

## Effect of Angle of attack on Stability Derivatives of a Delta wing in Hypersonic Flow with Straight leading edge

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**Abstract:** In the present paper effect of angle of attack on stability derivatives in pitch of delta wing for the attached shock case is been studied. A strip theory is used in which strips at different span-wise locations are independent. This combines with the similitude to give a piston theory. In the present theory, the similitude, and the piston theory have been extended to a flat wing with straight leading edges. The linear dependence of the stiffness derivative is seen for all parameters of the present study, however for higher angle of attack the non-linearity in the stiffness derivative is observed; since at very high angle of attack the flow separation will take place. It is also, observed that when angle of attack is small the variation in the stiffness and damping derivative in pitch remains in the range around thirty to thirty six percent, later for higher angles of attack, namely ten to twenty five degrees this variation in the stiffness and damping derivatives is in the range from ten to fifteen percent; and for angles of attack beyond twenty five degrees it remains independent of angle of attack in spite of variations in the Mach number and angles of attack. When the stiffness and damping derivatives are considered for  $h = 0.6$  which is also happens to be the center of pressure and for some cases the aerodynamic center the independency with angle of attack has been observed. The present theory is valid for large angle of incidence and Mach number. The present theory is simpler than both Lui and Hui and Hui et al and brings out explicit dependence of the stability derivatives on the similarity parameter. The present theory is not valid for a detached shock case. Future research can be done by taking into account the effects of shock motion, viscosity, wave reflections and the real gas effects.

**Keywords:** angle of attack, delta wing, Hypersonic flow, pitching derivatives, Piston theory

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### I. Introduction

The analysis of hypersonic flow over flat deltas (with straight leading edge and curved leading edge) over a wide incidence range is of current interest since the desire for high speed, maneuverability and efficiency has been dominating the evolution of high performance military aircrafts. The knowledge of aerodynamic load and stability for such types is a need for calculating simple but reasonably accurate methods for parametric calculations facilitating the design process. The computation of dynamic stability for these shapes at high incidence which is likely to occur during the course of reentry or maneuver is of current interest. Usually the shock waves are very strong when descending and they can either be detached or attached.

The theories for steady delta wings in supersonic/hypersonic flow with shock wave attached were given by Pike [1] and Hui [2]. Carrier [3] and Hui [4] gave exact solutions for 2-D flow in the case of an oscillating wedge and for an oscillating flat plate were given by Hui [5], which is valid uniformly for all supersonic Mach numbers and wedge angles or angles of attack with attached shock wave. Hui [5] also calculated pressure on the compression side of a flat delta.

The importance of dynamic stability at large incidence during re-entry or maneuver has been pointed out by Orlik-Ruckemann [6]. The shock attached relatively high aspect ratio delta is often preferred for its high lift to drag ratio.

Hui and Hemdan [7] have studied the unsteady shock detached case in the context of thin shock layer theory. Liu and Hui [8] have extended Hui's [5] theory to a shock attached delta wing in pitch. Light hill [9] has developed a "Piston Theory" for oscillating airfoils at high Mach numbers. A parameter  $\delta$  is introduced, which is a measure of maximum inclination angle of Mach wave in the flow field. It is assumed that  $M_\infty \delta$  is less than or equal to unity (i.e.  $M_\infty \delta \leq 1$ ) and is of the order of maximum deflection of a streamline. Light hill [9] likened the 2-D unsteady problem to that of a gas flow in a tube driven by a piston and termed it "Piston Analogy".

Ghosh [10] has developed a large incidence 2-D hypersonic similitude and piston theory. It includes Light hill's [9] and Mile's [11] piston theories. Ghosh and Mistry [12] have applied this theory of order of  $\phi^2$  where  $\phi$  is the angle between the attached shock and the plane approximating the windward surface. For a plane surface,  $\phi$  is the angle between the shock and the body. The only additional restriction compared to small disturbance theory is that the Mach number downstream of the bow shock is not less than 2.5.

Ghosh [13] has obtained a similitude and two similarity parameters for shock attached oscillating delta wings at large incidence. Crasta and Khan have extended the Ghosh similitude to Hypersonic/supersonic flows past a planar wedge [14] and [18] and Non planar wedge [20], [21], and [22]. Crasta and Khan have obtained stability derivatives in pitch and roll of a delta wing with straight leading edge [23] and [24] and curved leading edges for supersonic flows [15] and Hypersonic flows [16]. Crasta and Khan have studied the effect of angle of incidence on pitching derivatives and roll of a damping derivative of a delta wing with curved leading edges for an attached shock case [17] and [27]. Further in all cases stability derivatives in Newtonian limit have been calculated by Crasta and Khan [19], [25], and [26]. In the present analysis the effect of angle of attack on the stiffness and the damping derivative for hypersonic flows has been studied and results are obtained are shown in the section to follow. Further, the pressure on the lee surface is assumed zero.

## II. Analysis:

Consider a delta wing with straight leading edge. X-axis is taken along the chord of the wing and the Z-axis is perpendicular to the chord in the plane of the wing.

$$\text{Equation of x-axis is } z = 0 \tag{1}$$

$$\text{Equation of straight L.E } Z = x \cot \epsilon \tag{2}$$

Area of the wing:

$$\text{Area ABD} = \int_0^C Z dx, \text{ Let } k = \frac{\pi}{c} \tag{3}$$

A Ghosh strip theory is been used where the pressure ratio is given by

$$\frac{P}{P_\infty} = 1 + AM_p^2 + AM_p(B + M_p^2)^{\frac{1}{2}} \tag{4}$$

### Pitching moment derivatives:

Let the mean incidence be  $\alpha_0$  for the wing oscillating in pitch with small frequency and amplitude about an axis  $X_0$ . The piston velocity and hence pressure on the windward surface remains constant on a span wise strip of length  $2z$  at  $x$ , the pressure on the lee surface is assumed zero. Therefore, the nose up moment is

$$m = -2 \int_0^c p.z.(x - x_0) dx \tag{5}$$

**Stiffness Derivative:** The stiffness derivative is non-dimensionalized by dividing with the product of dynamic pressure, wing area and chord length.

$$-C_{m\alpha} = \frac{2}{\rho_\infty U_\infty^2 C^3 \cot \epsilon} \left( \frac{\partial m}{\partial q} \right)_{\substack{\alpha=\alpha_0 \\ q=0}} \tag{6}$$

Where  $\rho_\infty$  and  $U_\infty$  are density and velocity of the free stream, and  $q$  is the rate of pitch (about  $x = x_0$ ) defined positive nose up.

$$-C_{m\alpha} = \frac{2 \sin \alpha_0 \cos \alpha_0 f(S_1)}{c^3} \int_0^c (x) (x-x_0) dx \tag{7}$$

by solving the above equation, we get

$$-C_{m\alpha} = \sin \alpha_0 \cos \alpha_0 f(S_1) \left[ \frac{2}{3} - h \right] \tag{8}$$

$$\text{Where } f(S_1) = \frac{(\gamma + 1)}{2S_1} \left[ 2S_1 + (B + 2S_1^2) / (B + S_1^2)^{\frac{1}{2}} \right] \tag{9}$$

$$\text{Where } S_1 = M_\infty \sin \alpha_0$$

**Damping Derivative:** The damping derivative is non-dimensionalized by dividing with the dynamic pressure, wing area, chord length and characteristic time factor  $\left(\frac{C}{U_\infty}\right)$ .

$$-C_{mq} = \frac{2}{\rho_\infty U_\infty C^4 \cot \epsilon} \left( \frac{\partial m}{\partial q} \right)_{\substack{\alpha=\alpha_0 \\ q=0}} \tag{10}$$

Since m is given by integration to find  $\left(\frac{\partial m}{\partial q}\right)$  differentiation within the integration is necessary.

$$\left[ \frac{\partial p}{\partial q} \right]_{\substack{\alpha=\alpha_0 \\ q=0}} = A \frac{P_\infty (x - x_0)}{a_\infty} [2S_1 + (B + 2S_1^2) / (B + S_1^2)^{\frac{1}{2}}] \tag{11}$$

Substituting the value of the integral in the above equation

$$-C_{mq} = \sin \alpha_0 f(S_1) \left[ h^2 - \frac{4}{3}h + \frac{1}{2} \right] \tag{12}$$

The expressions obtained in equations (8) and (12), the results were computed for stiffness and the damping derivatives in pitch for various angle of attack and Mach numbers.

### III. Results And Discussions

Fig.1 presents the results for stiffness derivatives in pitch as a function of angle of attack for various Mach numbers for a fixed value of pivot position,  $h = 0$  of a delta wing which is nothing but the point at the leading edge. From the figure it is seen that there is a considerable decrement in the value of the stiffness derivative which is around thirty three percent, when the Mach number was increased from  $M = 5$  to 7. The variation in the values of the stiffness derivative is in the range from 0.25 to 0.45, which is around thirty three percent and more so this variation is for angle of attack five degrees. From the figure it observed that, as far as higher angles of incidence are concerned, namely from ten degrees to twenty degrees, these variation in the stiffness derivatives remains within 10 to twenty percent. It is also seen that further, increase in the Mach number results in the decrease in the value of the stiffness derivative, and it remains in the range around five, ten, and fifteen percent, however, the magnitude of decrease remains marginal, and the reason for this trend may be due to the increase in the Mach number generally stiffness derivative will tend to decrease, nevertheless, progressively, the decrement will not be appreciable and ultimately it will be become steady state. It is also seen that the stiffness derivative increases linearly in the range from five to thirty five degrees and later, further, increase in the angle of attack results in decrease of the stiffness derivative, this may be due to the stalling of the flow leading to flow separation at the wing.

Fig.2 shows the results for damping derivatives in pitch for a fixed value of pivot position ( $h = 0$ ) for Mach numbers in the range from  $M = 5$  to 15 as a function of angle of incidence. We see a linear behavior with angle of attack and there is no non-linearity in the trend as the pivot position is kept constant. For the present range of Mach number the value of damping derivative varies appreciably for the angles of attack in the range from five degrees fifteen degrees. It's value remains in the range from 0.2 to 0.35 and this variation is around thirty percent for angle of attack five degrees, whereas, for angle of attack in the range ten to twenty degrees this variation is limited within 10 to 15 percent. For higher Mach numbers and angle of attack all the values remains constant, this may be due to the Mach number independence principle.

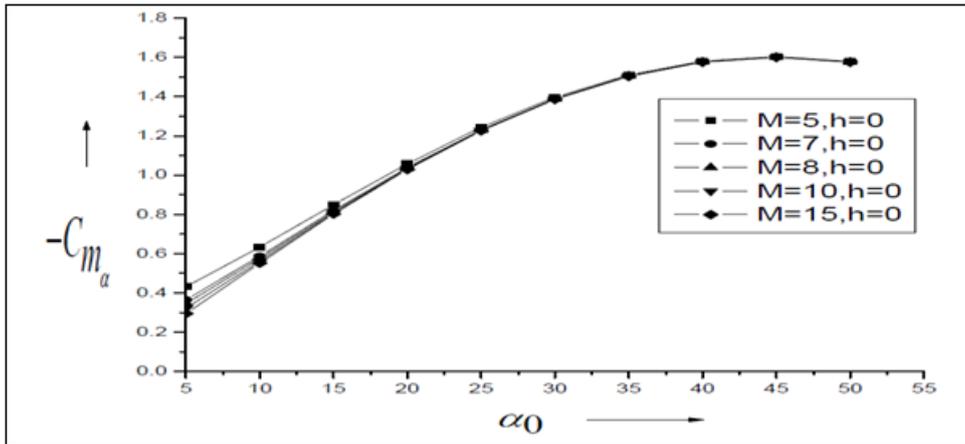


Fig. 1: Variation of Stiffness derivative with angle of attack

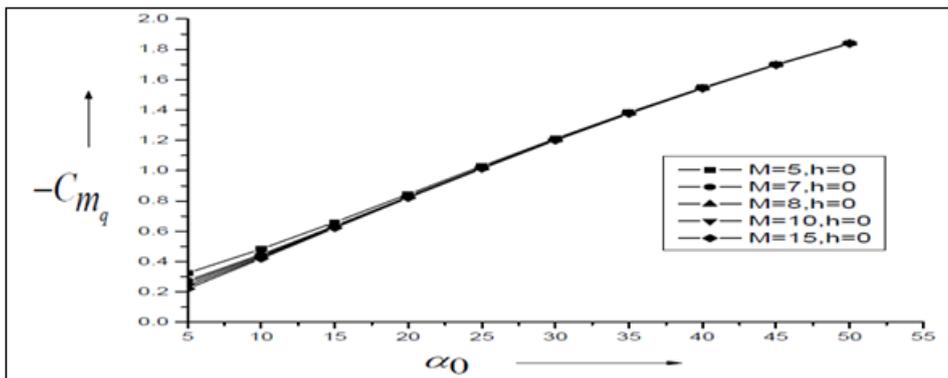


Fig. 2: Variation of damping derivative with angle of attack

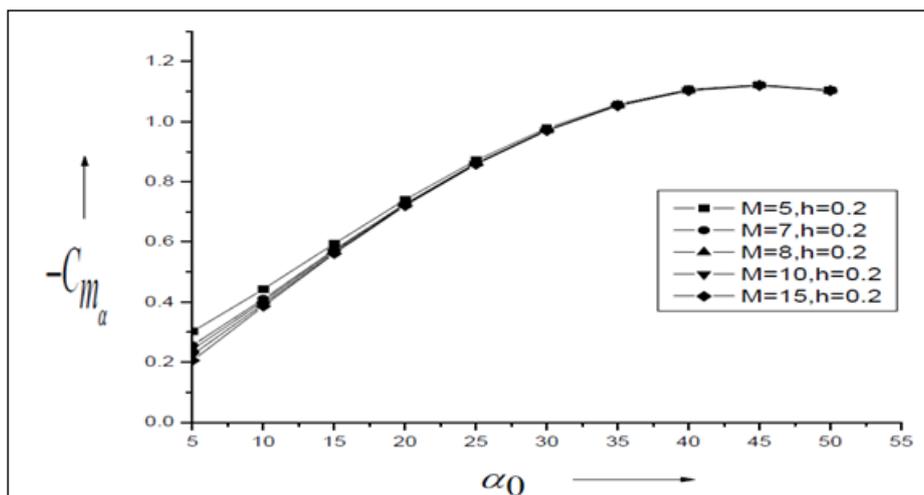


Fig. 3: Variation of Stiffness derivative with angle of attack

Fig. 3: Variation of Stiffness derivative with angle of attack

Figure 3 shows the Variation of Stiffness derivative with angle of attack for various Mach numbers in the range from  $M = 5$  to  $15$ . As the pivot position is shifted exactly from the leading edge towards the trailing edge (i.e. from  $h = 0$  to  $0.2$ ), it is clearly seen that the magnitude of Stiffness derivative decreases and the magnitude lies between  $0.2$  to  $0.3$  as compared with pivot position  $h=0.0$ . This decrease in the value of the stiffness derivative is due to the decrease in the moment arm of the wing. It is also seen that the decrease in the stiffness derivative is around thirty three percent at angle of attack of five degrees, however, for angle of attack in the range from ten to twenty degrees this decrement is in the range ten to fifteen percent. The trend of stiffness derivative with angle of attack remains linear up to thirty five degrees, for further increase in angle of attack results in decrease of stiffness derivative due to stalling of the flow field.

Fig. 4 presents the results for damping derivatives as a function of angle of attack for various Mach numbers in the range  $M = 5$  to  $15$  for pivot position  $h = 0.2$ . Here again the results show similar trends excepting that the magnitude has marginally reduced due to the reduction in the moment arm due the shift of the pivot position from  $h = 0$  to  $0.2$ . The trend of the damping derivative remains linear for all the values of the angle of attack of the present study.

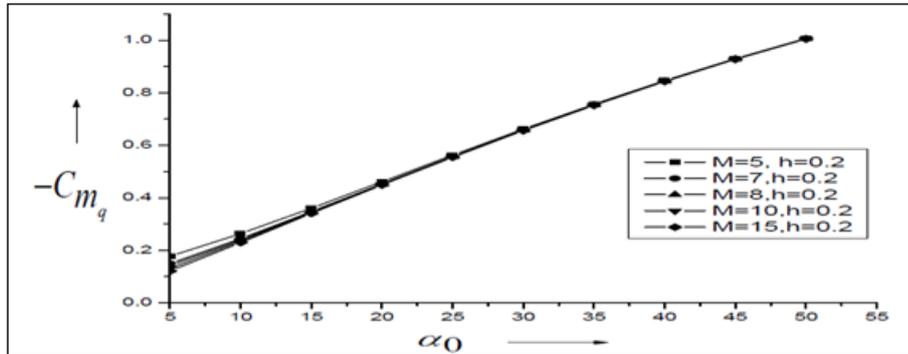


Fig. 4: Variation of damping derivative with angle of attack

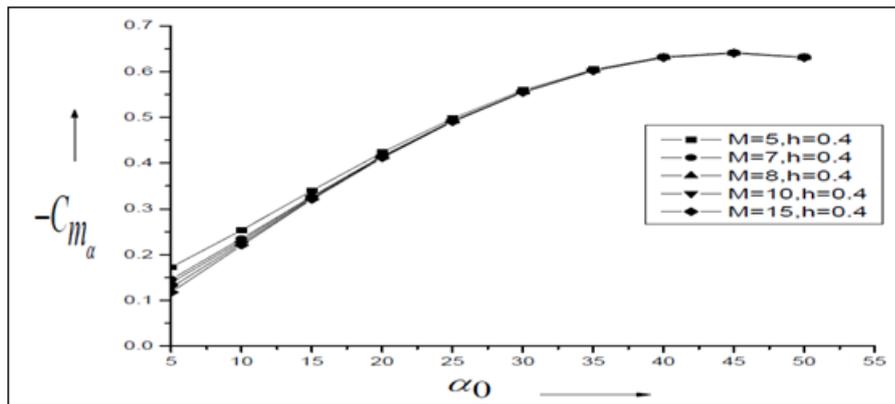


Fig. 5: Variation of Stiffness derivative with pivot position

For pivot position  $h = 0.4$ , the results for stiffness derivative are shown in figure 5. The trend remains the same however; due the shift of the fixed value of the pivot position from the leading edge towards the trailing edge resulting in further decrease in the moment arm for the pitching moment may be reasons for the low values of the stiffness derivatives. It is also seen that the decrease in the magnitude of the stiffness derivative is in the range around 25 percent for angle of attack five degrees, the behavior remains linear till angle of attack thirty five degrees. There is appreciable change in the value of the stiffness derivative for Mach numbers between 5 to seven, later on the reduction is only marginal, the reason for this trend is the same as discussed above.

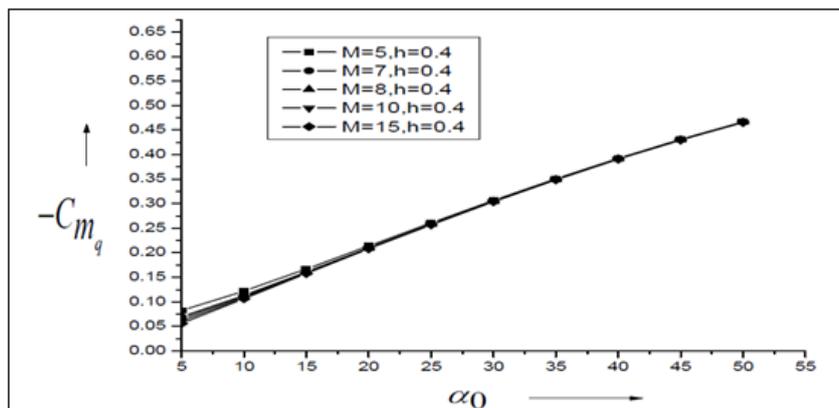


Fig. 6: variation of damping derivative with angle of attack

Results for damping derivatives for pivot position  $h = 0.4$  are presented in figure 6, here again, it is seen that the magnitude of the damping derivative has decreased considerably, which is around thirty eight percent, primarily due the increase in the Mach number and secondary due to the shift of the pivot position by fort percent towards the trailing edge resulting in the decrease of the moment arm and hence, the pitching damping derivatives, however; the behavior remains linear for all the values of the angle of incidence of the present study.

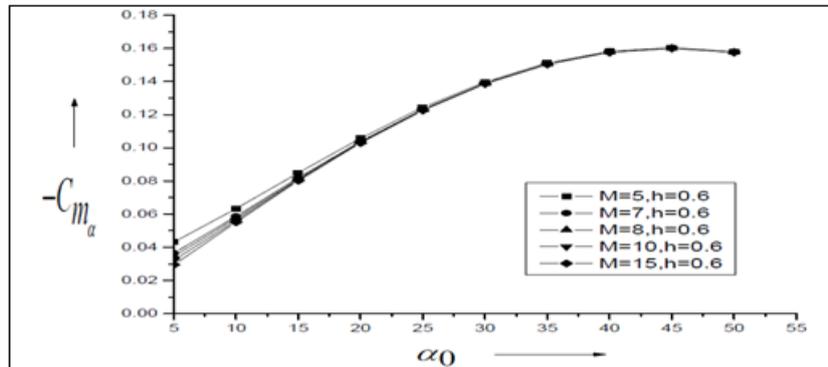


Fig.7: Variation of Stiffness derivative with angle of attack

Figure 7 shows the variation of Stiffness derivative with angle of attack for pivot position  $h = 0.6$ ; which happens to be very close to the center of pressure/aerodynamic center of the wing. It is seen that the magnitude of Stiffness derivative is very low as when compared to other pivot positions. It is well known from aerodynamics of the wings that in and around the aerodynamic center of the wing the aerodynamic derivatives will be independent of angle of attack.

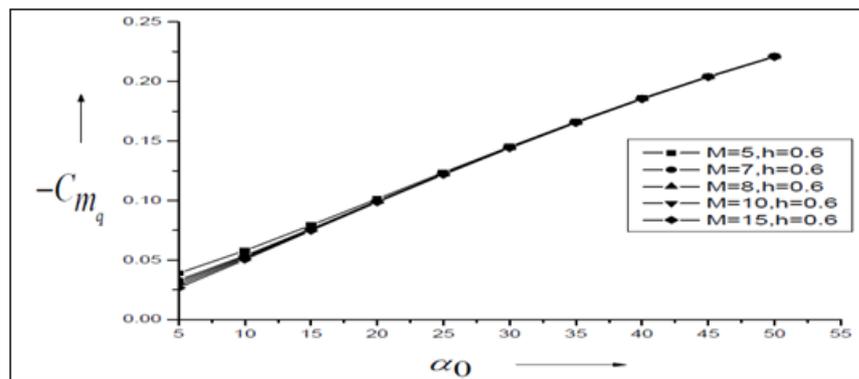


Fig. 8: Variation of damping derivative with angle of attack

Fig. 8 depicts the results of the damping derivatives in pitch for pivot position  $h = 0.6$ , which is within the range of the aerodynamic center and the magnitude is very small, as expected there is forty percent decrease in the damping derivative for angle of attack five degrees, later for higher values of the angle of attack this change comes down to the value around five to ten percent.

Results for stiffness derivatives for pivot position  $h = 0.8$  are shown in Fig. 9 keeping all other parameters namely the angle of attack and the Mach numbers of the present case remains the same. Here the trend is different as compared to the previous cases. The stiffness derivatives has become negative, however, the variation between Mach number  $M = 5$  to  $7$  at angle of attack five degrees remains around thirty five percent, this variation is around five to the percent for angle of attack in the range fifteen to ten degrees, and for higher values of the angle of attack the value of the stiffness derivative remains unchanged. This peculiar behavior may be due to the location of the pivot position which is eighty percent from the leading edge and is well behind the center of pressure leading to the unstable pitching moment. The other characteristics remain the same, needless to say that the variation in the stiffness stability derivative remains linear up to thirty degree later non-linearity crept in.

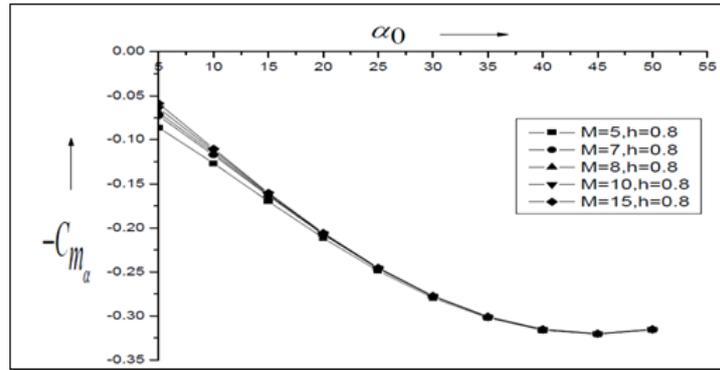


Fig. 9: Variation of stiffness derivative with angle of attack

Results for damping derivatives for pivot position  $h = 0.8$  is shown in Fig. 10, keeping the same inertia levels and the angles of attack. From the figure it is seen that the magnitude of the damping derivative is also very low due to the location of the pivot position which is well behind and very close to the trailing edge leading to the instability of the wing. The reduction in the magnitude of the damping derivative for the Mach numbers between  $M = 5$  to  $7$  is around thirty six percent at angle of attack five degrees, however, for the other Mach numbers in the range from  $M = 8$  to  $15$ , it remains in the range ten to fifteen percent and for the higher angle of attack, namely from twenty five degrees onwards it remains unaltered.

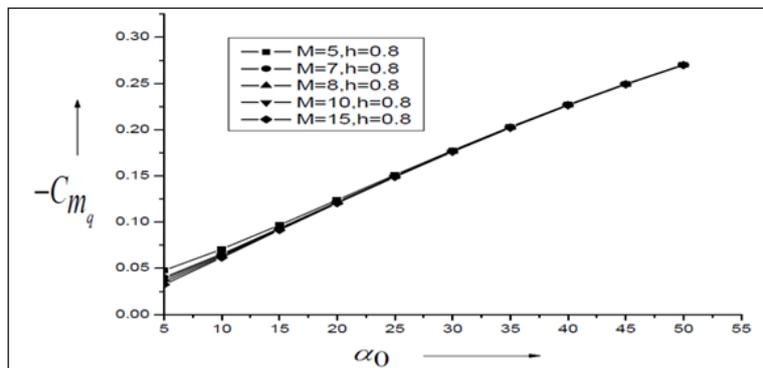


Fig. 10: variation of damping derivative with angle of attack

The results for stiffness derivative for the extreme pivot position  $h = 1.0$ , that is when the pivot position is exactly at the trailing edge leading to the highly unstable situation of the wing. Accordingly it leads to negative static margin. There is appreciable change in the stiffness derivative for angle of attack of five degrees for Mach number in the range  $M = 5$  to  $7$ . For other Mach numbers the variation in the stiffness derivative is marginal which is limited up to angle of attack of around twenty degrees and later for angles of attack beyond twenty degrees it remains independent of angle of incidence.

Figure 12 presents the results for damping derivatives for pivot position  $h = 1.0$ , which is at the trailing edge of the wing, and far away from the centre of pressure. Here, again the variation in the magnitude of the damping derivatives is around thirty six percent; it becomes independent of angle of attack for twenty five degrees and above.

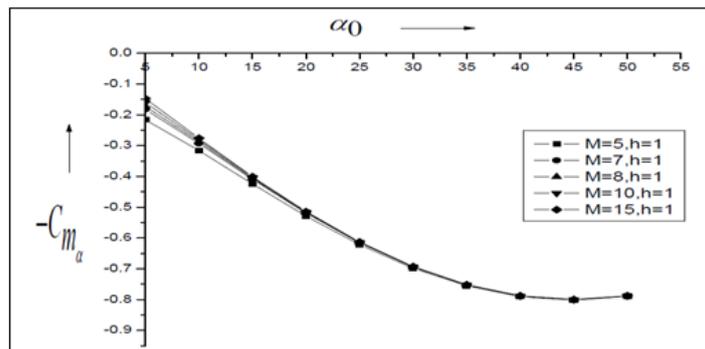


Fig. 11: Variation of Stiffness derivative with angle of attack

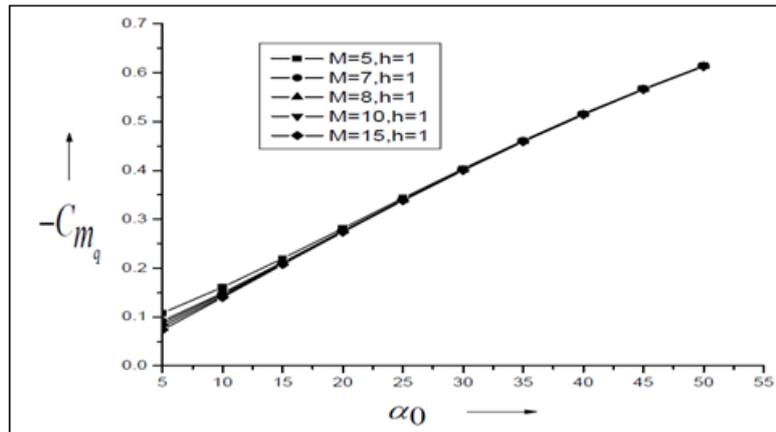


Fig. 12: Variation of damping derivative with angle of attack

#### IV. Conclusion

In the present theory, the similitude, and the piston theory have been extended to a flat wing with straight leading edges. The linear dependence of the stiffness derivative is seen for all parameters of the present study, however for higher angle of attack the non-linearity in the stiffness derivative is observed; since at very angle of attack the flow separation will take place. It is also, observed that when angle of attack is small the variation the stiffness and damping derivative in pitch remains in the range around thirty to thirty six percent, later for higher angles of attack namely ten to twenty five degrees this variation in the stiffness and damping derivatives remains in the range from ten to fifteen percent; and for angles of attack beyond twenty five degrees it remains independent of angle of attack in spite of variations in the Mach number and angles of attack. When the stiffness and damping derivatives are considered for  $h = 0.6$  which also happens to be the center of pressure and for some cases the aerodynamic center the independency with angle of attack has been observed. The present theory is valid for large angle of incidence and Mach number. The present theory is simpler than both Lui and Hui and Hui et al and brings out explicit dependence of the stability derivatives on the similarity parameter. The present theory is not valid for a detached shock case. Future research can be done by taking into account the effects of shock motion, viscosity, wave reflections and the real gas effects.

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