Vibrations of a mechanical system with inertial and forced disturbance

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Abstract: We present a mechanical system model of the inertial type vibrocompactor used in railroad building. In the paper we examine forced vibrations of the mechanical system excited by inertial disturbance. Using a dynamical model of the mechanical system and applying numerical methods, frequency response and frequency relations are found.

Keywords: vibrocompactor, dynamical model, frequency response, inertial forced vibrations.

I. Introduction

Vibrocompacting of bulk materials is widely used in the construction, repair, and maintenance of automobile and rail-roads. This ensures that the road’s overlays are dense enough for the needed strength of the road.

Vibrocompacting is a dynamic process that creates a specific level of density of the bulk materials through regulating the different parameters — frequency, amplitude, and force/pressure.

The dynamical modelling of the vibrating machine of inertial type enables to study working regimes depending on different frequencies of disturbance action.

The results of the study of the dynamical model are vital in the construction of vibrocompactors.

II. Dynamical model

The inertial vibrocompactor can be represented as a two-mass system (Fig. 1) with two degrees of freedom, that performs forced oscillations generated by an inertial disturbance [1]. The parameters of the system are focused — it has been assumed that the motion is only along one of the principal axes.

The second kind Lagrange equation is used to describe the forced fluctuations from the stable state of the mechanical system in the presence of potential and dissipative forces.

The system of differential equations, describing the model, follows:

\[ M\ddot{x} + B\dot{x} + Cx = \ddot{z} \]  (1)
where:
\[
(M) = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad (B) = \begin{pmatrix} b_1 + b_2 & -b_2 \\ -b_2 & b_2 \end{pmatrix}, \quad (C) = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{pmatrix}, \quad \bar{z} = \begin{bmatrix} P \sin \omega t \\ F \end{bmatrix}
\]

### III. Natural frequencies and frequency response

1. **The natural frequencies** are determined for the free vibrations of the undamped system [2]:

   \[
   M\ddot{x} + C\dot{x} = 0
   \]  
   \[ (2) \]

   The harmonic solutions for \( x \) mean that it has the form \( x = X \sin \omega t \) and substitution into the equation of motion (2) gives:

   \[
   \left(-\omega^2 M + C\right)x = 0
   \]  
   \[ (3) \]

   The natural frequencies are found by solving the eigenvalue problem:

   \[
   \text{det}[C - \lambda M] = 0, \text{ where } \lambda = \omega^2 (\lambda > 0, \lambda \in \mathbb{R}).
   \]  
   \[ (4) \]

   The natural frequencies follow as \( \omega_i = \sqrt{\lambda_i} \).

   Using MATLAB and the values of the parameters listed in the appendix, the following natural frequencies are calculated:

   \( \omega_1 = 138.8 \text{ s}^{-1} \text{ and } \omega_2 = 929.5 \text{ s}^{-1}. \)

2. **Frequency response** is determined for the forced damped system [2] after a Laplace transformation of equation (1) has been performed:

   \[
   \left(Ms^2 + Bs + C\right)x(s) = z(s) \text{ or } X(s) = \left(Ms^2 + Bs + C\right)^{-1}z(s), \text{ where } s = i\omega \text{ and therefore:}
   \]

   \[
   X(i\omega) = \left(-\omega^2 M + i\omega B + C\right)^{-1}z(i\omega)
   \]  
   \[ (5) \]

   The frequency response function (fig.2) is built using (5) in MATLAB.

![Fig.2 Frequency response of the damped system](image)

### IV. Numerical analysis

We show the numerical analysis of the system for four cases: free vibrations (\( \omega = 0 \)), dynamic multiplication (\( \omega = \omega_1, \omega = \omega_2 \)), beating regime (\( \omega \) is close to \( \omega_1 \)) and regime with frequency inbetween natural frequencies [3].

The numerical solutions are found in MATLAB (see the values of machine’s parameters are in the appendix).
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Fig. 3 Free Vibrations at $\omega = 0$

Fig. 4 Forced Vibrations at $\omega = \omega_1 = 138.8 \text{ s}^{-1}$
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Fig. 5 Forced Vibrations at $\omega = \omega_2 = 929.5 \text{ s}^{-1}$

Fig. 6 Forced Vibrations at $\omega = 130 \text{ s}^{-1}$ (close to $\omega_1$)
In all cases, during the stationary regime, there are steady harmonic vibrations. At frequencies 130 s\(^{-1}\) and 400 s\(^{-1}\) (fig. 6, fig. 7) the non-stationary processes are poly harmonic due to the beat on both coordinates \(x_1\) and \(x_2\).

V. Conclusions

The examination of vibrations of the inertial type vibrocompactor prove that the machine can work steady and properly inside a wide range of frequencies except these ones in the area near the first natural frequency where the beat breaks up steady work in non-stationary regimes.

Such examination could take part in the process of constructing of new machines or improve the performance of the existing ones.

Appendix

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>dimension</th>
<th>Parameter</th>
<th>Value</th>
<th>dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of vibrator incl. unbalanced mass (-m_1)</td>
<td>40</td>
<td>kg</td>
<td>Damping coefficient of the gravel (-b_2)</td>
<td>1900</td>
<td>N/m</td>
</tr>
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<td>Mass of the tampers (-m_2)</td>
<td>60</td>
<td>kg</td>
<td>Damping coefficient of the suspension (-b_1)</td>
<td>200</td>
<td>N/m</td>
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<tr>
<td>Stiffness coefficient of the suspension (-c_1)</td>
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<td>N/m</td>
<td>Force (-F)</td>
<td>10</td>
<td>kN</td>
</tr>
<tr>
<td>Stiffness coefficient of the gravel (-c_2)</td>
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<td>N/m</td>
<td>Radius of the unbalanced mass (-r)</td>
<td>0.05</td>
<td>m</td>
</tr>
</tbody>
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References