

An Investigation into the Factors Affecting the Dynamic Performance of Steel/Wood Sandwich Beams

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Abstract: Sandwich beams offer designers a number of advantages, such as high stiffness to weight ratio, high bending, and buckling resistances,etc. which are useful in aerospace, and mechanical engineering designs as well as in renewable-energy applications e.g. wind mill blades. In the present study the effects of the different sandwich beam parameters, such as; face and core thicknesses, beam width and length on the dynamic performance of steel/wood sandwich beams have been investigated as compared with a steel solid beam with the same dimensions. The damped-and undamped natural frequencies of the beams have been determined both theoretically and experimentally. The impulse excitation technique has been used to determine the frequency response. The present investigation has revealed that, the theoretical and experimental results are in a good agreement. The sandwich beam has shown higher stiffness-to weight ratio & higher damping capacity as well as higher damped-and undamped natural frequencies than the steel solid beam. The natural frequencies of the sandwich beam are increased by the increase of the beam thickness caused by the increase of the face layer thickness or the core thickness. On the other hand these frequencies are reduced by the increase of the beam length but are unaffected by the beam width.

However it has been concluded that the selection of the optimum sandwich beam parameters depends on the application requirements.

Keywords: Sandwich beam – Natural frequency – Damped natural frequency.

NOMENCLATURE

Symbols	Names	Unit
A_c	Cross sectional area of the sandwich beam core	m^2
A_f	Cross sectional area of the sandwich beam face	m^2
B	Width of sandwich beam	m
C	Neutral line offset	m
c_{eq}	Equivalent damping coefficient	N.s/m
E	Modulus of elasticity of material	MPa
E_c	Modulus of elasticity of sandwich beam core material	MPa
E_f	Modulus of elasticity of sandwich beam face material	MPa
E_{eq}	Equivalent modulus of elasticity of sandwich beam materials	MPa
f_d	Damped natural frequency	Hz
f_n	Undamped natural frequency	Hz
H	Total thickness of sandwich beam	m
h_c	Face thickness of sandwich beam	m
h_f	Core thickness of sandwich beam	m
I	Area moment of inertia of sandwich beam	m^4
I_c	Area moment of inertia of sandwich beam core	m^4
I_f	Area moment of inertia of sandwich beam face	m^4
K_c	Stiffness of the core of the sandwich beam	N/m
K_f	Stiffness of the face of the sandwich beam	N/m
K_{eq}	Equivalent stiffness of the cantilever sandwich beam	N/m
K_{eq}^*	Equivalent stiffness-to-weight ratio= (k_{eq}/m_{eq})	N/m/kg
L	Length of sandwich beam	m
m_{eq}	Dynamic equivalent mass	kg
ρ_c	Density of sandwich beam core material	kg/m^3
ρ_f	Density of sandwich beam face material	kg/m^3
η_{eq}	η equivalent damping ratio	-
ω_d	Angular damped natural frequency	rad/s
ω_n	Angular undamped natural frequency	rad/s

I. Introduction

Modern engineering requires the use of sophisticated and optimized structural design. One way to achieve this goal is to use materials in a way that will optimize their inherent properties. The application of

sandwich lamination is suitable for this purpose. Sandwich materials are frequently used wherever high strength and/or stiffness together with low weight are important criteria. A sandwich beam is a special class of composite materials which is fabricated as shown in **Figure 1**, usually composed of, face sheets, core and adhesive interface layers. The faces carry in-plane and bending loads, while the core resists transverse shear forces and keeps the faces in place [1]. The advantages of sandwich constructions, the development of new materials and the need for high performance low-weight structures insure that sandwich construction will continue to be in demand [2-3]. The purpose of the core is to maintain the distance between the laminates and to sustain shear deformations. By varying the core, the thickness and the material of the face of the sandwich structures; it is possible to obtain various properties and desired performances [4].

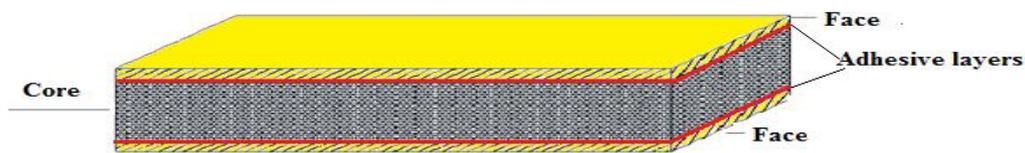


Figure 1: Components of the Sandwich Beam

The face of a sandwich beam can be made of steel or aluminum laminate with thickness (h_f), while polyamide or wood can be used as a core material with thickness (h_c). The third component is the adhesive layer usually epoxies the main function of which is bonding the two faces with the core as one unit. The characteristics of sandwich beam component materials and their effect on its dynamic behavior can be investigated by stress analysis [5]. The free vibration behavior of the sandwich beam with functionally graded core materials was analyzed using the mesh-less method [6]. Visco-elastic damping of structures is introduced by transverse shear in the core due to the difference between in-plane displacement of the elastic faces and the lower stiffness of the core [7-8]. Damping and vibration properties of polyethylene fiber core material are affected by temperature, where the natural frequency decreases with increasing temperature [9].

The face laminate dominates the stiffness of sandwich beams. Natural frequencies of sandwich beams are affected directly by the face material [10]. Increasing the adhesive amount used for bonding face sheets and cores increases the bending fatigue strength of Aluminum Honeycomb sandwich beams [11].

Mechanical systems are usually subjected to vibration and noise problems [12-14]. The dynamic characteristics of sandwich beams depend on their construction, aiming at increasing the beam bending stiffness, and increasing damping without adding excessive weight. The mechanical behavior of sandwich structures depends on the properties of the faces, core, and the adhesive bonding of the core to the faces [15]. Intuition of a particular failure mode depends on the constituent material properties, geometry, and type of loading.

In the present study sandwich beams specimens as shown in **Figure (2-a)** with steel face material and wood as a core material have been prepared. A steel solid beam specimens **Figure (2-b)** has been used for the sake of comparison with the sandwich beam specimens [16]. The sandwich beam dimensions have been changed to study their effects on the dynamic performance of the beams. Theoretical and experimental investigations have been carried out to determine the damped-and undamped natural frequencies of the sandwich beams.

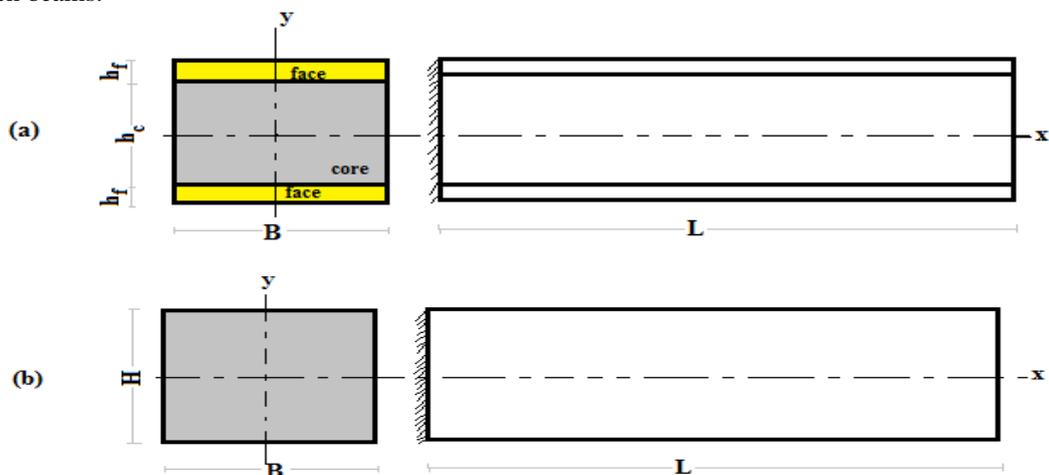


Figure 2, (a): Dimensions of the Sandwich Beam Specimen
(b): Dimensions of the Steel Solid Beam Specimen

II. Theoretical Analysis

1. Undamped Natural Frequency of the Sandwich Beam (f_n)

The equations used to calculate the natural frequency of a sandwich beam are given as follows [9], [15]:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K_{eq}}{m_{eq}}} \quad (1)$$

$$m_{eq} = 0.25 m = 0.25 (\rho A)_{eq} L = 0.25 B L (2\rho_f h_f + \rho_c h_c) \quad (2)$$

$$k_{eq} = k_c + k_f = \frac{3E_c I_c}{L^3} + 2\left(\frac{3E_f (I_f + A_f y^2)}{L^3}\right) \quad (3)$$

$$I_c = \frac{B h_c^3}{12} \quad (4)$$

$$I_f = \frac{B h_f^3}{12} \quad (5)$$

$$A_f = B * h_f \quad (6)$$

From equations (1) to (6) the value of (f_n) can be written as follows:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{3E_{eq} I}{0.25 B (2\rho_f h_f + \rho_c h_c) L}} \quad (7)$$

Where;

$$E_{eq} = \frac{\left(\frac{12}{H^3}\right) \left\{ \frac{E_f}{3} \left[\left(C - \frac{h_c}{2}\right)^3 - \left(C - \frac{h_c}{2} - h_f\right)^3 \right] + E_c C^2 h_c + \frac{E_c h_c^3}{12} + C + h_c \right\}}{23} + \frac{E_f}{3} \left[\left(C + \frac{h_c}{2} + h_f\right)^3 - \right] \quad (8)$$

$$C = \frac{E_{f1} h_{f1} (h_{f1} + h_c) - E_{f3} h_{f3} (h_{f3} + h_c)}{2(E_{f3} h_{f3} + E_c h_c)} \quad (9)$$

2. Damped Natural Frequency of the Sandwich Beam (f_d)

Damping is important for mechanical design to control vibrations. The damped natural frequency is determined as follows [15]:

$$f_d = \frac{\omega_d}{2\pi} = f_n \sqrt{1 - \xi_{eq}^2} \quad (10)$$

The damping ratio (ξ_{eq}) is equal the damping coefficient (c_{eq}) divided by the critical damping coefficient as given by;

$$\xi_{eq} = \frac{c_{eq}}{2\sqrt{k_{eq} m_{eq}}} = \frac{\pi f_n c_{eq}}{K_{eq}} \quad (11)$$

It is determined experimentally from the time response signal of the decaying vibration amplitude

$$c_{eq} = 2m_{eq} \xi_{eq} \omega_n \quad (12)$$

The aforementioned equations are used to calculate the undamped and damped natural frequencies of solid and sandwich beams. The frequencies of steel/wood sandwich beams are obtained for different values of the beam dimensions (h_f , h_c , B , and L). The output results of the applied the equations are tabulated in **Table 1**.

Table 1: Specifications and Natural Frequencies of Solid Steel and Steel/ Wood Sandwich Beams Used in the Present Work

Specimens	Dimension in mm					m_{eq} (kg)	K_{eq} (N/m)	f_n (Hz)	c_{eq} (N.s/m)	f_d (Hz)
	h_f	h_c	H	B	L					
Solid Steel Beam	-	-	16	25	180	0.1422	921811	405.22	368.724	348.625
Steel/ Wood Sandwich Beam	1.5	10	13	25	180	0.03476	285461	456.077	180.92	191
	3	10	16	25	180	0.06142	712834	542.178	371.835	248.8
	4.5	10	19	25	180	0.08808	1334651	619.51	575.588	336.8
	3	15	21	25	180	0.06547	1378906	780.38	530.557	343
	3	18	24	25	180	0.0145	109380	936	636.668	411.4
	3	10	16	20	180	0.04914	570267	542.178	294.468	248.8
	3	10	16	25	240	0.0819	300726	304.975	253.998	179.2

The tabulated results show the calculated values of undamped & damped natural frequencies of steel/wood sandwich beams and the solid steel beam for the different dimensions of steel faces and wood core of the used sandwich beams.

III. Experimental Work

Eight specimens one solid steel beam and seven steel/wood sandwich beams have been prepared as cantilever beams to investigate the dynamic performance experimentally. The seven sandwich beams have the dimensions of faces thickness, beam width, and length as indicated in the theoretical work.

The test rig used to determine the natural frequencies of the specimens is shown in **Figure 3**. It consists of:

- Rigid fixation of the beam end.
- Impact hammer with steel tip.
- Piezoelectric accelerometer (B&K 4515, frequency range 1Hz-10kHz, weight 8.3 gram).
- Line drive charge amplifier (ENDEVCO, frequency range (1Hz-1KHz)).
- Data acquisition system (DSO-2090 USB-100MS/s Real Sampling-40 MHz Bandwidth-Hantek).
- Laptop and software program (DSO 2090 USB V6).

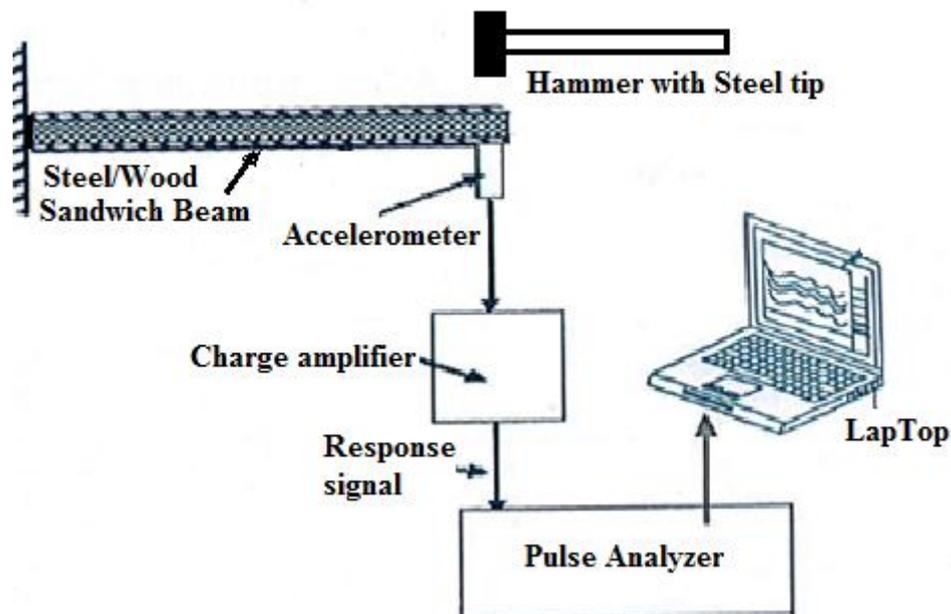


Figure 3: Layout of the Experimental Test Rig

Each beam is firmly clamped at one end in the tool holder of a heavy lathe as shown in **Figure 4**. An impulsive force is applied to the beam free end using an impact hammer to excite free damped natural vibrations.

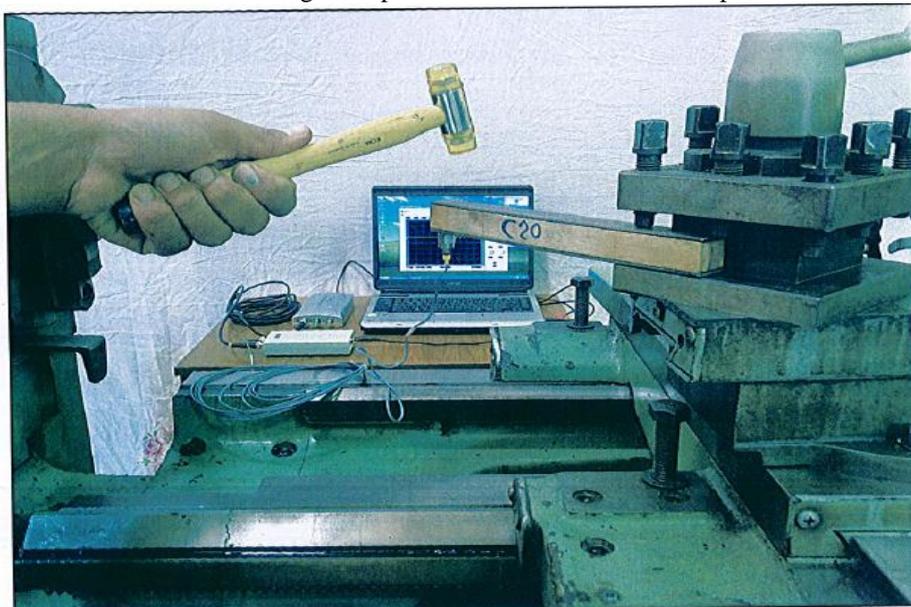


Figure 4: Experimental Test Rig

The vibration signals are fed into the data acquisition system to obtain the frequency response of the sandwich beam under test. The data acquisition system software program, based on digital system oscilloscope (DSO-2090 USB), is run on the laptop for determining the frequency response of the sandwich beams in order to get time domain or frequency domain spectra.

The frequency response spectrum of the solid steel beam with 16mm thickness, 25mm width, and 180mm length is shown in **Figure 5**.

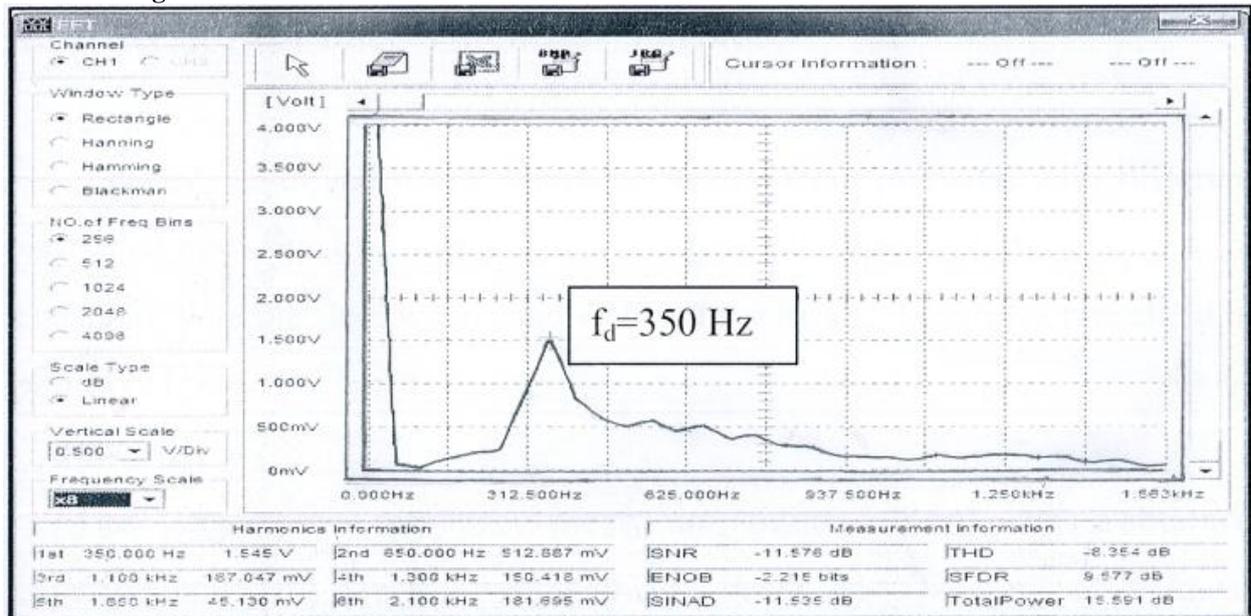


Figure 5: Frequency Response of the Solid Steel Beam

Figures (6-12) show the frequency response spectra of steel/wood sandwich beams with different dimensions.



Figure 6: Frequency Response Spectrum of Steel/Wood Sandwich Beam ($h_f = 1.5\text{mm}$, $h_c = 10\text{mm}$, $B = 25\text{mm}$, and $L = 180\text{mm}$)

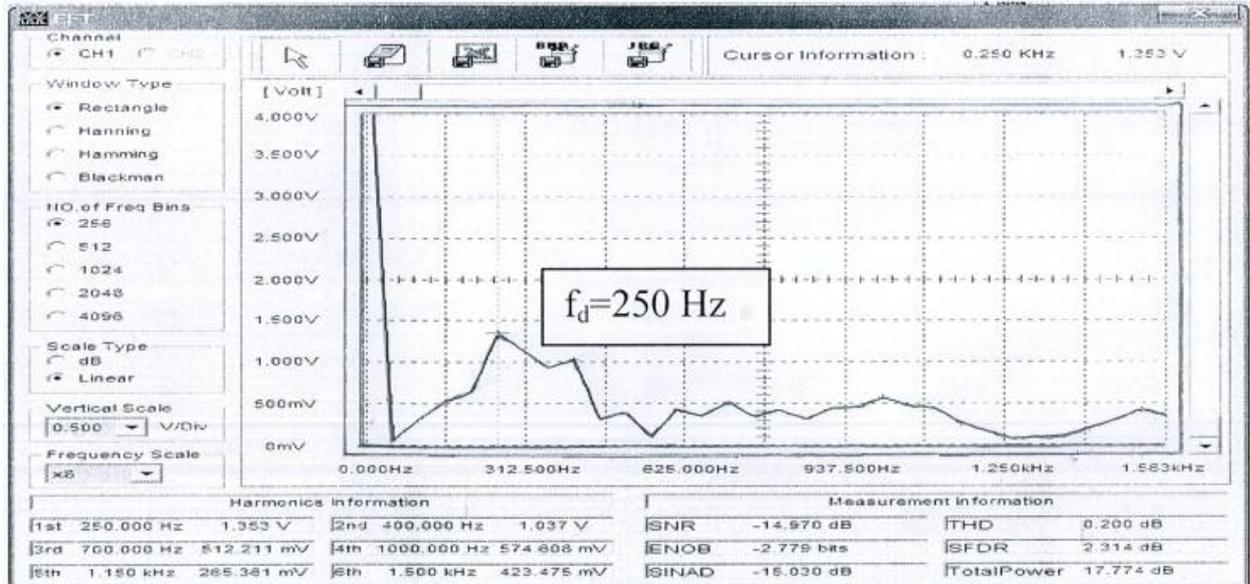


Figure 7: Frequency Response Spectrum of Steel/Wood Sandwich Beam ($h_f= 3\text{mm}$, $h_c= 10\text{mm}$, $B= 25\text{mm}$, and $L= 180\text{mm}$)

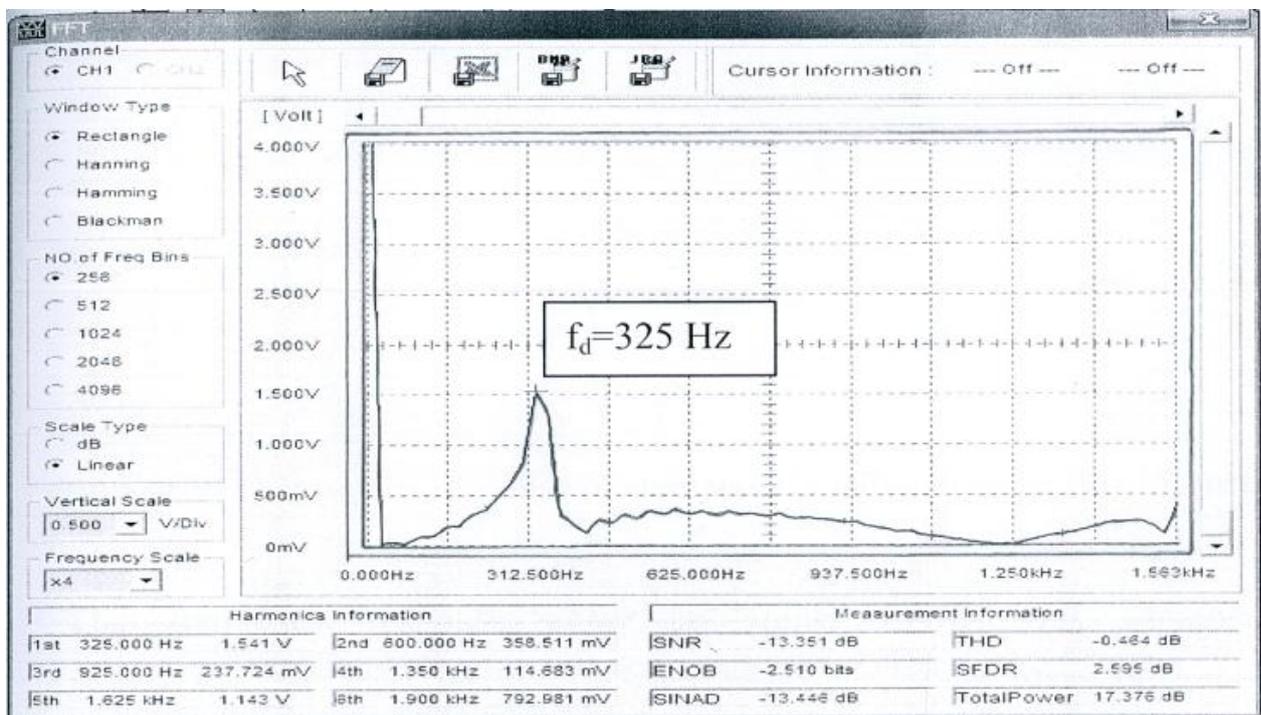


Figure 8: Frequency Response Spectrum of Steel/Wood Sandwich Beam ($h_f= 4.5\text{mm}$, $h_c= 10\text{mm}$, $B= 25\text{mm}$, and $L= 180\text{mm}$)

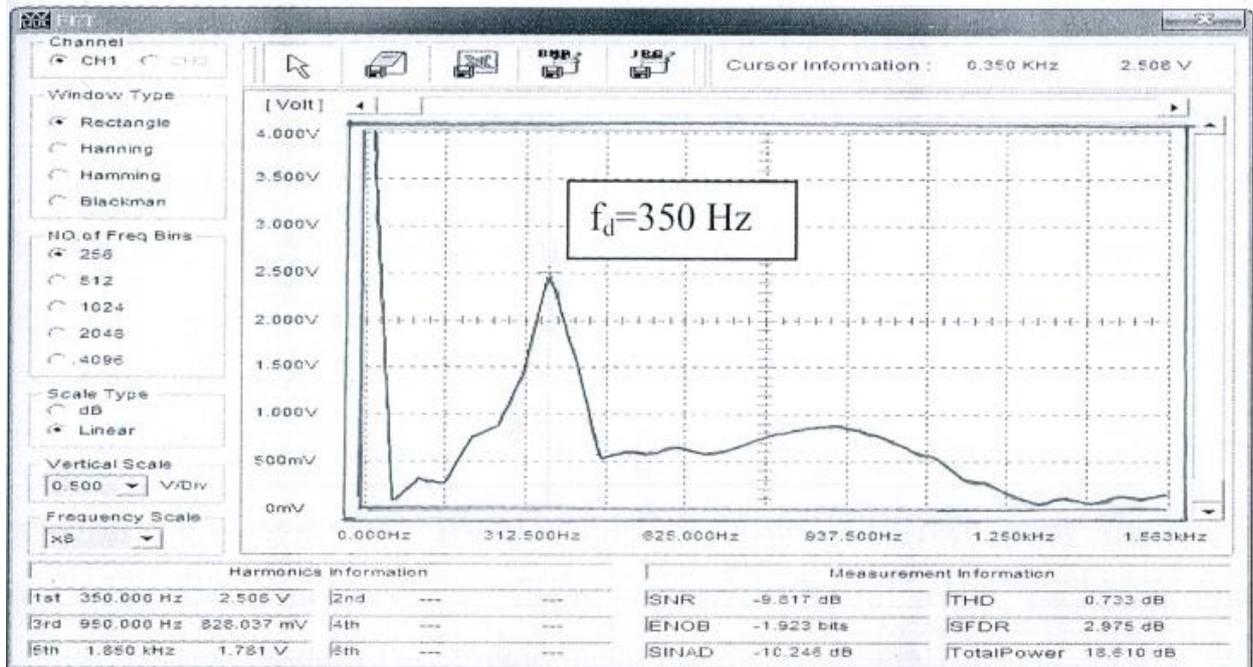


Figure 9: Frequency Response Spectrum of Steel Wood Sandwich Beam ($h_f= 3\text{mm}$, $h_c= 15\text{mm}$, $B= 25\text{mm}$, and $L= 180\text{mm}$)

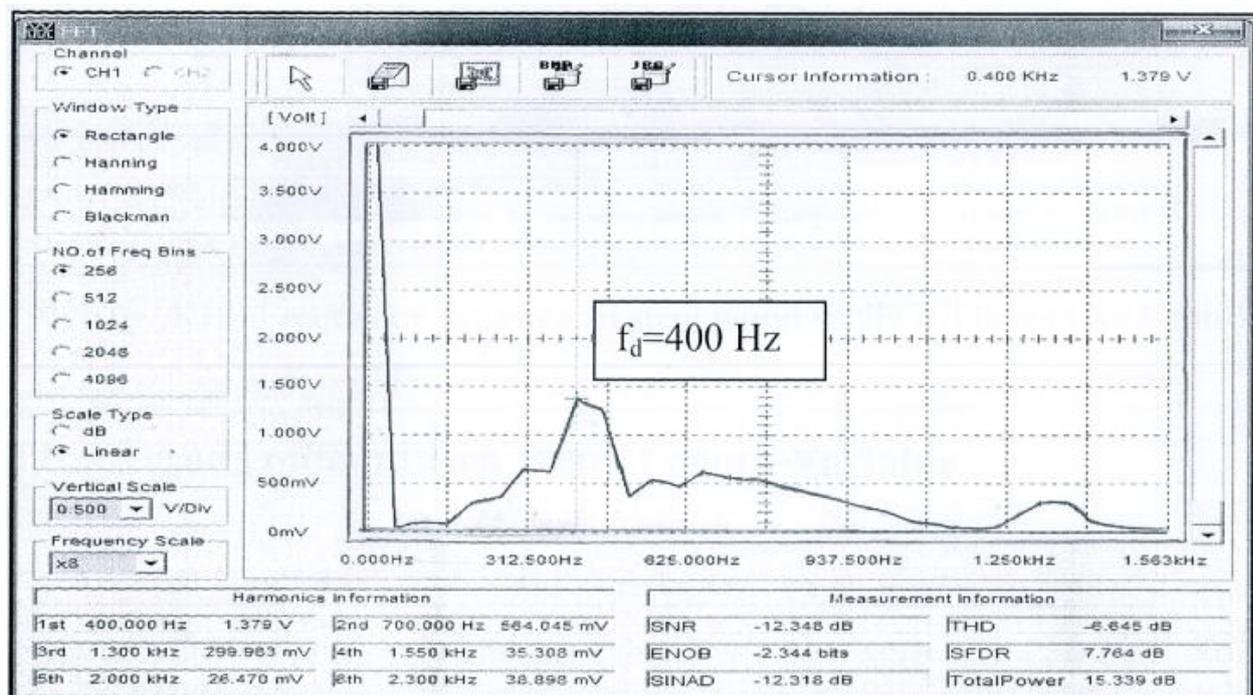


Figure 10: Frequency Response Spectrum of Steel Wood Sandwich Beam ($h_f= 3\text{mm}$, $h_c= 20\text{mm}$, $B= 25\text{mm}$, and $L= 180\text{mm}$)



Figure 11: Frequency Response Spectrum of Steel Wood Sandwich Beam ($h_f= 3\text{mm}$, $h_c= 10\text{mm}$, $B= 18\text{mm}$, and $L= 180\text{mm}$)

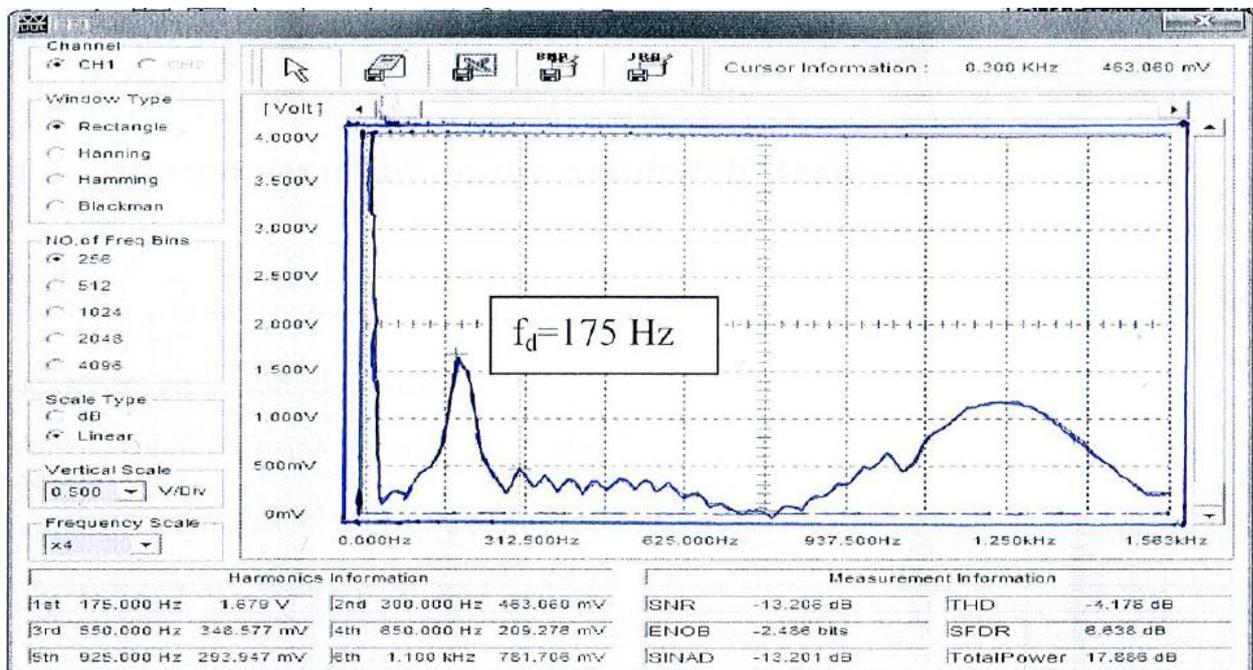


Figure 12: Frequency Response Spectrum of Steel Wood Sandwich Beam ($h_f= 3\text{mm}$, $h_c= 10\text{mm}$, $B= 25\text{mm}$, and $L= 240\text{mm}$)

IV. Experimental Results And Discussion

1. Comparison between theoretically and experimentally determined natural frequencies (f_a)

According to the results of the frequency response curves of solid steel and steel/wood sandwich beams, a comparison between theoretical and experimental damped natural frequencies (f_d) is given in Table 2.

Table 2: Theoretically and Experimentally Determined Natural Frequencies of the Solid Steel Beam and the Steel/Wood Sandwich Beams

Specimens	Dimensions in mm					Theoretical f_d (Hz)	Experimental f_d (Hz)
	h_f	h_c	H	B	L		
Solid Steel Beam	-	-	16	25	180	348.625	350
Steel/ Wood Sandwich Beam	1.5	10	13	25	180	191	200
	3	10	16	25	180	248.8	250
	4.5	10	19	25	180	336.8	325
	3	15	21	25	180	343	350
	3	18	24	25	180	411.4	420
	3	10	16	20	180	248.8	250
	3	10	16	25	240	179.2	175

2. Comparison between the dynamic characteristics of the solid steel beam and the steel/wood sandwich beam.

Table 3, gives the calculated values of the dynamic performance parameters of interest for the solid steel beam and the steel/wood sandwich beam having the same external dimensions (H , B , and L) and same end conditions as fixed end cantilever beams.

Table 3: Dynamic Characteristics of Solid steel beam and Steel/Wood sandwich Beam

Specimens	h_f	h_c	H	B	L	E_{eq}	K^*_{eq}	f_n	f_d
Solid Steel Beam	-	-	16	25	180	210	6.5×10^6	405.22	348.62
Steel/Wood sandwich Beam	3	10	16	25	180	162.39	11.6×10^6	542.17	248.8

In spite of having 23% lower equivalent stiffness (K_{eq}) due to lower modulus of elasticity (E_{eq}), the sandwich beam has 77% higher stiffness-to-weight ratio (K^*_{eq}) than the solid steel beam due to its considerably lighter weight leading to 34% higher undamped natural frequency (f_n). However, due to its higher damping ratio (75% larger than solid steel), the damped natural frequency of the sandwich beam is 28% lower than that of the solid steel beam, which limits its frequency application range.

3. Effect of the Sandwich Beam Thickness on its Dynamic Characteristics:

An increase of 50% of the sandwich beam thickness (H) has been attained by either each the following methods:

- The increase of the steel face layer thickness (h_f) from 1.5mm to 4.5mm at a constant wood core thickness of 10mm to detect the effect of increasing the equivalent modulus of elasticity (E_{eq}) associated with the increase of the beam equivalent mass (m_{eq}).
- The increase of the wood core thickness (h_c) from 10mm to 18mm at a constant face layer thickness (h_f) of 3mm to reduce the beam equivalent mass (m_{eq}) and hence to attain higher stiffness-to-weight ratio, inspite of the reduction of the equivalent modulus of elasticity (E_{eq}).

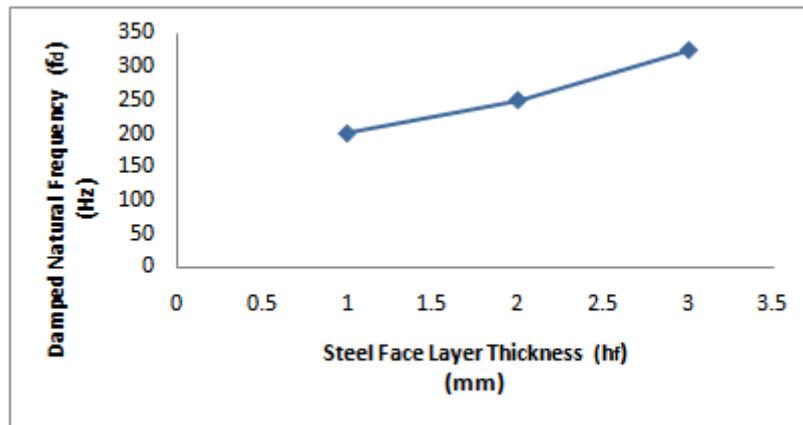
Each of the above mentioned methods is discussed in what follows;

3.1. Effect of the Increase of the Steel Face Layer Thickness

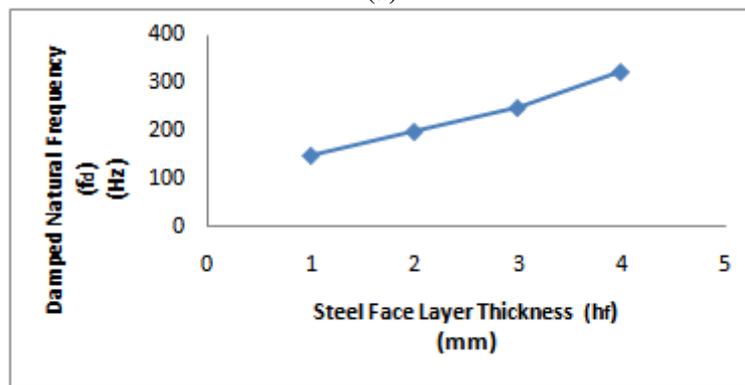
The increase of the steel face layer thickness (h_f) of the sandwich beam from 1.5mm to 4.5mm keeping the core thickness (h_c) at a constant value of 10mm (**Table 4**) leads to the increase of the damped natural frequency (f_d) from 200 Hz to 325 Hz by (62.5%), due to the increase of the equivalent stiffness (k_{eq}) to 4.6 times caused by the increase of both the equivalent modulus of elasticity (E_{eq}) by 50% and the increase of the section moment of inertia three times, while the stiffness-to-weight ratio is only increased by 85% as shown in **Table 4** and **Figure (13-(a), (b))** due to the 1.5 times increase of the equivalent mass (m_{eq}) caused by the relatively heavier steel face layers.

Table 4: Effect of the Increase of the Steel Face Layer Thickness (h_f) on the Dynamic Characteristics of the Steel/Wood Sandwich Beam

Specimens	h_f	h_c	H	B	L	E_{eq}	K^*_{eq}	f_n	f_d
Steel/Wood Sandwich Beam	1.5	10	13	25	180	121.242	8.2×10^6	456.07	200
	3	10	16	25	180	162.39	11.6×10^6	542.17	250
	4.5	10	19	25	180	181.570	15.2×10^6	619.51	325



(a)



(b)

Figure 13: Effect of Steel/Wood Sandwich Face Layer Thickness on k_{eq}^* & f_d

3.2. Effect of the Increase of the Core Thickness

The increase of the wood core thickness (h_c) from 10mm to 18mm leading to the same increase of the beam thickness (H) as in 3.1 causes the increase of the damped natural frequency (f_d) by 68% as shown in **Table 5** and **Figure 14**, (f_n) is increased by 72.7% due to the 2.27 times increase of the stiffness-to-weight ratio, which reveals that the increase of the sandwich beam thickness by increasing of the steel face thickness is slightly more beneficial than that attained by increasing of the wood core thickness with respect to the damped natural frequency, while the increase of wood core thickness is much more beneficial with respect to the beam stiffness-to-weight ratio.

Table 5: Effect of the Increase of the Wood Core Thickness on the Dynamic Characteristics of the Steel/ Wood Sandwich Beam

Specimens	h_f	h_c	H	B	L	E_{eq}	K_{eq}^*	f_n	f_d
Steel/Wood Sandwich Beam	3	10	16	25	180	162.39	11.6×10^6	542.178	250
	3	15	21	25	180	138.935	21.6×10^6	730.381	350
	3	18	24	25	180	166.722	25.9×10^6	936	420

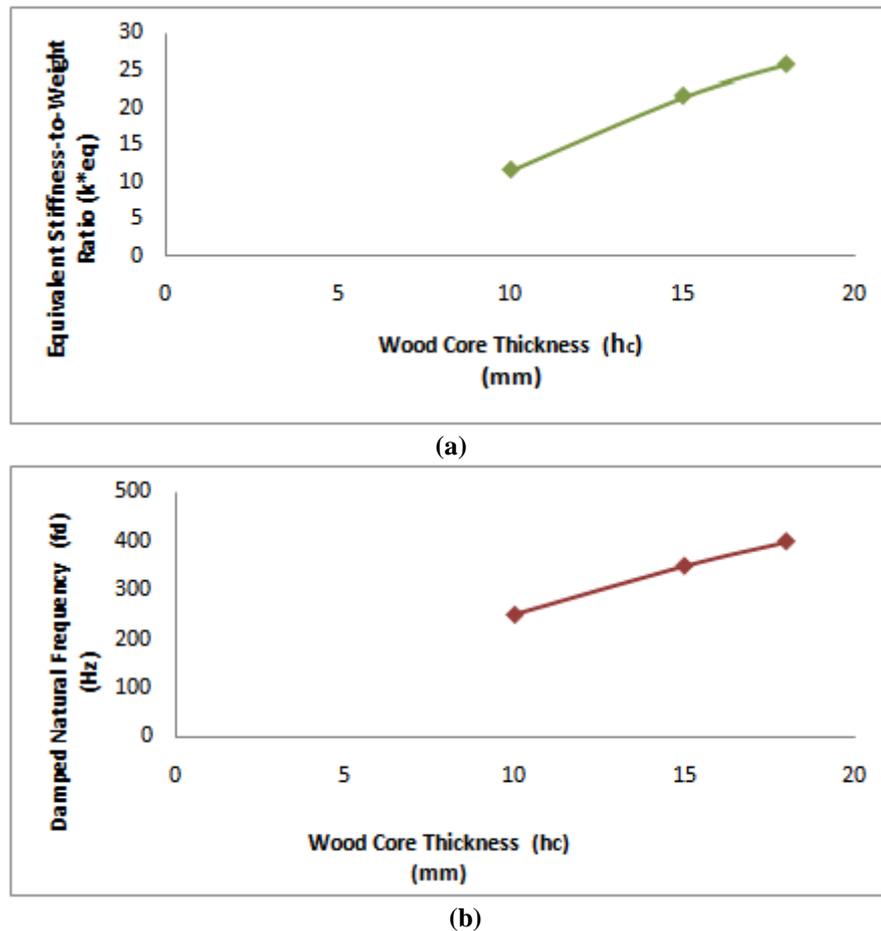


Figure 14: Effect of Wood Core Thickness on k_{eq}^* & f_d

4. Effect of the Beam Width on the Dynamic Characteristics of the Sandwich Beam.

The reduction of the sandwich beam width (B) by 20% (from 25mm to 20mm), leads to the decrease of each of the equivalent stiffness (K_{eq}) and equivalent mass (m_{eq}) by the same percentage since both of them varies proportionally with the beam width however the equivalent stiffness-to-weight ratio (k_{eq}^*) remains the same which leads to nearly the same undamped and damped natural frequencies (damping ratio is insignificantly affected). The variation of the sandwich beam width has therefore practically no effect on the beam dynamic characteristics.

5. Effect of the Increased of the Beam Length on the Dynamic Characteristics of the Sandwich Beam.

The increase of sandwich beam length (L) by 33% (from 180mm to 240mm) results in the reduction of the equivalent stiffness (K_{eq}) by 42% being inversely proportional to (L^3) and the decrease of the equivalent mass (m_{eq}) by 33% leading to 67% lower stiffness-to-weight ratio (k_{eq}^*) and hence 56% lower undamped natural frequency (f_n) and 28% lower damped natural frequency (f_d). The increase of beam length therefore impairs the dynamic characteristics of sandwich beams.

V. Conclusion

The dynamic performance of the steel/wood sandwich beams has been investigated in the present paper both theoretically and experimentally. Eight specimens of solid steel beam and different dimensions of steel/wood sandwich beams have been used to study their damped and undamped natural frequencies as an indication of the dynamic performance of the composite material.

The study has revealed that;

- The undamped natural frequency for steel/wood sandwich beam is higher while the damped natural frequency is lower than for the solid steel beam having the same dimensions.
- The damping ratio and the stiffness-to-weight ratio for the sandwich beam are better than for the solid steel beam having the same dimensions.
- The stiffness-to-weight ratio and the undamped natural frequency of the steel/wood sandwich beam can be remarkably increased by increasing the beam thickness through the increase of the core thickness or to a

less extent, by increasing the steel face layer thickness. The increase of the beam length has on the contrary an adverse effect on its dynamic performance while the increase of the beam width has no effect.

- The obtained theoretical and experimental results indicate a close agreement with a maximum deviation of $\pm 5\%$. Therefore the theoretical analysis can be used in the design stage to serve in the selection of the best sandwich beam parameters.
- The sandwich beam parameters must be carefully selected to increase its advantages to make it attractive for such applications, where vibration damping and high stiffness to weight ratio are required.

However it has been concluded that the selection of the optimum sandwich beam parameters depends on the application requirements.

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