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Finite grid method for deformation of thin rectangular plates

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Abstract: *In this study, it is proposed to extend analytical solutions of the discrete one-dimensional elements for solution of rectangular thin plate problems. The derivations of the governing differential equations used to obtain exact shape functions and stiffness terms of one dimensional beam elements can be useful tools for solution of complex plate problems. Finite grid method is a numerical method and provides advantages in the sense of variable plate geometry and local changeable thickness, local changeable boundary conditions and loading. The maximum displacement of plate under distributed or concentrated load on the middle are calculated and compared the finite grid method and some other methods for variable boundary condition cases. The results verified that the ease in arriving at results of engineering accuracy by this method outweighs small errors.*
Keywords: *Boundary conditions, finite grid method, shape functions, stiffness matrices, thin plate*

I. Introduction

Introducing the finite element method in 1960s and the developments in computers have a great importance for the developments in applied mechanics. A broad range of the engineering problems has been solved by computer-based methods such as finite element and boundary element methods. A broad range of the beam or plates as engineering problems has been solved by computer-based numerical methods such as finite element and boundary element methods [1-6]. However closed form solutions for plates have been published for a limited number of cases. Owing to its convenience in solution of plate problems as a numerical method the finite strip method have attracted much attention from many authors as [7-8]. In order to simplify the problem it is possible to use a grid of beam elements to model plates. After all, within limitations of simplified formulation as Wilson [9] indicated, plate bending is an extension of one dimensional beam theory. Some numerical and approximate methods, such as finite element, finite difference, boundary element and framework methods have been developed to overcome such complex plate problems. The governing equation for transverse displacement w(x,y) of plates subjected to lateral loads by using two-dimensional Laplacian operator given in Eqn 1. as follows:

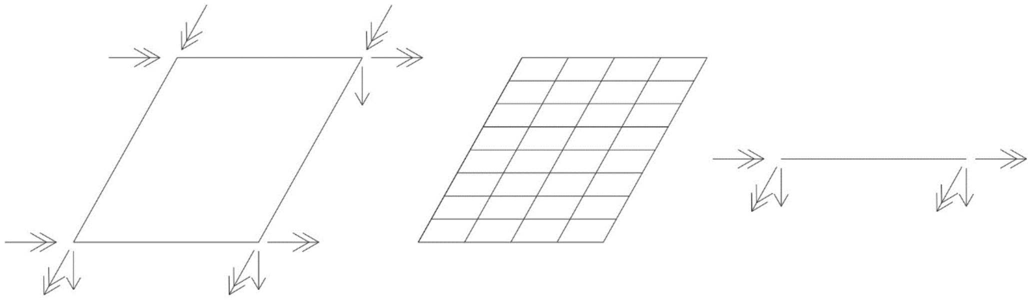
$$\nabla^2\nabla^2 w = \frac{q}{D}; \quad D = \frac{2h^3E}{3(1-v^2)} \tag{1}$$

where D is the flexural rigidity, E is modulus of elasticity, ν is passion ratio and h is thickness of the plate element and q is lateral loads

The closed form solutions have been published for a limited number of cases. In this study gridwork model of plates for general applications suggested solving a wide range of plate problems. A differential part of a plate is represented by two parallel sets of beam elements for rectangular plates [10-13]. The formulations based on interpolation (shape) functions have been used in solution by finite element method. The exact stiffness matrices of a beam element is used to solve general plate bending problems.

II. Formulation of plate elements

Hermitian polynomials into strain energy functions that has been derived to converge better solution, the beam needs to be divided into smaller segments. In this form, plates are idealized as a grillage of beams of a given geometry satisfying given boundary conditions as shown in Fig. 1.



a **b** **c**

Fig 1. The idealized discrete system **a)** the elements are connected at finite nodal points of a rectangular thin plate in flexure, **b)** Parallel sets of one-dimensional elements replaced by the continuous surface **c)** Typical node displacements and forces in a grid plane as a beam in local coordinates.

The framework method that replaces a continuous surface by an idealized discrete system can represent a two-dimensional plate. The plate through the lattice analogy at which the discrete elements are connected at finite nodal points can be represented by one dimensional elements have 3 degree of freedom (two rotations and one translation) at each node as shown in Fig.2.

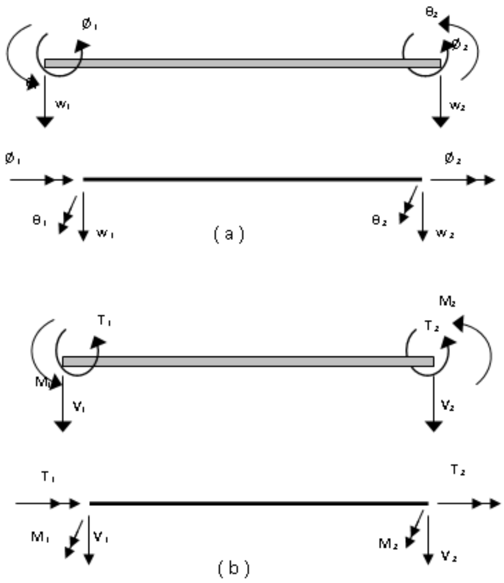


Fig. 2. The idealized discrete finite element of with 3 DOF's at each node (a) generalized displacements, (b) loads applied to nodes

Derivation of exact shape functions for a beam element related Fig.2 the homogeneous form of Eq. (1) is obtained by using q(x)=0. The generalized displacement vector which forms boundary conditions shown in Fig. 2 is obtained with x= 0 and x= L. From the figure:

$$\{\underline{d}\}^T = \{\phi_1, \theta_1, w_1, \phi_2, \theta_2, w_2\}$$
$$\{\underline{F}\}^T = \{T_1, M_1, V_1, T_2, M_2, V_2\} \qquad (2)$$

Then the arbitrary constant elements of the vector C can be related to the end displacements in matrix form as follows;

$$[\underline{d}] = [\underline{H}] \cdot [\underline{C}] \text{ or } [\underline{C}] = [\underline{H}]^{-1} \cdot [\underline{d}] \qquad (3)$$

where [H] is a 6x6 ,Substitute Eq. (3) into Eq. (1) then the closed form solution of the differential equation can be written in matrix form as:

$$[\underline{w}] = [\underline{B}]^T \cdot [\underline{H}]^{-1} \cdot [\underline{d}] \qquad (4)$$

Eq. (4) can be redefined by introducing vector N that includes six shape functions. Then the closed form of the solution in terms of shape functions and the generalized displacements defined in Fig. 2 can be defined as:

$$[\underline{w}]=[\underline{N}]\cdot\begin{Bmatrix}\phi(x=0)\\ \dfrac{dw}{dx}(x=0)\\ w(x=0)\\ \phi(x=L)\\ \dfrac{dw}{dx}(x=L)\\ w(x=L)\end{Bmatrix}\quad where\ [\underline{N}]=[\underline{B}]^T\cdot[\underline{H}]^{-1} \tag{5}$$

The non-dimensional forms of the shape functions as Hermitian polynomials for $\xi=\dfrac{x}{L}$ can be formed

$$\begin{aligned}\frac{\psi_2}{L}&=\xi-2\xi^2+\xi^3\\ \psi_3&=3\xi^2-2\xi^3-1\\ \frac{\psi_5}{L}&=\xi^3+\xi^2\\ \psi_6&=2\xi^3-3\xi^2\end{aligned} \tag{6}$$

The element stiffness matrix of a beam element, which relates the nodal forces to the nodal displacements by the exact shape functions for the prismatic beam element shown in Fig. 2. can be obtained from the minimization of strain energy as follows,

$$[\underline{K_e}]=EI\int_0^L\left\{\frac{d^2\{N\}}{dx^2}\right\}^T\left\{\frac{d^2\{N\}}{dx^2}\right\}dx \tag{7}$$

where N is, a 6x1 matrix of the exact shape functions. This equation aggregates the stiffness terms as follow;

$$\begin{aligned}k_{22}&=\frac{4EI}{L}\\ k_{23}&=-\frac{6EI}{L^2}\\ k_{25}&=\frac{2EI}{L}\\ k_{26}&=\frac{6EI}{L^2}\\ k_{33}&=\frac{12EI}{L^3}\\ k_{36}&=-\frac{12EI}{L^3}\end{aligned} \tag{8}$$

The element stiffness terms of the one dimensional beam element obtained with procedures by Karasin et al.[13], which relates the nodal forces to the nodal displacements, the conventional stiffness terms are verified as expected.

III. Results and discussion

In order to check the validity of the solution techniques some plate problems solved by the finite grid solution (FGM) evaluated with known analytical [14-16] and other numerical solutions such as SAP2000 and ANSYS Workbench [17-19]. As case study rectangular thin plates with various boundry conditions and loading types evaluated. The type of rectangular plate and analysis type are shown in Table 1 and 2 respectively.

Table 1. Types of plate examples

Code	Boundary conditions	Loading Conditions	ratio (b/a)
SSSS-P-1	All edges are simple supported	Concentrated Load at Centre	1
SSSS-P-2			2
CCCC-P-1	All edges are fixed	Concentrated Load at Centre	1
CCCC-P-2			2
SSSS-Q-1	All edges are simple supported	Uniform Distributed Load	1
SSSS-Q-2			2
CCCC-Q-1	All edges are fixed	Uniform Distributed Load	1
CCCC-Q-2			2

Table 2. Types of Analysis

Analysis code	Analysis type
Ref	Analytical Solution (Timoshenko, 1959)
FEM	Sap2000 solution by 8 subdivisions
ANSYS	Ansys Workbench solution by 8 subdivisions
FGM	Finite Grid Solution by 8 subdivisions

The plates for the two loading conditions as concentrated load at the center and uniform distributed loading are investigated. In the analysis Modulus of elasticity, E, is 87360 kN/m^2, poisson ratio, n, is 0,3 and plate thickness accepted as 0,05 meters. For b/a=1 and b/a=2 plate dimensions are given as 1x1m and 2x1 m. the normalized maximum deflection tabulated in Table 3 and the corresponding error percentages given in Table 4.

Table 3. Normalized max defection (w_{max})

Code	Ref	FEM	ANSYS	FGM
SSSS-P-1	11.601	11.937	12.752	12.474
SSSS-P-2	16.524	17.087	17.473	17.414
CCCC-P-1	5.600	5.895	6.416	6.091
CCCC-P-2	7.220	7.656	8.056	7.784
SSSS-Q-1	4.062	4.060	4.289	4.329
SSSS-Q-2	10.129	10.135	10.407	10.575
CCCC-Q-1	1.260	1.319	1.327	1.372
CCCC-Q-2	2.540	2.602	2.623	2.748

Table 4. Error Percentage, %

Code	FEM	ANSYS	FGM
SSSS-P-1	2.90	9.92	7.53
SSSS-P-2	3.41	5.74	5.39
CCCC-P-1	5.27	14.56	8.76
CCCC-P-2	6.04	11.57	7.81
SSSS-Q-1	0.06	5.57	6.57
SSSS-Q-2	0.06	2.75	4.41
CCCC-Q-1	4.68	5.33	8.92
CCCC-Q-2	2.44	3.28	8.17

From the table one can see that the plane-grid system as finite grid method with respect to relative error for deflections of points located on the axis passing through the centre of the plate reflects a high degree of accuracy.

IV. Conclusion

A grid work analogy called the Finite Grid Method involving discretized plate properties mapped onto equivalent one dimensional elements with adjusted parameters and matrix displacement analysis are used to develop a more general simplified numerical approach for plates. For particular plate problems, closed form solutions have been obtained. It has been verified the validity of the solution with applications of plates by comparing the other finite element and ANSYS results.

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