

## A Dynamic Approach to the Lifting Of the Railway Track

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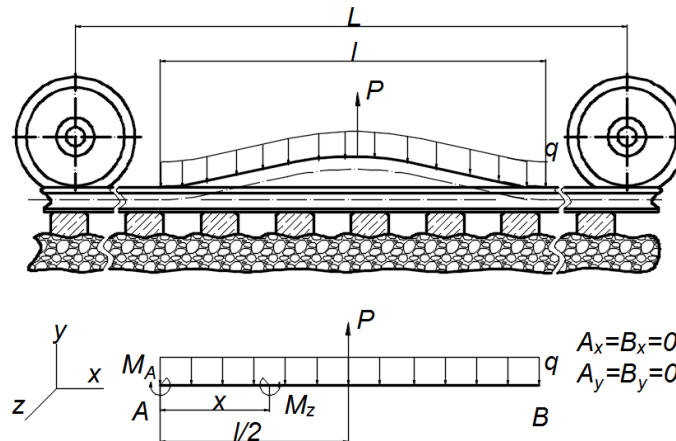
**Abstract :** The aim of this paper is to create a method for dynamical modelling of railway lifting which is presented as a one-mass system. There is analysis of important parameters based on the numerical solution of the differential equation of motion. Applications of this method are suitable for the synthesis and choice of basic parameters of devices which lift railway track.

**Keywords:** railway track, lifting, dynamical model

### I. Introduction

The lifting of railway track has a wide range of applications: during lifting work on switches; during tamping the track; during cleaning the ballast; maintaining the insulated joints; for lifting rails to replace rail pads and for lifting and slewing entire track panels.

There is a well-known [1] static method for determining the parameters of railway lifting. It includes integration of the differential equation (2) of the elastic line of the railway track presented as a beam (fig.1). Here, the assumption is that the length  $l$  of the lifted part of the track is in-between the wheels of the vehicle and the value of the lift  $y$  is rather small. The distributed load  $q$  consists of the rail's weight per meter and the resistance which the ballast applies to the sleepers during the lifting.



**Fig.1.** Calculational scheme of the parameters of lifting the railway track

The following expressions refer to the calculational scheme:

$$(1) \quad M_z(x) = M_A - \frac{qx^2}{2}$$

$$(2) \quad \frac{d^2y}{dx^2} = \frac{M_z(x)}{EJ_z} = \frac{1}{EJ_z} \left( M_A - \frac{qx^2}{2} \right)$$

after integration in the limits  $x=(0 \ l)$ :

$$(3) \quad M_A = \frac{ql^2}{24} \text{ and}$$

$$(4) \quad l(y) = \sqrt[4]{\frac{384EJ_z}{q} y}$$

## II. Dynamic Model

The lifted part of the railway track might be substituted with one-mass system (fig.2) which moves along the  $y$  axis pulled by the force  $P$ . The mass  $m$  is variable and depends on the value of the lift  $y$ . The stiffness  $c$  and damping  $b$  coefficients determine dynamical properties of the system.

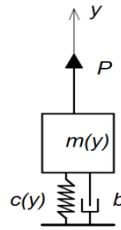


Fig.2. Dynamic model of lifting the rail track

The equation, describing the motion of the corresponding dynamical model is derived from the second kind Lagrange equation. It can be presented as follows:

$$(5) \quad \frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{y}} \right) - \frac{\partial E_k}{\partial y} = - \frac{\partial U}{\partial y} - \frac{\partial R}{\partial \dot{y}} + Q, \text{ where:}$$

$E_k$  is the kinetic energy of the system;  $U$  – potential energy;  $R$  – Rayleigh's dissipative function;  $Q=P$  – generalized force;  $y$  – generalized coordinate.

The left side of (5) after transformation is:

$$(6) \quad \frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{y}} \right) - \frac{\partial E_k}{\partial y} = m(y)\ddot{y} + \frac{1}{2} \frac{dm(y)}{dy} \dot{y}^2, \text{ where: } m(y) = \frac{ql(y)}{g}$$

The potential energy is:

$$(7) \quad U = \frac{1}{2} cy^2$$

The elastic force  $Q_e$  can be derived from (7):  $Q_e = - \frac{\partial U}{\partial y} = -cy$  or:

$$(8) \quad y = - \frac{1}{c} Q_e$$

As it is known from Theoretic Mechanics [2,3] the potential energy is also:

$$(9) \quad U = \frac{1}{2EJ_z} \int_0^l M_z^2(x) dx, \text{ where:}$$

$E$  is the Young's modulus (modulus of elasticity);  $J_z$  – the area (second) moment of inertia.

Using (1) and (3) and substituting in (9) we can derive:

$$(10) \quad U = \frac{1}{53EJ_z} q^2 l^5$$

As we know that  $P=ql(y)=Q_e$  we can present (10) as:

$$(11) \quad U = \frac{l^3(y)}{53EJ_z} P^2 = \frac{l^3(y)}{53EJ_z} Q_e^2$$

From (4), (7), (8) and (11) we can derive the stiffness coefficient  $c$  which depends on the value of the lift  $y$ :

$$(12) \quad c(y) = \frac{53EJ_z}{l^3(y)} = \frac{53EJ_z}{\left(\frac{384 EJ_z}{q}\right)^{\frac{3}{4}}} y^{\frac{3}{4}} = Sy^{\frac{3}{4}}$$

According to (7) and (12) :

$$(13) \quad \frac{\partial U}{\partial y} = 0,375Sy^{-\frac{1}{4}}y^2 + Sy^{\frac{3}{4}}y$$

The dissipative function is:

$$(14) \quad R = \frac{1}{2}by^2$$

The damping coefficient  $b$  varies in large range of values depend on the quantity of the ballast covering the sleepers.

The dissipative force is:

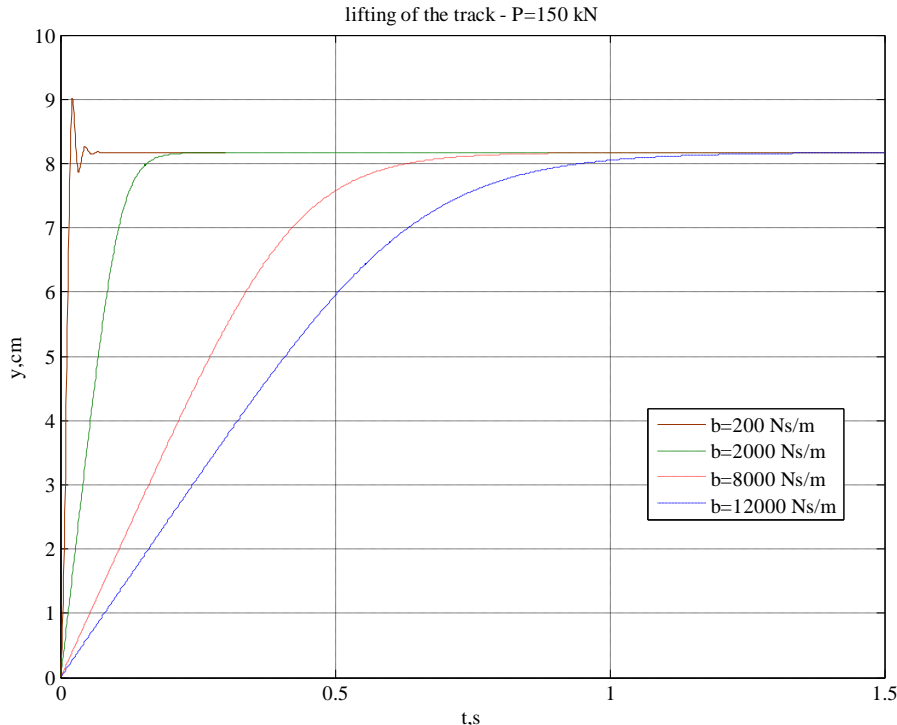
$$(15) \quad \frac{\partial R}{\partial y} = by$$

From (5), (6), (13), (14) we derive the equation of motion of the dynamic model:

$$(16) \quad \ddot{y} = -(Ky^{-1}\dot{y}^2 + 0,375Dy^{2,5} + Dy^{1,5} - P)$$

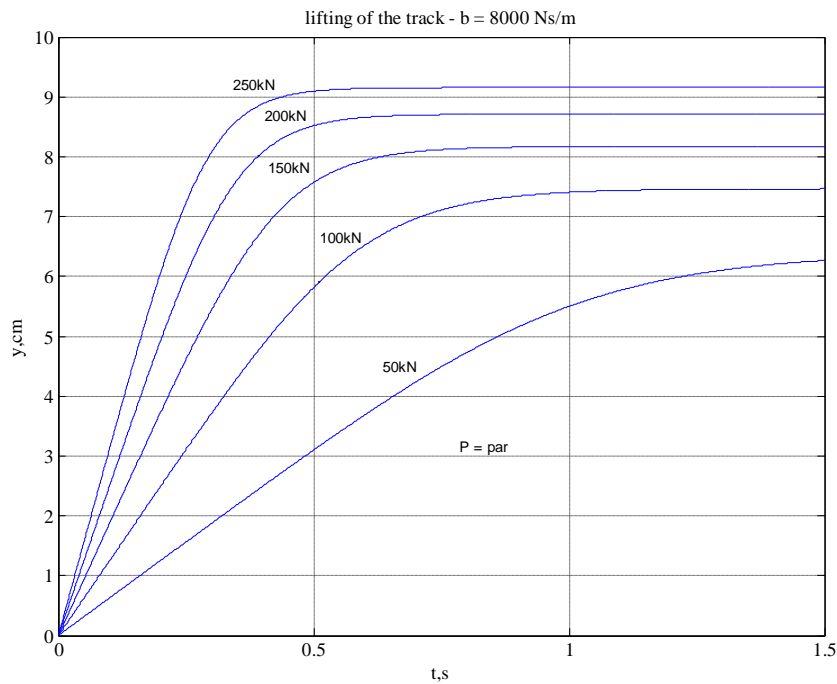
### III. Numerical Solution

The equation of motion (16) is essentially non-linear so it is solved numerically by using Runge-Kutte integration scheme (ode45 in MATLAB) [4] for various values for the damping coefficient  $b$  and lifting force  $P$ . The solutions have been plotted in Figures 3 and 4.



**Fig.3.** Lifting with constant force and various damping coefficients

These solutions show that damping coefficients have an effect on the duration and on the quality of the non-stationary process when the value of the lift is constant.



**Fig.4.** Lifting with constant damping coefficient and various forces

These solutions show that lifting forces have an effect on value of the lift and on the duration of the non-stationary process.

#### IV. Conclusions

The dynamic model of the railway track lifting gives opportunity to study the influence of the parameters of the system upon its behavior and stability.

#### References

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