Vibrations and Their Avoidance in Chip Removal Mechanical Processing

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Abstract: There are several procedures where the vibrations are necessary and their existence is limited. This does not apply in chip removal processing case as their existence is harmful and should be eliminated as much as possible. This article investigates cases of processing where high precision and surface quality are required and raising the chip removal system in order to increase the productivity and lower the costs.

Keywords: Vibrations; Chip removal; Mechanical Process; Amplitude frequency; Cutting tool.

I. Introduction

Due to the complex characteristics of vibrations, almost all the researchers explained and proved in different ways the causes of their appearance. It is known that during cutting of metals forced vibrations, auto-vibrations and revaluation vibrations may appear [1]. In order to explain the oscillator characteristics of vibrations, many theories were produced but until today there is no theory to explain the causes of auto-vibrations and their intimate phenomenon and to suggest the practical measures in order to avoid them or to completely eliminate them during chip removal processing.

II. Study of Vibrations

Forced vibrations appear because of dynamical external factors (that are not in connection with the intimate phenomenon of cutting process) such as: (1) periodical cutting forces appearing at input of cutter teeth, (2) periodical cutting forces appearing when the revolution surfaces are processed in case the semi-finished product has non-uniform processing addition or it has longitudinal channels, (3) periodical forces due to transmission of the motion of the electrical motor and others which appear when the electrical motor and part are not perfectly equilibrated and (4) periodical forces transmitted by neighbor machines through the foundation. Once the factors that produce forced vibrations are discovered, in a concrete situation, the measures to eliminate them are obvious [2]. Auto-vibrations appear when forced vibrations are completely removed and they are produced and entertained by a variable force that appears and driven by means of the vibrating motion itself. The force vanishes when the motion stops. In order to explain the auto-oscillator characteristics of vibrations many theories were produced, but until today there is none to elucidate the causes of auto-vibrations and to recommend practical measures in order to eliminate them [2,3]. However, auto-vibrations are some relative complementary movements in the technological system such as machines, devices, parts and tools. The vibration process of an element from the above chain can be described by a non-linear equation system that is very difficult to solve. In order to explain the creation of vibrations, it is proposed the theory regarding the cutting tool and part relative oscillatory movement, like a joint of the oscillations reciprocally bonded by generalized coordinates of a plane system. The part is considered an absolute hardness body having a uniform rotation motion. The entire oscillating process is determined only by moving the cutting tool. The mass of the oscillating system is considered to be concentrated in the top of the cutting tool, as can be seen in fig.1 (ξ and η are the stiffnesses) [4].

Figure 1: Mass of the oscillating system concentrated in the peak of the cutting tool.
The movement equations for the considered system, reported to ones y and z are:

\[ m\ddot{Z} + h_y\dot{Z} + C_y Z + C_{yz} Y = F \cos \alpha \] .............................................. (1)

\[ m\ddot{Y} + h_y\dot{Y} + C_y Y + C_{yx} Z = -F \sin \alpha \] .............................................. (2)

Where: \( m \): the mass of the oscillator system concentrated in the peak of the cutting tool.
\( h_y, h_z \): coefficients defining the resistance forces.
\( C_y, C_z \): stiffness coefficients showing the relation between the forces applied to mass "m", towards the deformations produced by them, considered equal with unity.
\( C_{yz}, C_{zy} \): coefficients for completing the elastic complainings, overlapping mass "m" thus avoiding its movement on the coordinate axes after the principle.

\( F \): cutting force.
\( \alpha \): angle between the direction of the cutting force and z axis.

Using the energetic method solves the equations (1), representing then as follows:

\[ \ddot{Z} + \omega^2 Z = \left( \omega^2 - \xi_z^2 \right) Z - 2\delta_z Z - \alpha_z Y + F \cos \alpha = \sum R_z \] .............................................. (2)

\[ \ddot{Y} + \omega^2 Y = \left( \omega^2 - \xi_y^2 \right) Y - \alpha_y Y + F \sin \alpha = \sum R_y \] .............................................. (2)

Where:
\( \xi_z, \xi_y, \delta_z, \delta_y, \alpha_z, \alpha_y \) are coefficients obtained when transforming equations (1) in (2).
\[ \xi_z^2 = \frac{C_z}{m}; \xi_y^2 = \frac{C_y}{m}; 2\delta_z = \frac{h_z}{m}; 2\delta_y = \frac{h_y}{m}; \alpha_z = \frac{C_{yz}}{m}; \alpha_y = \frac{C_{zy}}{m} \]

Given the shape of the movement for the case when the cutting speed is much bigger than the oscillator movement speed

\[ Y = \alpha \sin \omega t \] ............................................................... (3)
\[ Z = b \sin \omega t - \cos \omega t \] ............................................................... (3)

Introducing relation (3) in (2) and making zero the numbers of the members of resonance frequency, V.A. Kudinov obtained a four algebraic equation system defining the stationary amplitudes and frequencies [5, 6].

The frequency of the oscillation is

\[ \omega_z^2 = \frac{p}{2} \pm \sqrt{\left( \frac{p}{2} \right)^2 + q} \] .............................................. (4)

\[ p = 2\xi_z^2 + \alpha_y \delta_z \cos \alpha - 4\delta_z^2 + \alpha_y \delta_y \sin \alpha; \quad q = \xi_z^2 + \left( \xi_y^2 + \delta_z^2 \right) \alpha_y \delta_y + \alpha_y \alpha_z \delta_z \delta_y \]

The amplitude of the oscillation is

\[ A_y = \alpha = 2 \sqrt{\delta_z \sin \alpha} \] .............................................. (5)

The displacement angle of the oscillation phase is

\[ \tan \phi = \frac{\omega^2 - \xi_y^2}{2\delta_z \omega} \] .............................................. (6)

The amplitudes ratio is

\[ \frac{A_z}{A_y} = \frac{\delta_y}{\delta_z \alpha_y} \sqrt{\left( \omega^2 - \xi_z^2 \right) + 4\omega^2 \delta_z} \] .............................................. (7)

Using the obtained equations, all parameters indicate that the movement of the cutting tool peak during cutting with vibrations can be computed.

This movement represents the sum of two oscillatory movements having a phase difference between them, as we can see from relations (3) and (5).

\[ Y = A_y \sin \omega t; \]
\[ Z = A_y \sin(\omega t - \phi) \] .............................................................. (8)
During cutting, the cutting tool is removed from the equilibrium state by a certain factor, it starts implementing oscillatory movements in two directions, such as the relative resulting movement between the cutting tool and the part represents a closed curve, usually an ellipse (Fig. 2).

![Figure 2: The relative resulting movement between the cutting tool and the part represents a closed curve.](image)

Such a curve can be described also by the part rotation axis or can be a result of the difference of the coordinates related to cutting tool peak movement and part rotation axis. When the cutting tool peak moves on the direction cutting force “F”, meaning from point 1 to point 4; the cutting depth increases and in the same time chip section and cutting force value increases. For the movement in reverse all these are decreasing. Fig. 3 represents the variation of cutting force “F” as function of cutting tool peak movements during cutting with vibration [7, 8].

![Figure 3: The variation of cutting force F as function of cutting tool peak movements during cutting with vibration.](image)

The surface between the abscissa axis and the upper curve branch (Fig. 4) is representing the mechanical work consumed when transmitting the cutting tool movement on the direction of cutting force. The surface between the lower branch of the elliptical curve and abscissa axis is representing the mechanical work steeld from the vibratory movement of the cutting tool. The difference between the above surfaces (meaning the area of the ellipse) is representing the mechanical work consumed during a cycle of the oscillation in order to maintain the oscillatory movement of the cutting tool. Thenecessary energy was taken fromthe machine main driving system; the one that is making the part to rotate and the tool to advance longitudinally. Thus, the source compensates the consumed energy in order to maintain the oscillatory movement of the technological system [9, 10].
The cause of auto-vibrations is still unknown. Thus it is sustained and proved that the appearance of auto-vibrations is due to the following factors:

- Variable friction between the recess face of the tool and chips and also between the positioning face and the surface to be processed of the part.
- Variation of the cutting force value when the peak of the cutting tool enters the non deformed material and when it is rejected because of the action of the high durability layer formed in front of the cutting edge of the cutting tool.
- Variation of the cutting force value because of geometry change of tool cutting side during vibration cutting process. The change of recess angle ($y$) and ($\alpha$) is shown in Fig.4.
- Small rigidity of the technological system. The biggest stability belongs to the technological system that has a big rigidity in the direction of the normal to the generated surface and in the direction of the cutting force.
- Variation of the cutting force because of the irregularity of the part surface that has been obtained in the previous processing, having as a result the vibration of the cutting depth. If the cutting depth is increasing, the cutting force will increase and so on. So because of these oscillatory movements of the part symmetry axis and of the cutting tool peak during the processing with vibrations, and the surface of the part some wavy traces will appear (Fig. 5).

![Figure 4: The surface between the abscissa axis and the upper curve branch.](image1)

![Figure 5: Oscillatory movements of the part symmetry axis and of the cutting tool peak during the processing.](image2)

When it is considered that because of the vibration of "F" component of the cutting force, the peak of the cutting tool executes an oscillatory movement only on the direction of this component, the relation of the vibratory wave amplitude can be obtained:

$$A = \frac{2}{3} a \sqrt{\frac{v}{\omega}} \sqrt{\frac{\alpha - \frac{v h}{B}}{\beta}}$$  \hspace{1cm} (9)

Where:
- $a$ and $c$: positive constant coefficients
- $v$: cutting speed, m/min
- $h$: coefficient that increases when the cutting force increases
- $B$: width of the cutting edge of the cutting tool that is found in the chips
- $W$: is the circular frequency of the oscillation and has the value:

$$\omega = \sqrt{\alpha} = \sqrt{\frac{r+k}{m}}$$  \hspace{1cm} (10)

in this relation

$$r = q \frac{R}{s} = \text{const} \ tan \ t;$$ \hspace{1cm} (11)

Where:
- $q$: index (equal with 0.75). $R$: radial component of the cutting force for cutting without vibrations.
- $s$: feed. $m$: vibrating mass (the cutting tool, blade holder support and table saddle). $k$: elasticity or support give up coefficient (inverse of rigidity). One of the factors that most influence the size of the amplitude is the cutting
speed \( v \). In order to find the cutting speed for which we have maximum amplitude, we have to make the derivative \( \frac{dA}{dV} = 0 \), obtaining

\[
\nu_1 = \frac{2/3 \times \alpha B}{h} \tag{12}
\]

If the term under the square root in the expression (9) is equal to zero, then \( A = 0 \) meaning that, theoretically, the vibrations would disappear when

\[
\nu_2 = \frac{\alpha \times B}{h} \tag{13}
\]

The vibration of the vibratory light \( H = (A/2) \) is experimentally determined and represented in the diagram from fig.6. From the diagram we can see that the processing without vibrations is possible at small speeds, but than the processing has small productivity. This thing is resulting also from relationship (9) that shows that amplitude "A" drops when "w" increases and "w" increases when "r" increases and "r" increases when the advance "s" is small. It is recommended that low speed processing to be performed only in special cases (the part is big and eccentric).

Small amplitude of vibrations is obtained also when the processing is performed at high speeds, when both high quality of the processed surface and high productivity are guaranteed. Another factor that influences the size of the vibrations amplitude is the circular frequency of vibrations \( (w) \). According to the relation (10), we can see that the frequency of vibrations depends on "m", "k", and "r". Once the mass "m" is reduced and the system elasticity coefficient "k" is increased, the frequency increases. Also the frequency increases when "r" increases that is when the cutting width of the cutting tool "B" increases and the advance "s" decreases (because \( q < 1 \)) as can be seen from the expression of "r"

\[
\begin{align*}
\nu &= \frac{R}{s} = \frac{\nu}{s} = \frac{B}{s^{1-q}} = \frac{B}{s^{0.25}} \\
\text{where } C &= 0.75C_r
\end{align*}
\]

From the expression of "r", it can be seen that the frequency is also influenced by the characteristics of the metal from which the part is made of and geometrical shape of the cutting end of the cutting tool. This influence is expressed by the constant "C_r".

Relation vibrations are auto-vibrations of different nature where the exciting and resistance forces influence the shape and frequency of vibrations. These vibrations are produced when the rigidity or the mass of the system are small. The shape of relaxation vibrations is represented in fig. 7.

Figure 6: The vibration of the vibratory light \( H = (A/2) \).
The shape of the relaxation vibrations depends on the parameter, that is:
\[ \varepsilon = \frac{B}{\omega_0} = \frac{B}{u} \left( \alpha - \frac{u \cdot h}{B} \right) \frac{1}{\sqrt{m(k+r)}}. \]  
\[ \text{where } \beta = \frac{\alpha B - u \cdot h}{u \cdot m}. \]
(14)

After examining the curves in fig. 7, the following results are highlighted:
- \( \varepsilon = 0.1 \): the shape of vibrations is almost sinusoidal, the amplitude growing relatively slow. The vibratory motion will stabilize after a considerable period of time.
- \( \varepsilon = 0 \): corresponds to purely harmonic vibrations.
- \( \varepsilon = 1 \): the deviation from the sinusoidal shape becomes visible and the motion stabilizes faster.
- \( \varepsilon = 10 \): the vibrations have an options non-linear character and the vibratory motion stabilizes almost from the stars.

The amplitude of vibrations is given by:
\[ A = \frac{2}{3 \sqrt{\pi} \tau_0} \left( \delta - \frac{h \cdot u}{B \cdot \tau_0} \right) \left( \delta - \frac{h \cdot u}{B \cdot \tau_0} \right). \]  
(15)

Where: \( \delta \) is the excitation coefficient and
\[ \tau_0 = \frac{C}{0.25} = \frac{0.75Cr}{0.25} = \frac{128}{0.25} \]  
(16)

Once can see that the amplitude is more influenced by system rigidity (k) and not by the mass “m”.

The frequency of the relation vibrations is given by
\[ \omega = \frac{1}{T} = \frac{(k + r)\nu}{1.58 \left( \varepsilon \cdot \tau_0 - \frac{\nu \cdot h}{B} \right)}. \]  
(17)

So in case of relation vibrations, the frequency sudden increases when the system rigidity and cutting speed increases.

### III. Conclusions

The appearance of vibrations during the metal chip removal process can be avoided by taking adequate measures over:

1. Manufacture equipment: magnification of the rigidity of the machine tool by:
   - Eliminating the clearance from the main axle bearings decreasing the length in the bracket knee, using rotary peaks, fixing on elastic bearings;
   - Decrease of oscillatory or rotation masses without reducing the rigidity;
   - Reduction of external excitatory forces intensity by decreasing the centrifugal forces of various components in rotating motion, choosing an adequate speed for the part or tool, and
   - Isolation of the machine-tool from the vibrations of neighbor machines.

2. Manufacture tool:
   - Angels of incidence as high as possible (between 750 and 900);
   - Positive back- rake angels- angels of clearance as small as possible;
   - The radius from the cutting edge peak to be as small as possible;
   - The length in the bracket knee of the tool to be as small as possible;
   - When turning the external surfaces, the peak of the cutting tool has to be placed at the height of the part rotation axis;
   - Worm tools will not be used;
   - Use cutting tools with the cutting edge placed on the neutral axis of cutting tool body;
   - When processing through planning is performed buckled cutting tools should be used, and
   - Utilization of arched cutting tools also called swan throat.

3. Chip removal system:
   - The cutting speed to be in the area of small speeds or big speed;
   - The chip should not be too large and thin, and
• Utilization of vibration attenuator devices that absorb the energy of vibratory motion.

4. Manufacture part:

Having a geometrical shape of the semi-finished product closed to the one of the finished part, the layer of metal removed during chip removal is constant, the cutting depth is more constant and the cutting force more constant, resulting in minimum of vibrations and minimum errors for the part shape.

References