Analysis of Thermal Characteristics of Flared and Rectangular Fin Profiles by Using Finite Element Method

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Abstract : In this study, temperature distribution and heat flux through various fin surfaces have been investigated. Here only steady state heat transfer was discussed. The temperature distribution has been estimated by taking the assistance of finite element method. In order to do so, automatic meshing method has been used. The meshing process was done by using ANSYS meshing utility. Later these results were compared to find out the relatively more efficient fin profile. Finally, the success of this simulation based experiment was mainly dependent on the refinement of generated mesh as well as its quality. Both hexahedral and tetrahedral mesh element were utilized but considering the capability of the available device (processing unit), available time, complexity and accuracy, finally automatic method was chosen. For this experiment, flared and rectangular fin arrays were considered. Based on the variations in fin profiles, different temperature distribution as well as heat flux have been found.

Keywords: ANSYS Workbench, FEM, flared fin array, hex dominant element, rectangular fin array, SolidWorks, tetrahedral element.

I. Introduction

Heat transfer describes the exchange of thermal energy, between physical systems depending on the temperature and pressure, by dissipating heat. The fundamental modes of heat transfer are conduction or diffusion, convection and radiation. The exchange of kinetic energy of particles through the boundary between two systems which are at different temperatures from each other or from their surroundings. In past, various machineries frequently failed because of inefficient heat transfer. And then the idea of fin introduced. In the study of heat transfer, a fin is a surface that extends from an object to increase the rate of heat transfer to or from the environment by increasing convection. Experimental analysis were done and are being done around the world to determine fin heat transfer rate as well as efficiency. By using simulation software, analysis has become easier and more effective. Finite element method is a method which mainly deals with precise mathematical calculation for determining the approximate solution. It takes simpler elements from the main problem and calculate separately and thus minimizes the probabilities of errors.

The purpose of this experiment is to conduct numerical analysis on some complex fin profiles such as flared fin and rectangular fin. In actual practice, numerous experiments had been concluded on other fin profiles but finally these were chosen as they have complex geometries and are not used widely. Such fin arrangements are difficult to prepare for lab experiment. It is one of the main purposes of choosing these profiles in order to avoid this constraint and numerical analysis was performed to visualize and obtain the temperature distribution as well as heat flux.

1.1 Literature Review

Previously, various work has been done using finite element method for solving heat transfer based problem. Malatip et al carried an analysis of conjugate heat transfer between solid and unsteady viscous flow using finite element analysis in 2009 [1]. In their work, a fractional four-step finite element method for analyzing conjugate heat transfer between solid and unsteady viscous flow is presented. The method uses a three-node triangular element with equal-order interpolation functions for all the variables of the velocity components, the pressure and the temperature. Four test cases, which are the lid-driven cavity flow, natural convection in a square cavity, transient flow over a heated circular cylinder and forced convection cooling across rectangular blocks, are selected to evaluate the efficiency of the method presented. Huang et al has performed an analysis to obtain the fin performance under dry, wet and partial wet conditions. They used Numerical models for micro channel heat exchanger [2]. Abdullah H. AlEssa and Mohamad I. Al-Widyan tried to enhance the natural convection heat transfer from a fin by triangular perforation of bases parallel and toward its tip [3]. This study examined the heat transfer enhancement from a horizontal rectangular fin embedded with triangular perforations (their bases parallel and toward the fin tip) under natural convection. In this study, assistance of finite element analysis had been taken. Abdul Majeed and S. AL-Ghamdi theoretically investigated

a Cascaded Fin Thermal Behavior [4]. Three different methods of solution are implemented in this study. Abdul Majeed and S. AL-Ghamdi also numerically investigated the result using MATLAB. The results of this study show that MATLAB can be used effectively and efficiently to solve challenging heat transfer problems. Luo et al performed a stress study on different designs of ceramic high temperature heat exchangers. In this study, the thermal stress performances of common and new designed ceramic plate heat exchangers are presented [5]. The temperature distribution and thermal stresses are calculated using ANSYS. Singh et al compared the performance of different profiles of fins using thermal analysis [6]. The numerical analysis has been carried out for computing the temperature and thermal stress distributions in circular fins using ANSYS. The models were constructed of same dimensions and only outer profiles were changed using CATIA were made rectangular, tapered and helical. Later the boundary conditions were set up using ANSYS. Mohsen Torabi and Hessameddin Yaghoobi analyzed thermal analysis of the convective-radiated fin with a step change in thickness and temperature dependent thermal conductivity [7]. This paper provides heat transfer analysis in a straight fin with a step change in thickness and variable thermal conductivity, which is losing heat by simultaneous convection and radiation. The calculations are carried out by using the differential transformation method (DTM) that can be applied to various types of differential equations. The results obtained employing DTM are compared with an accurate numerical solution to verify the accuracy of the proposed method. Sorathiya et al reviewed the effect of cylinder block fin geometry on heat transfer rate of air-cooled 4S SI Engine [8]. Computational Fluid Dynamic analysis and Wind tunnel experiments have shown improvements in fin efficiency by changing fin geometry, fin pitch, number of fins, fin material and climate condition. Abdullah H. M. AlEssa, studied the augmentation of fin natural convection heat dissipation by square perforations [9]. This study examined heat transfer enhancement from a horizontal rectangular fin embedded with square perforations under natural convection compared to the equivalent solid (non-perforated) fin. The parameters considered were geometrical dimensions and thermal properties of the fin and of the perforations. The study considered the gain in fin area and extent of heat transfer enhancement due to perforations. It showed also that for certain range of square dimension and spaces between perforations there is an improvement in perforated fin heat dissipation over that of the equivalent solid one. Christopher L. Chapman and Seri Lee analyzed the thermal performance of an elliptical pin fin heat sink [10]. Comparative thermal tests have been carried out using aluminum heat sinks made with extruded fin, cross-cut rectangular pins, and elliptical shaped pins in low air flow environments. The elliptical pin heat sink was designed to minimize the pressure loss across the heat sink by reducing the vortex effects and to enhance the thermal performance by maintaining large exposed surface area available for heat transfer. The performance of the elliptical pin heat sink was compared with those of extruded straight and crosscut fin heat sinks, all designed for an ASIC chip. Abdullah H. AlEssa, Ayman M. Maqableh1 and Shatha Ammourah tried to enhance the natural convection heat transfer from a fin [11]. In this study, the enhancement of natural convection heat transfer from a horizontal rectangular fin embedded with rectangular perforations of aspect ratio of two has been examined using finite element technique. The results for perforated fin have been compared with its equivalent solid one. A parametric study for geometrical dimensions and thermal properties of the fin and the perforations was carried out. The study investigated the gain in fin area and of heat transfer coefficients due to perforations. K. A. Rajput and A. V. Kulkarni performed a finite element analysis of convective heat transfer augmentation from rectangular fin by circular perforation [12]. Vinay Pal examined fin heat sink with scales on fins cooled by natural convection [13]. He used a high power LED array as uniform heat load of 65 watt. Rupali V. Dhanadhya, Abhay S. Nilawar and Yogesh L. Yenarkar observed the heat transfer augmentation from horizontal rectangular fin with circular perforation [14]. This study examines the heat transfer augmentation from horizontal rectangular fins with circular perforations under natural convection compared with solid fins. Fins with different thickness keeping length constant are also examined. The parameters considered were geometrical dimension and thermal properties of fin such as material properties, convective heat transfer coefficient. P. Sai Chaitanya, B. Suneela Rani and K. Vijaya Kumar analyzed by Varying Its Geometry and Material [15]. Transient thermal analysis determines temperatures and other thermal quantities that vary over time. Hiechan Kang evaluated of fin efficiency and heat transfer coefficient for fined tube heat exchanger [16]. This study discussed the estimation of the fin efficiency and the pure-heat transfer coefficient in the heat exchanger. One hundred twenty cases of plate fins having known heat transfer coefficients were tested numerically to investigate the validity of the previous classical theory on the fin efficiency. Qusay R. Al-Hagag, Hameed K. Al Naffiey and Hayder Krady R. analyzed convection and radiation in a triangular fins heat exchanger [17]. The effects of convection and radiation on triangular heat exchanger fins of a catalytic reactor are investigated. Heat is transferred by conduction along the fin and dissipated from the surface by convection and radiation. The base of the fin is maintained at a variable elevated temperature, while the tip loses heat due to convection and radiation. Due to symmetry and simplicity of analysis, one of the walls is considered. M. Sudheer, G. Vignesh Shanbhag, Prashanth Kumar and Shashiraj Somayaji carried a finite element analysis of thermal characteristics of annular fins with different profiles [18]. Finite element method (FEM) was used to compute the temperature and the stress fields. An extensive study was carried out using ANSYS. It was found

that the radius ratio and fin profiles are the significant parameters affecting the temperature and thermal stress distribution in annular fins. W. Lewis, P. Nithiarasu and Kankanhalli N. Seetharamu concluded an extensive writing to obtain the best quality element using finite element method for heat and fluid flow [19].

1.2 Objectives

- > The sole purpose of this study is to analyze temperature distribution and heat flux of flared fin and rectangular fin profiles.
- Generating solid models by using CAD package (for this experiment, SolidWorks 2014 was used).
- Choosing the best physics preference for solving this problem. In this experiment, ANSYS Mechanical has been selected.
- > Determining the best meshing element.

II. Methodology And Theoretical Analysis

While proceeding with the simulation process, flared and rectangular fin profiles were considered. As the analysis is too extensive, the whole working procedure can be broadly subdivided in the following sections:

- Geometry construction
- Mesh generation
- Specifying analysis type and solver
- Setting up the initial condition
- Finally obtaining result.

2.1 Geometry construction

ANSYS workbench has its own CAD (Computer Aided Design) software named as Design Modeler. But for the sake of convenience, fin geometry has been designed by using SolidWorks. SolidWorks itself is a widely popular CAD package. Following fin geometry has been considered for this analysis.

- Rectangular fin array
- Flared fin sample.



Figure 1: Computer aided design of rectangular fin array.

Rectangular fin array in Fig. 1 was designed in such way, so that rectangular shaped geometry has been extended from a hollow cylinder. The inside of the cylinder was used as heating surface in this analysis. The outer diameter of this hollow cylinder is 25 mm and thickness is 1 mm. the height is 100 mm. total number of fin attached with the surface is twelve. Whereas their dimension is $25 \times 2 \times 100 \text{ mm}^3$.



Figure 2: Computer aided design of flared fin array.

Flared fins were installed on a base plate with dimension $100 \times 60 \times 20 \text{ }mm^3$. Total surface area of the shortest fin profile is 4803 mm^2 and the longest fin profile has a total surface area of 6633.87 mm^2 .

2.2 Mesh generation

Three types of meshing methods were used in this experiment. Primarily automatic method was chosen to perform this analysis including tetrahedron element and hex dominant element. But automatic method was preferred as it is least time consuming. Refinement of mesh and minimum mesh generating time are of paramount importance but due to processing unit's limitation, automatic method was selected.

2.3 Finite element method

Finite Element Analysis is based on the premise that an approximate solution to any complex engineering problem can be reached by subdividing the problem into smaller, more manageable (finite) elements. Using finite elements, complex partial differential equations that describe the behavior of structures can be reduced to a set of linear equations that can easily be solved using the standard techniques of matrix algebra. Finite element analysis is used for optimizing new designs, verifying the fitness of existing facilities, predictive performance and evaluating new concepts. In addition, it has been used extensively for accident reconstruction and forensic investigations. The basic idea in the finite element method is to find the solution of a complicated problem by replacing it by a simpler one. Since the actual problem is replaced by a simpler one in finding the solution, it would be possible to find only an approximate solution rather than the exact solution. The existing mathematical tools will not be sufficient to find the exact solution (and sometimes, even an approximate solution) of most of the practical problems. Thus, in the absence of any other convenient method to find even the approximate solution of a given problem, it's convenient to prefer the finite element method. Moreover, in the finite element method, it will often be possible to improve or refine the approximate solution by spending more computational effort. In the finite element method, the solution region is considered as built up of many small, interconnected sub regions called finite elements. As an example of how a finite element model might be used to represent a complex geometrical shape.

The finite element analysis was based on the following common assumptions for this experiment:

- Steady state heat flow.
- The materials are homogeneous.
- The convection heat transfer coefficient is same all over the surface.
- The temperature of the surrounding fluid is uniform.
- The thermal conductivity of the material is constant.

A three-dimensional (3D) solid element can be considered to be the most general of all solid finite elements because all the field variables are dependent of x, y and z. An example of a 3D solid structure under loading is shown in Fig.3. As can be seen, the force vectors here can be in any arbitrary direction in space. A 3D solid can also have any arbitrary shape, material properties and boundary conditions in space. As such, there are altogether six possible stress components, three normal and three shear, that need to be taken into consideration. Typically, a 3D solid element can be a tetrahedron or hexahedron in shape with either flat or curved surfaces. Each node of the element will have three translational degrees of freedom. The element can thus deform in all three directions in space.

Since the 3D element is said to be the most general solid element, the truss, beam, plate, 2D solid and shell elements can all be considered to be special cases of the 3D element. Theoretically, the 3D element can actually be used to model all kinds of structural components, including trusses, beams, plates, shells and so on. However, it can be very tedious in geometry creation and meshing. Furthermore, it is also most demanding on computer resources. Hence, the general rule of thumb is, that when a structure can be assumed within acceptable tolerances to be simplified into a 1D (trusses, beams and frames) or 2D (2D solids and plates) structure, always do so. The creation of a 1D or 2D FEM model is much easier and efficient. Use 3D solid elements only when there're no other choices. The formulation of 3D solids elements is straightforward, because it is basically an extension of 2D solids elements. All the techniques used in 2D solids can be utilized, except that all the variables are now functions of x, y and z.



Figure 3: Example of 3D solid under loadings.

Consider the same 3D solid structure as Fig. 3, whose domain is divided in a proper manner into a number of tetrahedron elements in Fig. 4 with four nodes as well as four surfaces, as shown in Fig. 5. A tetrahedron element has four nodes, each having three DOFs (u, v and w), making the total DOFs in a tetrahedron element twelve, as shown in Fig. 5.



Figure 4: Solid block divided into four-node tetrahedron elements.



Figure 5: A tetrahedron element.

Now consider a 3D domain, which is divided in a proper manner into a number of hexahedron elements with eight nodes and six surfaces, as depicted in Fig. 6. Each hexahedron element has nodes numbered from 1 to 8 in a counter-clockwise manner, as shown in Fig. 7. As there are three DOFs at one node, there is a total of 24 DOFs in a hexahedron element. It is again useful to define a natural coordinate system (ξ , η , ζ) with its origin at the center of the transformed cube, as this makes it easier to construct the shape functions and to evaluate the matrix integration. The coordinate mapping is performed in a similar manner as for quadrilateral elements. Like the quadrilateral element, shape functions are also used to interpolate the coordinates from the nodal coordinates.



Figure 6: Solid block divided into eight-nodal hexahedron elements.



Figure 7: An eight-nodal hexahedron element and the coordinate systems.

III. Results And Discussions

As mentioned before, three types of meshing methods were chosen for further study. But automatic method was preferred as it exhibited better uniformity and accuracy over other methods.



Figure 8: Generated mesh for flared fin array.

Fig. 8 is the generated mesh for flared fin profile. Coarse, medium and finer sized elements have been used. But in order to reduce the processing time, medium sized elements were chosen for concluding the simulation for both bodies. The final number of nodes in this case was 34777 and elements was 5040. It is imperative to have better and refined mesh in order to generate sufficient number of nodes and elements to make the computing much better.



Figure 9: Generated mesh for rectangular fin array.

Also for the mesh of rectangular fin profile in Fig. 9, medium sized elements were selected for the same reason as discussed above. The total number of nodes and elements was set at 92560 and 13570 respectively.

3.1 Rectangular fin array:

From Fig. 10, it can be seen that for 600 °C base temperature, the temperature at the outmost area of every fin was found 409.77 °C for selected fin geometry.



Figure 10: Temperature distribution of rectangular fin array.

And the minimum and maximum heat flux were found 419.04 W/m^2 and 8.9994e + 005 W/m^2 respectively which occurred at different portions of the same geometry which can be observed from Fig. 11.



Figure 11: Total heat flux of rectangular fin array.

Fins are used to transfer excess amount of heat from a body to the surroundings. It is observed that due to convective heat transfer, the heat transfer from the base to fin tip is decreasing gradually. Base temperature is 600 °C and at the middle of fin, the temperature was found around 500 °C as depicted in Fig. 10. Thus this experiment successfully satisfied the theoretical concept of fin.

3.2 Flared fin array

This is a special type of fin. In this geometry temperature distribution varied in different fin profile. For 600 °C base temperature, the temperature in middle fin was found 342.19 °C which is visualized in Fig. 12. And this temperature further decreased with the increase in length of fin.



Figure 12: Temperature distribution of flared fin array.

From the temperature distribution which was found from the solver, it is interesting to observe that the farthest fin has a tip temperature of 238.96 °C. As fin efficiency increases with length, the expected outcome has been observed for the farthest fin with longest fin length.



Figure 13: Total heat flux of flared fin array.

The minimum and maximum amount of heat flux were found 8018.2 W/m^2 and 1.149e + 006 W/m^2 respectively which occurred at different portions of the same geometry.

IV. Conclusion

Heat transfer distribution through rectangular and flared profiles was observed. In addition to that, heat flux distribution was also analyzed. The simulation was done by using finite element method based solver. Besides, for the meshing process, in order to conclude numerical analysis, ANSYS meshing utility has been used. From the simulation the following decision can be considered:

- With the increase in thermal conductivity of fin material, temperature distribution is increased.
- The ratio of the perimeter to the cross-sectional area of the fin should be as high as possible to increase the heat transfer.
- Two different types of mesh were used while performing this experiment. Later it was found that tetrahedral mesh had certain superiorities over hexahedral mesh due to less time to generate mesh, uniformity, non-extensive calculation. The simulation could be done by using hexahedral mesh but it was avoided because of the reasons stated above.

Simulation concludes that:

- Fin performance can be varied under various circumstances like, length of fin profile, coefficient of thermal conductivity, ambient temperature etc.
- Different fin profile needs to be chosen for different purpose. Not all fin profile will be effective for accomplishing the same objective.
- But in general fin performance depends upon the length of the cross-sectional area of selected fin profile. In case of flared fins (Fig. 12 and Fig. 13), it was observed that, different temperatures were obtained at different tips. For the longest profile, temperature was found 238.96 °C and for the shortest profile, temperature was found 342.19 °C.
- The temperature at the outmost area of every fin was found 409.77 °C for rectangular fin geometry (Fig. 10 and Fig. 11).

Before producing fin profile for commercial purpose, these can be checked via finite element analysis based solver. This simulation procedure can be concluded before producing fins in a large scale for different purpose using different software packages e.g. ANSYS, COMSOL Multiphysics, ABAQUS etc. in these thesis, ANSYS has been chosen to carry out the simulation procedure. Then the result will be checked if it's satisfactory for actual use considering different factor. If not, then the geometry can be easily modified and also the fin material can be easily changed. Then the simulation will be run again under different conditions. Finally, one will be easily able to choose the fin material. The cost will decrease in a dramatic level by taking the assistance of these simulations before mass production. Also huge time will be saved, which was used before for trial and error based production.

• The reliability of the whole thesis can be further verified by executing actual lab project.

- These simulations can also be done by refining the meshing i.e. constructing more nodes and elements in the geometry.
- There are better package available for creating a refined mesh e.g. HyperMesh, Pointwise, Gridgen. Although, it's noteworthy to mention that a geometry having smaller element, consume considerably large amount of times to complete the numerical analysis.
- It also requires the support of a relatively powerful device, which can be able to provide better processing speed.

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