

# An Approach of Source-to-Target Mapping for Illumination with Discrete Uniformities

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**Abstract:** The key component in designing a freeform lens for LED lighting is the mapping grids of light rays from one mesh to another. For different mappings and illumination with discrete uniformities, the design was based on iteration and modification. However, the light utilization efficiency is typically limited. An approach for freeform lens design of illumination with discrete uniformities is proposed in this study. The proposed approach is composed of smooth source-to-target mapping for arbitrarily-shaped illumination without iteration modification. The adequacy of the proposed approach is demonstrated by the Monte Carlo ray tracing simulation and one experimental case study. The light utilization efficiency can be as high as 0.9 or above.

**Keywords:** Freeform lenses, Illumination, Grid mapping, Discrete uniformities

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## I. Introduction

A freeform lens has high degree of design freedom that can be used to achieve a compact design with an excellent optical performance. The lenses have been widely studied and implemented for LED lighting with uniformity or discrete uniformities [1-7]. In most of the studies, the key component in the design is the mapping of light rays to target illumination, from one grid mesh to another. The source-to-target mapping is required to be smooth to ensure that the edge-ray principle and the integrability condition hold [8-10]. The crossing of light rays before hitting the illumination target should be prevented in the design to gain the light utilization efficiency.

Smooth freeform surface can be obtained with an integrable mapping. It is straightforward to design smooth mapping if the meshes of source and target are of the same [11]. However, it is very difficult to find such an integrable mapping if the meshes are different or illumination with discrete uniformities is desired.

Frankot *et al.* [10] designed a reflector with smooth mapping from a circular grid mesh of source to a rectangular one of target. The mapping of equal luminous flux radially varies from ellipse-alike to rectangle-alike contours on a target plane. Luo *et al.* [12] proposed an iteration method with a modification mechanism for the design of a freeform lens for rectangular illumination. The modification mechanism was based on the difference between simulation and the desired illumination. Mao *et al.* proposed an iteration method for arbitrarily-shaped illumination [7]. The iteration process includes the source-target mapping establishment, freeform surface construction, ray-tracing simulation, and iterative feedback modification. The iteration method is applicable when the size of the light source cannot be neglected. However, smooth mappings may not be achieved, and it could result in a freeform surface with discontinuous normal. The light utilization efficiency is limited.

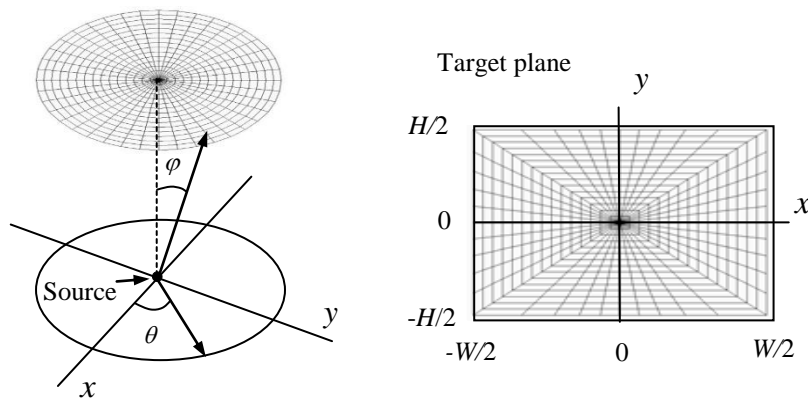
In this study, an approach for freeform lens design of illumination with discrete uniformities is proposed. The proposed approach is composed of smooth source-to-target mapping for arbitrarily-shaped illumination. The horizontal span of illumination is a single-valued function of  $y$  and the vertical span is a single-valued function of  $x$ . No iteration is required, and the light utilization efficiency is high. The adequacy of the proposed approach is demonstrated by the Monte Carlo ray tracing simulation.

## II. Uniform Illumination Design

In this study, an LED light is assumed to be a point source and the emitting pattern is characterized by a circular one. Only one-freeform surface is concerned in design. The luminous intensity distribution of the source is rotationally symmetric. The luminous intensity varies only with the inclination angle between emitting rays and the optical axis.

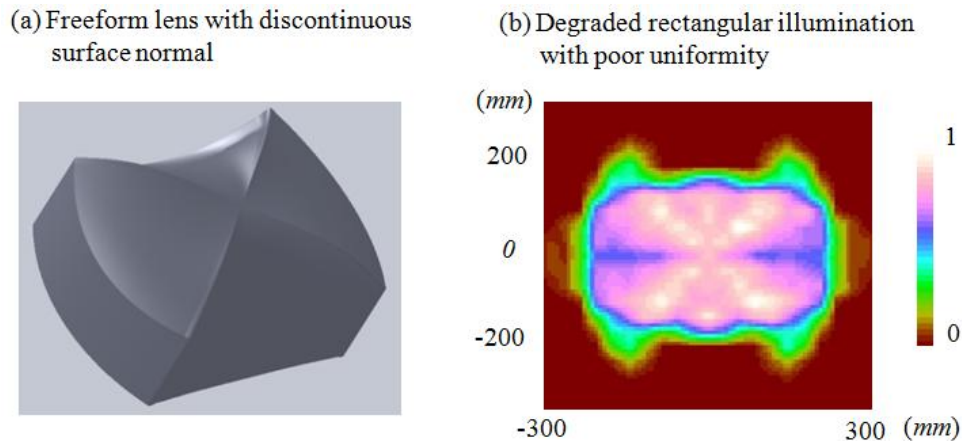
### 1. Previous non-smooth mapping

An example of mapping from a circular mesh of source to a rectangular one of target is shown in Fig. 1 where the width of the plane is  $W$  and the height is  $H$ . The mapping is described by the correspondence of the grids. The  $\varphi$ -contours of mapping are precisely from circles to rectangles. The contours are not differentiable at the corners of the rectangles, so it is not smooth.



**Fig. 1** Mapping from a circular mesh of source to a rectangular one of target.

Under the circles-to-rectangles mapping, a freeform lens achieved is shown in Fig. 2(a) where  $W=600$  mm,  $H=400$  mm, and  $\varphi - \theta$  grids are  $50 \times 200$ . The candela distribution of the source used in this study is given in [6]. The freeform lens has an obvious discontinuous surface normal. The normalized luminance distribution for 50,000-rays tracing using the LightTools software is shown in Fig. 2(b). The uniformity of illumination is poor, and the light utilization efficiency is less than 0.6.



**Fig. 2** Freeform lens and illumination achieved under the circles-to-rectangles mapping.

## 2. Proposed smooth mapping

The smooth and uniform mapping proposed in this study is given below:

$$x = \frac{\bar{\varphi} \cos \theta}{\sqrt{1 - \bar{\varphi}^2 \sin^2 \theta}} s_h(y) \quad (1)$$

$$y = \frac{\bar{\varphi} \sin \theta}{\sqrt{1 - \bar{\varphi}^2 \cos^2 \theta}} s_v(x) \quad (2)$$

where  $\bar{\varphi}$  denotes the normalized inclination angle of equal luminous flux ( $\bar{\varphi} = \varphi / \varphi_{\max}$ ) and  $\bar{\varphi} \in [0, 1]$ ;  $\varphi$  the inclination angle,  $\varphi_{\max}$  the maximal inclination angle;  $\theta$  the azimuth angle;  $s_h(y)$  the horizontal span of illumination,  $s_v(x)$  the vertical span of illumination.

It can be shown that the inverse mapping of Eq. (1) and Eq. (2) is

$$\bar{\varphi} = \sqrt{\frac{\bar{x}^2 + \bar{y}^2 - 2\bar{x}\bar{y}}{1 - \bar{x}^2\bar{y}^2}} \quad (3)$$

$$\theta = \tan^{-1} \left[ \frac{\bar{y}}{\bar{x}} \sqrt{\frac{(1 - \bar{x}^2)}{(1 - \bar{y}^2)}} \right] \quad (4)$$

where  $\bar{x} = x / s_h(y)$  and  $\bar{y} = y / s_v(x)$ .

For planar and rectangular illumination, the spans  $s_h(y)$  and  $s_v(x)$  are  $W/2$  and  $H/2$ , respectively. The mapping of Eq. (1) and Eq. (2) is shown in Fig. 3. The contours of equal luminous flux are similar to those achieved in design of a reflector [10]. The contours are smooth except at the extremes where  $\bar{x} = \bar{y} = \pm 1$  and  $\bar{\varphi} = 1$ .

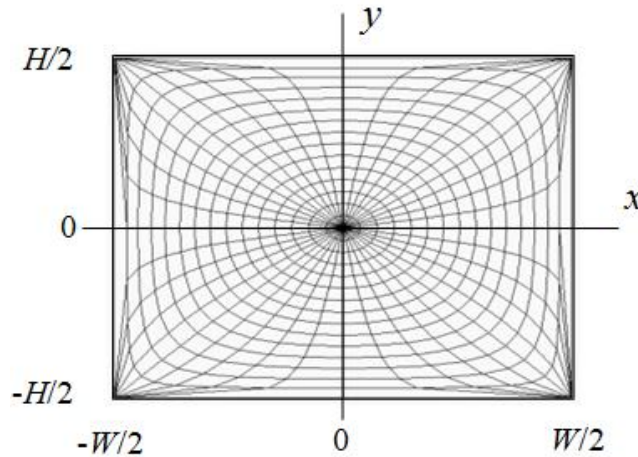
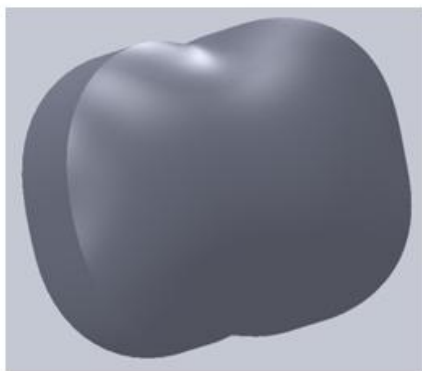


Fig. 3 Proposed uniform mapping for planar and rectangular illumination.

The resulting freeform lens using the proposed mapping is shown in Fig. 4(a). The whole freeform surface is smooth. Illumination with good uniformity is shown in Fig. 4(b), and the light utilization efficiency achieved is more than 0.9.

The proposed mapping is applicable for arbitrarily-shaped mesh where the horizontal span is assumed to be a single-valued function of  $y$  and the vertical span is a single-valued function of  $x$ . The spans of the mapping can be calculated and updated by iteration. Examples for triangular- and trapezoidal-shaped meshes are given in [6].

(a) Freeform lens with continuous surface normal



(b) Rectangular illumination with good uniformity

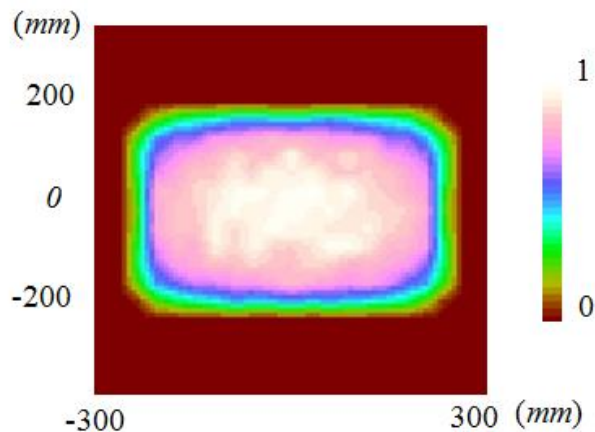


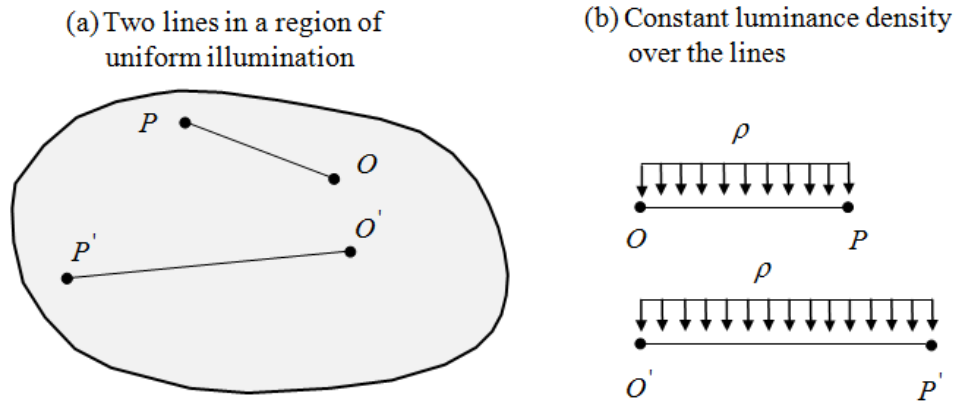
Fig. 4 Freeform lens and illumination achieved under the proposed mapping.

### III. Proposed Approach For Illumination With Discrete Uniformities

A region of uniform illumination shown in Fig. 5 has constant luminance density within the region. The luminance density over the lines  $\overline{OP}$  and  $\overline{OP'}$  in the region is

$$\rho = \frac{e}{OP} = \frac{e'}{OP'} \quad (5)$$

where  $\rho$  denotes the luminance density;  $e$  and  $e'$  the accumulated luminance over the lines, respectively.

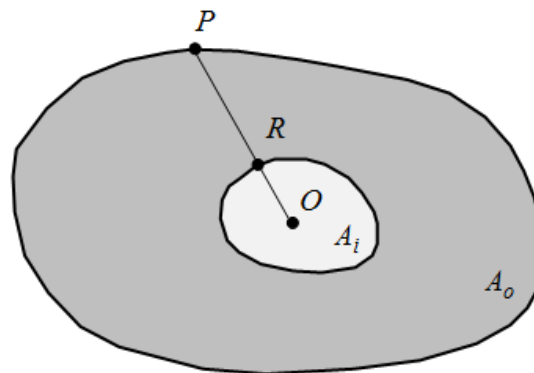


**Fig. 5** Uniform luminance density over two lines.

The proposed approach to design a lens for illumination with discrete uniformities is composed of two steps. The first step is to achieve uniform mapping grids over a region extended from the desired one using Eq. (1) and Eq. (2). For example, the desired region of illumination with discrete uniformities is shown in Fig. 6. The region is composed of two sub-regions (*i.e.*,  $A_i$  and  $A_o$ ) with different luminance densities, where one is a multiple of the other by a factor of  $m$ . Then, the accumulated luminance over the line  $\overline{OP}$  is

$$e = \rho(m\overline{OR} + \overline{RP}) \tag{6}$$

where  $\rho$  denotes the luminance density of  $A_o$ , and  $m\rho$  the luminance density of  $A_i$ .



**Fig. 6** Desired region of illumination with discrete uniformities.

The desired region of illumination is radially extended from the point  $O$  in the omni-directions. For example, the proposed extension along the line  $\overline{OP}$  is

$$\overline{PQ} = (m - 1)\overline{OR} \tag{7}$$

The region after the extension along the omni-directions is shown in Fig. 7. The extended region is denoted by  $A_e$ . The extended region normally has a complex and compound profile over which Eq. (1) and Eq. (2) can be applied to achieve the uniform mapping grids.

The second step of the proposed approach is to relocate the uniform mapping grids. Each of the mapping grids is relocated under conservation of luminance density such that  $A_e$  is contracted back to  $A_o$ . The relocation of the mapping grids under the conservation of luminance density is performed as follows.

For a mapping grid  $G'$  in Fig. 7, the line  $\overline{OQ'}$  through the grid is constructed. Then, grids distributed over the lines  $\overline{OQ'}$  and  $\overline{OP'}$  are defined in arrays as below:

$$[d_i] = [i\Delta d | i = 0, 1, \dots, n] \tag{8}$$

$$[d_j^*] = [j\Delta d^* | j = 0, 1, \dots, n] \tag{9}$$

where  $i$  denotes a thresholding index;  $j$  an interpolating index;  $n$  the number of total intervals of an array;  $\Delta d$  a fixed interval size,  $\Delta d^*$  a size with a step change. The interval sizes are defined as follow:

$$\Delta d = \overline{OQ'} / n \quad (10)$$

$$\Delta d^* = \begin{cases} \Delta d_1 & , \text{ if } j \leq n_1 = \overline{OR'} / \Delta d_1 \\ \Delta d_2 & , \text{ otherwise} \end{cases} \quad (11)$$

where  $\Delta d_2 = m \Delta d_1$ ,  $n_1 \Delta d_1 + n_2 \Delta d_2 = \overline{OP'}$ , and  $n = n_1 + n_2$ .

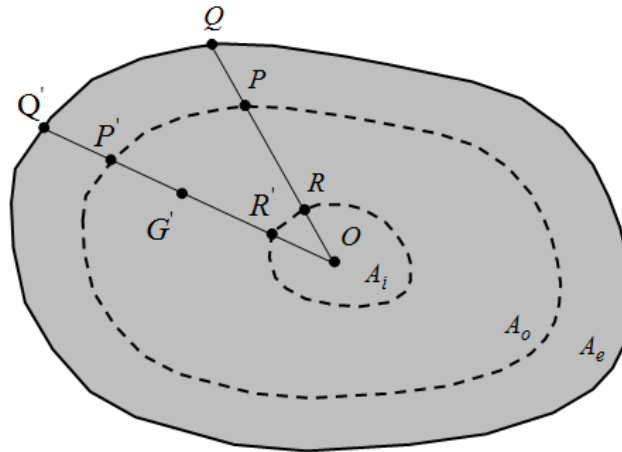


Fig. 7 Schematic as supplemental description of the proposed approach.

The elements of the array  $[d_i]$  denote the distances of  $\Delta d$  multiples. The array can be conceived as a union of grids which are uniformly distributed over the line  $\overline{OQ'}$ , while the array  $[d_j^*]$  is a union of grids, with a step-change distribution over the line  $\overline{OP'}$  that is in compliance with the desired illumination with discrete uniformities.

The grid arrays defined above for each of the mapping grids are used as references to relocate the grid. This relocation starts by thresholding the array  $[d_i]$  for  $\overline{OG'}$ , finding the corresponding index  $i$ , and then interpolating the index of the array  $[d_j^*]$  by letting  $j=i$ . The mapping grids after relocation have a discrete distribution of density. The grids are used to create a lens for illumination with discrete luminance uniformities.

The accumulated luminance over the line  $\overline{OQ'}$  in the extended region  $A_e$  is

$$e' = \rho' \overline{OQ'} \quad (12)$$

where  $\rho'$  denotes the luminance density of  $A_e$  with uniform illumination.

The same accumulated luminance is re-distributed over the line  $\overline{OP'}$ , and

$$e' = m \rho^* \overline{OR'} + \rho^* \overline{RP'} \quad (13)$$

where  $\rho^*$  denotes the luminance density of  $A_o$  of the desired illumination with discrete uniformities.

Eqs (7), (12) and (13) give

$$\rho^* = \rho' \quad (14)$$

The above conservation of luminance density holds in the omni-directions. Then, the luminance density of the whole region of  $A_o$  is conserved after the contraction of the extended region. This demonstrates that the region extension using Eq. (7) is essential and why the design of illumination with discrete uniformities can be achieved by the proposed approach.

#### IV. Case Studies

Refer to the study of Fig. 3 and Fig. 4 for design of illumination with discrete uniformities. The desired profile of illumination is defined by the images of Fig. 8, where different gray levels denote different levels of luminance to design. The region  $A_i$  is a circle with luminance two times higher than that of  $A_o$  (i.e.,  $m = 2$ ). Two cases are investigated for different locations of the circle. For Case 1, the circle is near the center of illumination; for Case 2, it is off-centered.

First, the outer contour of  $A_o$  is equi-spaced into 2000 grids in the computation of the extended region  $A_e$ , using Eq. (7). Then the mapping grids for uniform illumination over the extended region are calculated using Eq. (1) and Eq. (2) where the spans  $s_h(y)$  and  $s_v(x)$  are calculated by interpolation, based on the contour grids of  $A_e$ . The mapping grids for both cases are shown in Fig. 9.

The number of total intervals of the arrays set for the region contraction is  $n = 300$ . The mapping grids after the contraction are shown in Fig. 10. The contours of equal luminous flux are smooth for both cases, and used for the lens design. The resulting freeform lenses are shown in Fig. 11. Shown on the lenses are clear, convex freeform surfaces with the  $A_i$  profile. The resulting illumination by the Monte Carlo ray tracing simulations is shown in Fig. 12, respectively. Illumination with desired discrete uniformities is achieved, and the light utilization efficiency is 0.97 and 0.92 for Case 1 and Case 2, respectively.

For each of the cases, it takes only a few minutes for a design cycle, from a given illumination image and design parameters as inputs to the generation of a prototype freeform lens and running the ray tracing simulations. Fair experimental result for Case 1 is given in Fig. 13.

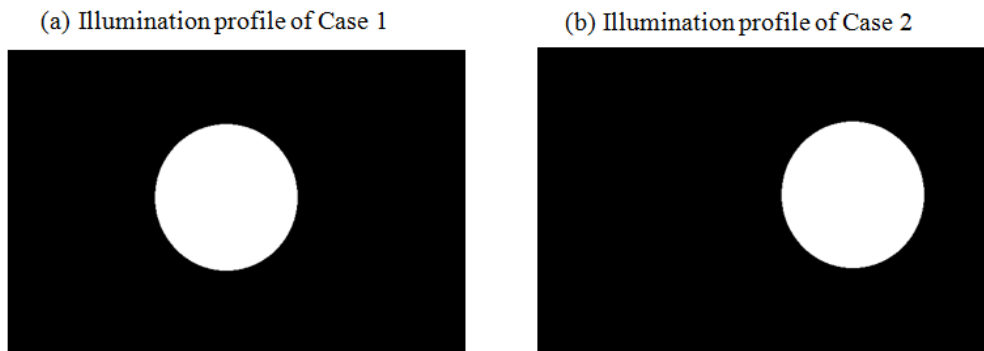


Fig. 8 Illumination profiles with discrete uniformities.

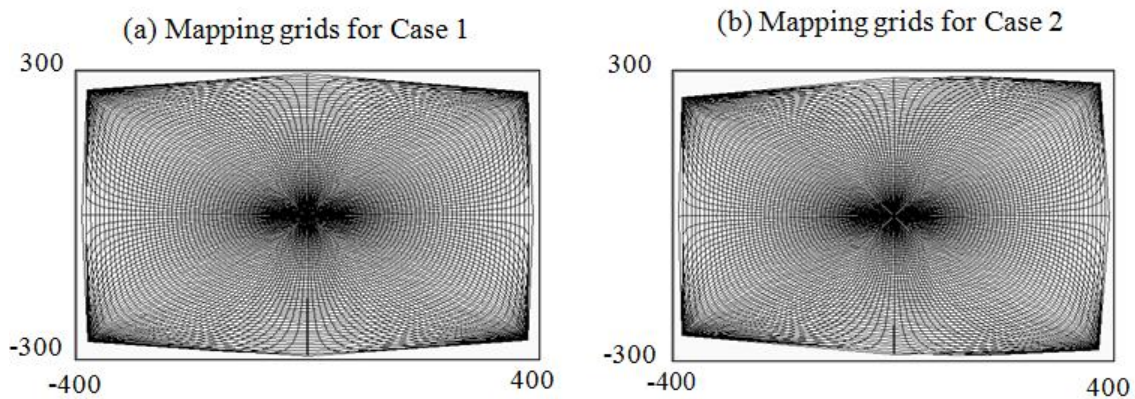


Fig. 9 Mapping grids for uniform illumination over the extended region.

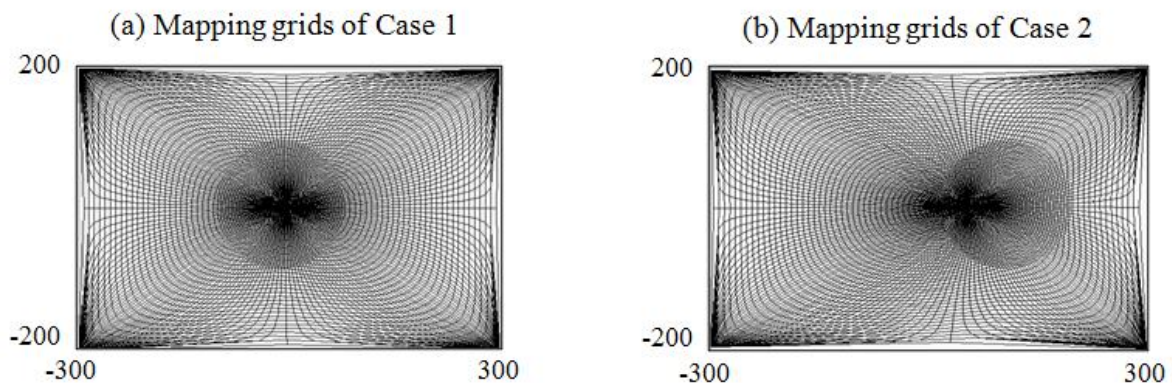


Fig. 10 Mapping grids for illumination with discrete uniformities.

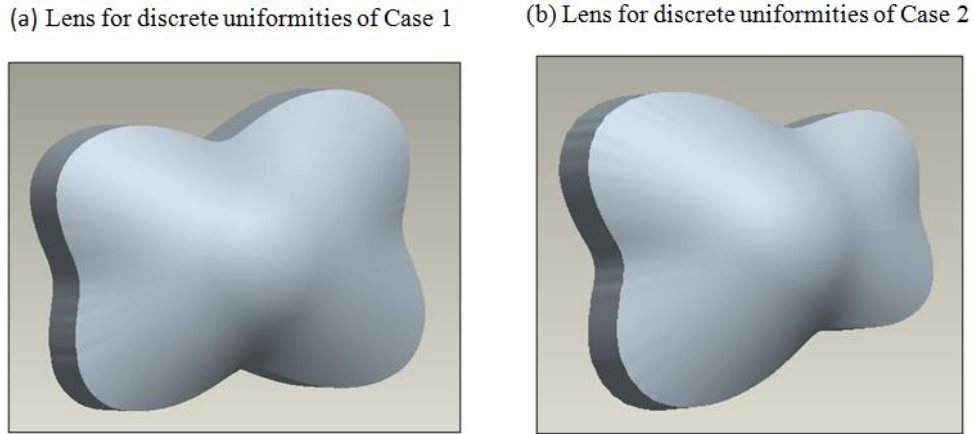


Fig. 11 Freeform lenses for illumination with discrete uniformities.

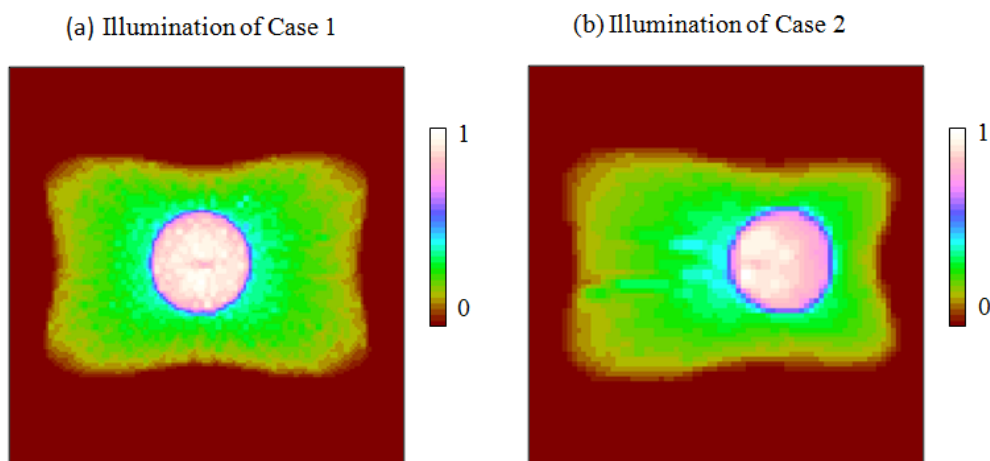


Fig. 12 Illumination with discrete uniformities by simulation.



Fig. 13 Illumination with discrete uniformities for Case 1 by experiment.

## V. Conclusions

The mapping grids of light rays from one mesh to another are the key component to design a freeform lens for LED illumination with uniformity and discrete uniformities. The current cutting edge for the design is based on iteration of simulation and modification. However, the light utilization efficiency is limited. There are two contributions of this study. One is the proposed smooth mapping for uniform arbitrarily-shaped illumination; the mapping grids are formulated in Eq. (1) and Eq. (2), where the horizontal span of illumination is assumed to be a single-valued function of  $y$  and the vertical span is a single-valued function of  $x$ . The spans of the mapping can be calculated and updated by iteration. The other contribution is the proposed concept of

relocating the mapping grids under luminance density conservation. It can be applied for illumination with discrete uniformities and for more advanced optics design.

The adequacy of the proposed approach is demonstrated by the Monte Carlo ray tracing simulation and one experimental case study. The light utilization efficiency of the case studies is high because of smooth mapping and smooth freeform surface. From a given illumination image and design parameters as inputs to the generation of a prototype freeform lens and running ray tracing simulations, the design cycle takes little time to accomplish.

In principle, the proposed approach is also applicable in case the  $A_i$  has multiple regions, with non-smooth contours. This is currently under investigation.

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