

## Estimation of Damping Derivative of a Delta Wing with Half Sine Wave Curved Leading Edges

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**Abstract:** In the Present paper effect of angle of incidence on Damping derivative of a delta wing with Curved leading edges (for a half sine wave) for attached shock case in Supersonic Flow has been studied. A Strip theory is used in which strips at different span wise location are independent of each other. This combines with similitude to give a piston theory which gives closed form solutions for damping derivatives at low to high supersonic Mach numbers. From the results it is seen that with the increase in the Mach number, there is a progressive decrease in the magnitude of damping derivatives for all the Mach numbers of the present studies; however, the decrease in the magnitude is variable at different inertia level. It is seen that with the increase in the angle of attack the damping derivative increases linearly, nevertheless, this linear behavior limit themselves for different Mach numbers. For Mach number  $M = 2$ , this limiting value of validity is fifteen degrees, for Mach 2.5 & 3, it is twenty five degrees, whereas, for Mach 3.5 & 4 it becomes thirty five degrees, when these stability derivatives were considered at various pivot positions; namely at  $h = 0.0, 0.4, 0.6, \text{ and } 1.0$ . After scanning the results it is observed that with the shift of the pivot position from the leading edge to the trailing edge, the magnitude of the damping derivatives continue to decrease throughout. Results have been obtained for supersonic flow of perfect gases over a wide range of angle of attack and Mach number. The effect of real gas, leading edge bluntness of the wing, shock motion, and secondary wave reflections are neglected.

**Keywords:** angle of attack, delta wing, Supersonic flow, pitching derivatives, Piston theory.

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### I. Introduction

The analysis of hypersonic and supersonic flow over flat deltas (with straight leading edge and curved leading edge) over a wide incidence range is of current interest since the desire for high speed, maneuverability and efficiency has been dominating the evolution of high performance military aircrafts. The knowledge of aerodynamic load and stability for such types is a need for calculating simple but reasonably accurate methods for parametric calculations facilitating the design process. The computation of dynamic stability for these shapes at high incidence which is likely to occur during the course of reentry or maneuver is of current interest. Usually the shock waves are very strong when descending and they can either be detached or attached.

The theories for steady delta wings in supersonic/hypersonic flow with shock wave attached were given by Pike [1] and Hui [2]. Carrier [3] and Hui [4] gave exact solutions for 2-D flow in the case of an oscillating wedge and for an oscillating flat plate were given by Hui [5], which is valid uniformly for all supersonic Mach numbers and wedge angles or angles of attack with attached shock wave. Hui [5] also calculated pressure on the compression side of a flat delta.

The importance of dynamic stability at large incidence during re-entry or maneuver has been pointed out by Orlik-Ruckemann [6]. The shock attached relatively high aspect ratio delta is often preferred for its high lift to drag ratio. Hui and Hemdan [7] have studied the unsteady shock detached case in the context of thin shock layer theory. Liu and Hui [8] have extended Hui's [5] theory to a shock attached delta wing in pitch. Light hill [9] has developed a "Piston Theory" for oscillating airfoils at high Mach numbers. A parameter  $\delta$  is introduced, which is a measure of maximum inclination angle of Mach wave in the flow field. It is assumed that  $M_\infty \delta$  is less than or equal to unity (i.e.  $M_\infty \delta \leq 1$ ) and is of the order of maximum deflection of a streamline. Light hill [9] likened the 2-D unsteady problem to that of a gas flow in a tube driven by a piston and termed it "Piston Analogy".

Ghosh [10] has developed a large incidence 2-D hypersonic similitude and piston theory. It includes Light hill's [9] and Mile's [11] piston theories. Ghosh and Mistry [12] have applied this theory of order of  $\phi^2$  where  $\phi$  is the angle between the attached shock and the plane approximating the windward surface. For a plane surface,  $\phi$  is the angle between the shock and the body. The only additional restriction compared to small disturbance theory is that the Mach number downstream of the bow shock is not less than 2.5.

Ghosh [13] has obtained a similitude and two similarity parameters for shock attached oscillating delta wings at large incidence. Crasta and Khan have extended the Ghosh similitude to Hypersonic/supersonic flows past a planar wedge [14] and [18] and Non planar wedge [20], [21], and [22]. Crasta and Khan have obtained stability derivatives in pitch and roll of a delta wing with straight leading edge [23] and [24] and curved leading

edges for supersonic flows [15] and Hypersonic flows [16]. Crasta and Khan have studied the effect of angle of incidence on pitching derivatives and roll of a damping derivative of a delta wing with curved leading edges for an attached shock case [17] and [27]. Further in all cases stability derivatives in Newtonian limit have been calculated by Crasta and Khan [19], [25], and [26]. In the present analysis the effect of angle of attack on the stiffness derivative for supersonic flows with curved leading edge has been studied and results are obtained are shown in the section to follow.

## II. Analysis

In the present analysis Ghosh's (1981) unified supersonic/hypersonic similitude has been used in combination with a strip theory for a supersonic delta wing whose leading edge is curved. A thin strip of the wing, parallel to the centerline, can be considered independent of the Z dimension when the velocity component along the Z direction is small. This has been discussed by Ghosh's(1984). The strip theory combined with Ghosh's large incidence similitude leads to the piston analogy and pressure P on the surface can be directly related to equivalent piston mach no.  $M_p$ . In this case both  $M_p$  and flow deflections are permitted to be large. Hence light hill piston theory cannot be used but Ghosh's piston theory will be applicable.

$$\frac{p}{p_\infty} = 1 + AM_p^2 + A M_p (B + M_p^2)^{1/2}, \text{ where } p_\infty \text{ is free steam pressure} \dots \dots \dots (1)$$

Since strips at different span wise location are assumed independent of each other, the strip can be considered as a flat plate at an angle of attack. The angle of incidence is same as that of the wing. Angle  $\phi$  is the angle between the shock and the strip. A piston theory which has been used in eqn.(1) has been extended to supersonic flow. The Expression is given below

$$\frac{p}{p_\infty} = 1 + A \left( \frac{M_p}{\cos \phi} \right)^2 + A \left( \frac{M_p}{\cos \phi} \right) \left( B + \left( \frac{M_p}{\cos \phi} \right)^2 \right)^{1/2} \quad (2)$$

Where  $p_\infty$  is free stream pressure,  $A = \frac{(\gamma + 1)}{4}$ ,  $B = (4/(\gamma + 1)^2)$ ,  $\gamma$  is the specific heat ratio and  $M_p$  = the local piston Mach number normal to the wing surface.

## III. Pitching moment derivatives

Let the mean incidence be  $\alpha_0$  for the wing oscillating in pitch with small frequency and amplitude about an axis. The piston velocity and hence pressure on the windward surface remains constant on a span wise strip of length  $2z$  at  $x$ . The pressure on the lee surface is assumed Zero. Therefore the nose up moment is

$$m = -2 \int_0^c p.z(x - x_0) dx \quad (3)$$

## IV. Damping Derivative

The damping derivative is non-dimensionalised by dividing with the product of dynamic pressure, wing area,

chord length and characteristic time factor  $\left( \frac{c}{U_\infty} \right)$

$$\therefore -^c m_q = \frac{2}{\rho_\infty U_\infty c^4 (\cot \varepsilon)} \left( - \frac{\partial m}{\partial q} \right)_{\alpha=\alpha_0, q=0} \quad (4)$$

Since m is given by an integration (3) to find  $\left( - \frac{\partial m}{\partial q} \right)_{\alpha=\alpha_0, q=0}$  the differentiation within the integration is necessary.

$$\therefore \left[ \frac{\partial p}{\partial q} \right]_{\alpha=\alpha_0, q=0} = \frac{A p_\infty (x - x_0)}{a_\infty \cos \phi} F(S^1_1)$$

From equations 2-4 solving the integral we get

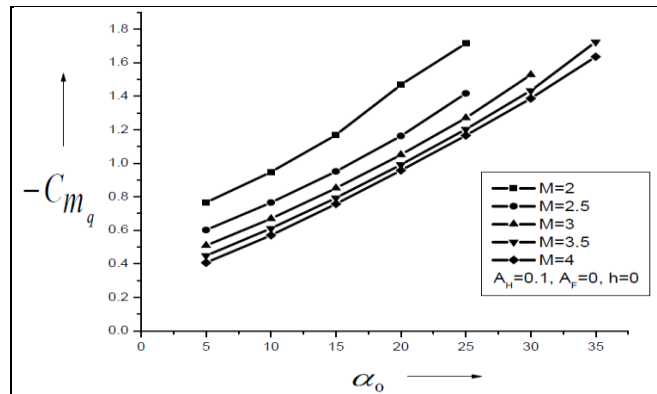
$$-C_{m_q} = \frac{\sin \alpha_0 f(S^1_1)}{\cos^2 \phi} \left[ \left( h^2 - \frac{4}{3} h + \frac{1}{2} \right) \right] \quad (5)$$

Where

$$f(s_1^1) = \frac{(\gamma + 1)}{2s_1^1} F(S_1^1) = \frac{(\gamma + 1)}{2s_1^1} [2s_1^1 + (B + 2s_1^2)/(B + S_1^2)^2]^{\frac{1}{2}} \tag{6}$$

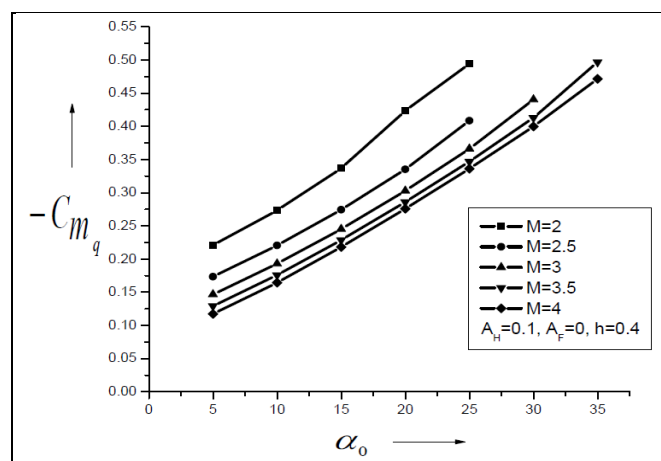
By using above expression, stiffness derivative calculations have been carried out and some of the results have been shown.

**V. Results And Discussions**



**Fig. 1:** Variation of Damping derivative with angle of attack for a half sine wave.

Figure 1 represents the results for Damping derivatives for full sine wave with amplitude being zero and for half sine wave with amplitude of 0.1 with pivot position  $h = 0$ . From the results it is seen that there is a continuous increase of Damping derivative linearly with angle of attack, nevertheless, this linear behavior limit themselves for different Mach number. For Mach number  $M = 2$ , this limiting value of validity is fifteen degrees, for Mach 2.5 and 3 it is twenty five degrees and for Mach 3.5 to 4 it is thirty five degrees. From the figure it is also seen that there is a continuous decrease in the magnitude of the Damping derivatives in the range of twenty three percent, fifteen percent, twelve percent and eleven percent for the Mach numbers in the range 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4. However inspite of pivot position being at  $h = 0$ , the low values of the Damping derivatives are attributed due to combined effect of variation in the wing plan form area as in this case we have kept the amplitude of half sine wave 0.1 and amplitude of full sine wave as zero resulting in change in the shape of the wing, due to this the location of the center of pressure will also change due to the change in the pressure distribution on the surface of the wing and also, the pivot position which is exactly at the leading edge; which is far away from the center of pressure.



**Fig. 2:** variation of Damping derivative with angle of attack for a half sine wave.

Figure 2 represents the results for Damping derivatives for full sine wave with amplitude being zero and for half sine wave with amplitude 0.1 with pivot position  $h = 0.4$ . From the results it is seen that there is a continuous increase of Damping derivative linearly with angle of attack, nevertheless, this linear behavior limit themselves for different Mach number. From the figure it is also seen that there is a continuous decrease in the magnitude of the Damping derivatives in the range of twenty three percent, fifteen percent, twelve percent and eleven percent for the Mach numbers in the range 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4. The change in the values of the Damping derivatives are attributed due to combined effect of variation in the wing plan form area

due to the change in the amplitude of sine waves, and the location of the pivot position which is forty percent aft of the leading edge; which is not far away from the center of pressure.

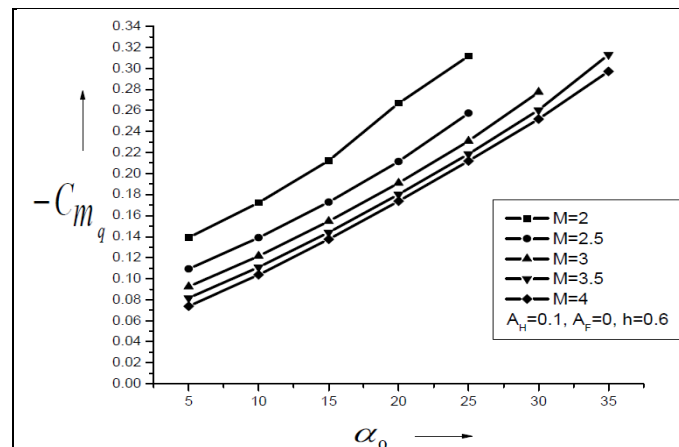


Fig. 3: variation of Damping derivative with angle of attack for a half sine wave.

Figure 3 represents the results for Damping derivatives for full sine wave with amplitude being zero and for half sine wave with amplitude 0.1 with pivot position  $h = 0.4$ . From the results it is seen that there is a continuous increase of Damping derivative linearly with angle of attack, nevertheless, this linear behavior limit themselves for different Mach number as discussed earlier. From the figure it is also seen that there is a continuous decrease in the magnitude of the Damping derivatives in the range of twenty four percent, sixteen percent, thirteen percent and eleven percent for the Mach numbers in the range 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4. The extremely low values of the Damping derivatives are attributed due to combined effect of variation in the wing plan form area due to the change in the amplitude of sine waves, and the location of the pivot position which is in the vicinity of the center of pressure leading to low values because of small moment arm of the wing mean aerodynamic chord is available.

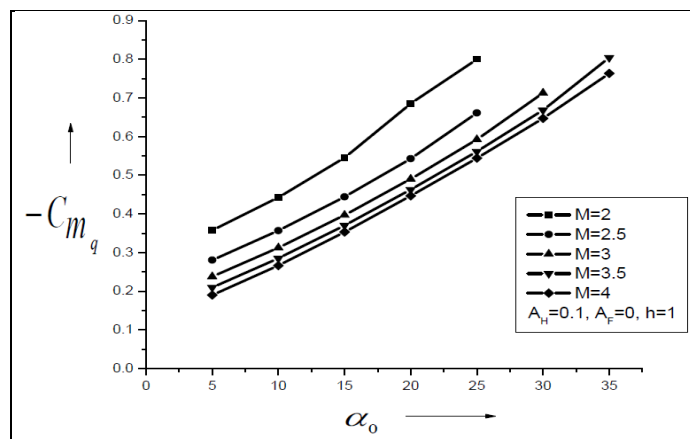


Fig. 4: Variation of Damping derivative with angle of attack for a half sine wave.

Figure 4 shows the variation of damping derivative with angle of attack for the amplitude of half sine wave as 0.1 and amplitude of full sine wave being zero for pivot position  $h = 1$ . The trend is similar as above and the trend is attributed due to the combined effect of variation in the wing plan form area due to change in the amplitude of sine waves and the location of the pivot position which is exactly at the trailing edge which is behind the center of pressure.

## VI. Conclusions

From above discussions we can draw the following conclusions;

- It is observed that the magnitude of the damping derivative is continuously decreasing with the increase in the Mach number for all the Mach number in the range from  $M = 2$  to 4.

- It is also observed that when we change the pivot position from  $h = 0, 0.4, 0.6,$  and  $1,$  the damping derivative decreases throughout for the present range of Mach numbers, wing plan form area and the pivot position.
- It is seen that with the increase in the angle of attack the damping derivative increases linearly, nevertheless, this linear behavior limit themselves for different Mach numbers. For Mach number  $M = 2,$  this limiting value of validity is fifteen degrees, for Mach  $2.5 \& 3,$  it is twenty five degrees, whereas, for Mach  $3.5 \& 4$  it becomes thirty five degrees.
- Results have been obtained for supersonic flow of perfect gases over a wide range of angle of attack for a half sine wave and the Mach numbers in the range from 2 to 4. The effect of real gas, leading edge bluntness of the wing, shock motion, and secondary wave reflections are neglected.

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