain trends and thus may prove useful in suggesting which foundation parameters may require more reliable data.

In summary, it was found that the soil inertia contribution will have great effect on the pile frequency, time period and deformations. Also it will have great effect on pile ultimate design moments and design shears.

From the above study, we can also concluded that as the soil mass increases the frequency on the pile decreases and the time period, design forces like design moments and shear will increases.

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