

Smart Response Surface Models using Legacy Data for Multidisciplinary Optimization of Aircraft Systems

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Abstract: *One of the key challenges in multidisciplinary design is integration of design and analysis methods of various systems in design framework. To achieve Multidisciplinary Design Optimization (MDO) goals of aircraft systems, high fidelity analysis are required from multiple disciplines like aerodynamics, structures or performance. High Fidelity Analysis like Computer-Aided Design and Engineering (CAD/CAE) techniques, complex computer models and computation-intensive analyses/simulations are often used to accurately study the system behaviour towards design optimization. Due to high computational cost and numerical noise associated with these analyses, they cannot be used effectively. The use of surrogates or Response Surface Models (RSM) is one approach in Multi Disciplinary design optimization to avoid the computation barrier and to take care of artificial minima due to numerical noise. This paper brings out a method based on use of "Smart Response Surface Models" to generate surrogate models, with its validated subspace, in the design space around the point of interest with the use of legacy data for MDO. The method has been evaluated on three test cases, which are created based on High Speed Civil Transport (HSCT) Multidisciplinary Design Optimization Test Suite*

Keywords: *Optimization, DOE, Surrogate Modelling, Multidisciplinary Design, Aircraft*

I. Introduction

Present generation multi-role combat aircraft with y by wire and state of the art weapons systems are complex systems in nature, which need specialists. Complexity of combat aircraft mandates the need for design teams to have multidisciplinary experience in the entire aircraft design with core expertise in their respective domains. Today aerospace design and development is not only multidisciplinary but also global in nature with design and engineering teams deployed around the world [1]. It requires a high level of technical and techno-managerial expertise across various engineering disciplines to cater for very stringent reliability, safety and performance requirements. This would enable design and development of an optimal multidisciplinary system in a collaborative and cohesive integrated environment of various engineering domains.

Multidisciplinary system design is a complex, computationally intensive process that combines discipline analysis with design-space search and decision making. The decision making is based on engineering judgment and is greatly assisted by computer automation. Towards this systems engineering provides holistic approach for integrated design and development of aircraft and its associated systems [2]. One of the key challenges in collaborative design is integration of design and analysis methods of various systems in system engineering framework. With the advances in Computer-Aided Design and Engineering (CAD/CAE) techniques, complex computer models and computation-intensive analyses/simulations (discipline analysis) are often used to accurately study the system behaviour towards design improvements. This design optimization process normally requires a large number of iterations before the optimal solution is identified. Design optimization, with high fidelity design tools, is computationally very expensive and time consuming. The use of approximation models or surrogates to replace the expensive high fidelity computer analysis, in Multi Disciplinary Optimization (MDO), is a natural approach to avoid the computation barrier and to take care of numerical noise[3]. Typically approximation models or surrogates of high fidelity design tools are used to reduce this computational effort and time during multidisciplinary design optimization process. This paper brings out a method based on use of "Smart Response Surface Models" to generate surrogate models in the design space around the point of interest with the use of legacy data for (MDO).

II. Response Surface Models (Rsm)

Complex aircraft engineering design problems are solved using high fidelity analysis/simulation software tools. The high computational cost associated with these analyses and simulations prohibits them from being used as performance measurement tools in the optimization of design for combat aircraft. Another major drawback in using high fidelity analysis is numerical noise, which occurs as a result of the incomplete convergence of iterative processes, the use of adaptive numerical algorithms, round-o_ errors, and the discrete representation of continuous physical objects (fluids or solids)[4]. The use of surrogates or Response Surface

Models (RSM) to replace the expensive high fidelity computer analysis, in MDO, is a natural approach to avoid the computation barrier and to take care of artificial minima due to numerical noise. Renaud and Gabriele developed Response Surface Modelling (RSM) of multidisciplinary systems during concurrent subspace optimizations (CSSOs) [5] [6]. Korngold and Gabriele addressed discrete multidisciplinary problems using the RSM [7].

Expensive high fidelity computer analysis can be represented as a blackbox function. In a simplest form the high fidelity analysis tools takes vector X as input and gives Y as the output as shown in Figure 1.



Figure 1: Representing it mathematically with limits on the design space

$$Y = f(\bar{x}) \quad \text{where } \bar{x} \in R^n \tag{1}$$

$\bar{x}_{min} < \bar{x} < \bar{x}_{max}$ defines the design space

This function would be replaced by polynomial based surrogate model. A typical second order surrogate model is shown below

$$y = \beta_0 + \sum_{i=1}^n \beta_i x_i + \sum_{i=1}^n \beta_{ii} x_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{kn} \beta_{ij} x_i x_j \tag{2}$$

$\bar{x}'_{min} < \bar{x}' < \bar{x}'_{max}$ defines the model subspace

Where β_i , β_j and β_{ij} are regression coefficients, \bar{x} is the input vector and y is the output. The subspace surrogate model is defined by the side constraints \bar{x}'_{min} and \bar{x}'_{max} .

III. Smart Response Surface Models

Smart response surface models is a methodology that develops a response surface model and identifies the subspace for which model is valid. In the conventional methods, of implementing response surface models (RSM) for Multidisciplinary Design, the model subspace is de_fined prior to generating the model and the accuracy of the model is not predefined [4]. The accuracy of the RSM generated is assumed to be acceptable apriori. An algorithm for developing surrogate models to pre-defined accuracy was developed by Gabbur & Ramchand is described in [3]. As the accuracy becomes more stringent there would be a reduction of model subspace with concomitant increase in number of iterations. The algorithm creates knowledge database for functions calls and surrogates models. Legacy or historical data if available would also form a part of this knowledge database. This database would reduce the number of times a high fidelity analysis/simulation software tool is run for model generation. The methodology has been tested on five different optimization test function and the result have been brought out in [3].

IV. Algorithm

The flow chart for smart RSM is shown in figure 2. The smart RSM comprises of six processes repeated iteratively to generate the validated surrogate models with its design space. The iterative steps are follows

1. Identifying the design space of the model
2. Design of Experiments
3. Analysis of DOE points
4. Generation of Response surface models based on DOE
5. Model Validation
6. Design space reduction

4.1 Identifying the design space of the model

The domain of the optimization problem is defined as the initial design space for the surrogate model. Mathematically it can be represented as

$$\text{Minimize } F(\bar{X}) \quad \text{where } (\bar{X}) \in R^N \quad (3)$$

$$X_{imin} \leq X_i \leq X_{imax} \quad \text{where } i = 1,2,3 \dots N \quad (4)$$

Subject to

$$\bar{G}(\bar{x}) \geq 0 \quad \text{where } \bar{G} \in R^m$$

$$\bar{H}(\bar{x}) = 0 \quad \text{where } \bar{H} \in R^p$$

Then the initial design space of the model, for N dimension, would be defined by equation 4.

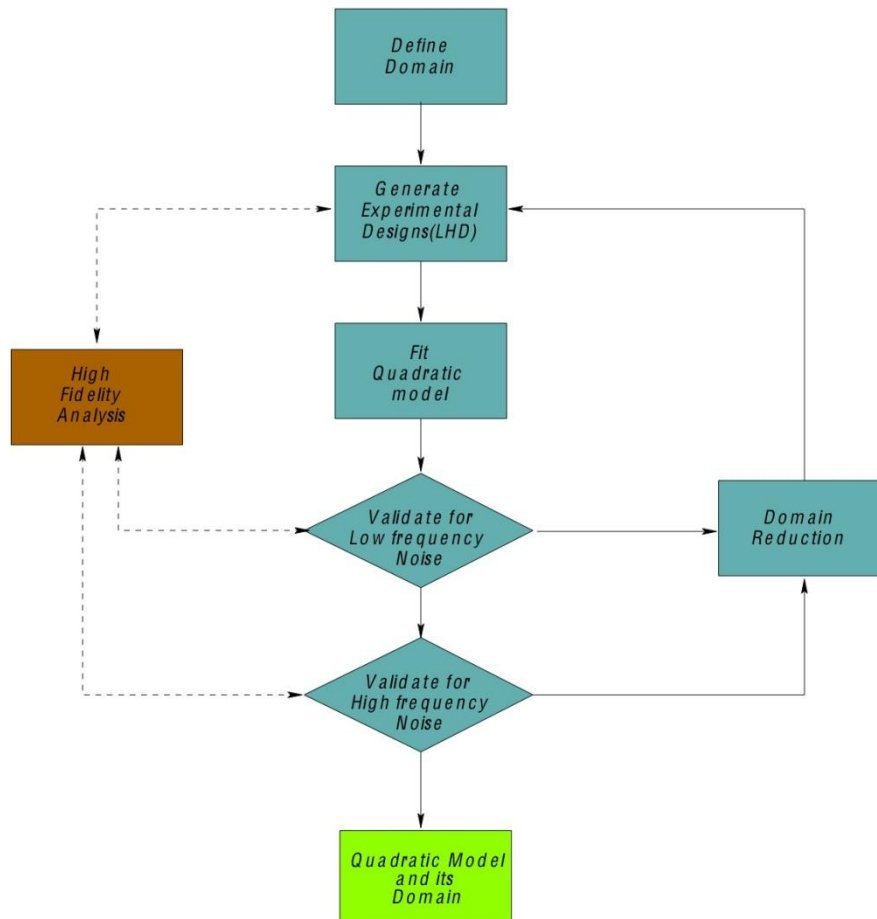


Figure 2: Smart RSM Alogorithm

4.2 Design of Experiments

Experimental design techniques, which were initially developed for physical experiments, are finding considerable use for the design of computer experiments/analyses. In Design of Experiment (DOE) techniques developed for analysis of physical experiments, random variation is accounted for by spreading the sample points out in the design space and by taking multiple data points (replicates). Among various classical experimental designs, Central Composite Design (CCD), alphabetical optimal designs, especially D-optimal designs, are also widely used [8, 9]. Sacks, et al. state that the classical techniques of experimental blocking, replication, and randomization are irrelevant when it comes to deterministic computer experiments [10]. Therefore sample points should be chosen to fill the design space for computer experiments. Koch, Mavris and Mistree [11] investigate the use of a modified central composite design (CCD) that combines half-fractions of an inscribed CCD with a face-centered CCD to distribute points more evenly throughout the design space. Koehler and Owen [12] describe several Bayesian space filling designs, including maximum entropy designs, mean squared-error designs, minimax and maximin designs, Latin Hypercube, randomized orthogonal arrays, and scrambled nets. Widely used space filling sampling methods are Orthogonal Array (OA) and Latin Hyper cube Design (LHD). OA can generate a sample with better space-filling property than LHD. However, the generation of an OA sample is more complicated than LHD [13, 14]. In addition, OA demands strict level

classification for each variable, which might bring difficulty in real design. In real design, not all combinations of variable level lead to realistic design solutions, and some may cause the crash of the analysis or simulation, which is not uncommon in finite element analysis. In that case, the engineers must manually adjust variables to an appropriate number, deviating from one of the defined levels. Thus the property of OA might be undermined [15]. Therefore for this algorithm LHD is used as DOE method.

4.3 Analysis of DOE points

Design analysis is carried out on the points selected by DOE and values of the objective function are evaluated through the computation intensive analysis and simulation processes.

4.4 Generation of Response surface models based on DOE

Based on the analysed above design points a quadratic response surface model is fitted to the data using the usual least square method. As an initial test R^2 and R^2 adjusted are metrics used to estimate and understand the quality of RSM.

4.5 Model Validation

The surrogate model is validated for acceptable fit in two stages. The first stage is to check for low frequency errors (gross misfit of the model). This is carried out around a check point in the design space. The check point is generated in such a way that if the model validation fails then the point would be in the new reduced domain. This point is then perturbed for low frequency error. The direction of perturbation is such that the perturbed point also lies in the reduced design space. This is shown in the Figure 3 for a two dimension function.

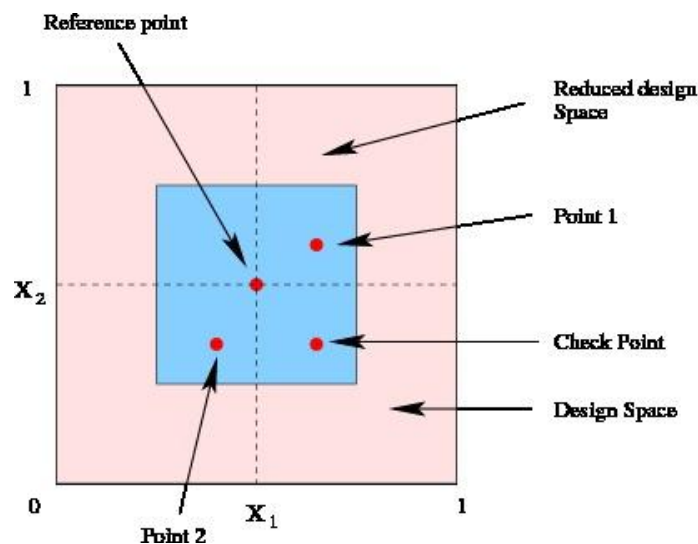


Figure 3 : Low Frequency validation points

Negative perturbation is given in the X_1 direction to get point 1 and positive perturbation is given along X_2 to get point 2. The perturbation value d is 50% of the reduced domain width for each input. The number of points needed for carrying low frequency error is $(k+1)$, where k is the dimension of the input vector. The error (residual) between actual value and the predicted value is calculated at each of these points. This value should be less than predetermined value (typically around 1%) for the model to be acceptable. Once the model is validated for low frequency then it is checked for high frequency error. For carrying out high frequency validation the above process is repeated with a change in perturbation value. The perturbation value d is changed to 5 % of the reduced design space. The error residuals are calculated as

$$\text{Error Residuals} = \frac{Y' - Y}{Y} * 100$$

where Y = Actual Value

Y' = Predicted Value

4.6 Design Space Reduction

For a complex analysis tool / function a single quadratic model may not satisfactorily represent the analysis tool for the full design space. When surrogate model does not accurately represent the analysis tool then the design space needs to be reduced. A selective reduction of design space is employed. The strategy is to halve the domain for that design variable for which the error residual is more than 1%. Further more design space is reduced (zoomed in) around the reference point. Mathematically x_{ri} is the reference point for i^{th} input and x_{li} and x_{ui} are its lower and upper limits respectively then the new design space lower limit x'_{li} and upper limit x'_{ui} are

$$x'_{li} = x_{ri} - \frac{x_{ui} - x_{li}}{4} \tag{5}$$

$$x'_{ui} = x_{ri} + \frac{x_{ui} - x_{li}}{4} \quad \text{for } i = (1, 2, \dots, k) \tag{6}$$

if $x'_{li} < x_{li}$ then

$$x'_{li} = x_{li} \text{ and } x'_{ui} = x_{ri} + \frac{x_{ui} - x_{li}}{2} \tag{7}$$

if $x'_{ui} < x_{ui}$ then

$$x'_{ui} = x_{ui} \text{ and } x'_{li} = x_{ri} + \frac{x_{ui} - x_{li}}{2} \tag{8}$$

Figure 4 shows Design space reduction for two dimension design space ($k=2$). In figure 4(a) and figure 4(b) both the domain are reduced and in figure 4(c) domain for one variable only is reduced. It is proposed to test the algorithm on a higher dimension (10^d or higher) realistic design problem. High speed civil transport (HSCT) data is chosen to create a synthetic problem of 25 design variable. The HSCT data used for optimization test problem consists of one objective function and 66 inequality constraints. Noisy functions are created for the objective function and constraints for proving the effectiveness of the smart RSM algorithm in filtering out numerical noise for use in Multidisciplinary Design Optimization.

V. High Speed Civil Transport (Hsct)

High speed civil transport (HSCT) is an example of extremely challenging aircraft designs, where the disciplines are highly coupled and results from high fidelity design analysis are critical to establishing the feasibility of the aircraft design. The design concept of HSCT is to fly the aircraft with more than 300 passengers at speeds in excess of 1,500 miles per hour. The aircraft development is by NASA and its industry partners as a next generation supersonic passenger jet of the future [16]. HSCT aircraft configuration is shown in figure 5

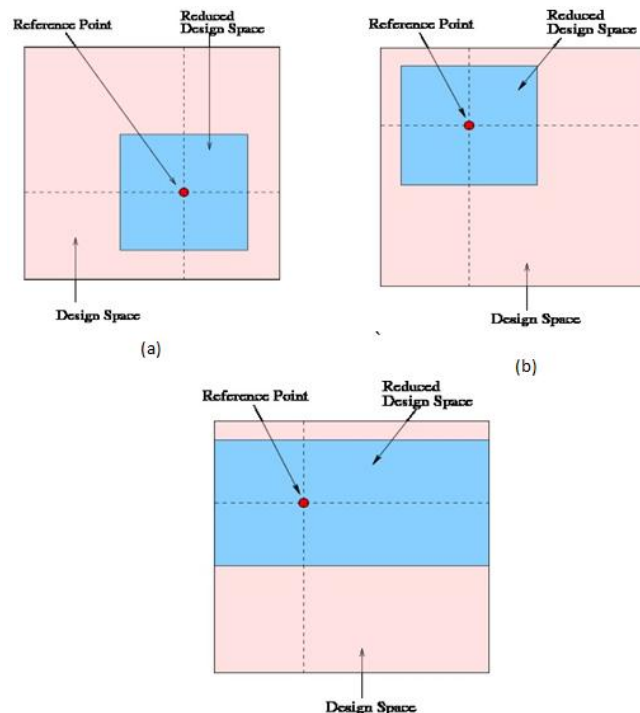


Figure 4: Design Space Reduction



Figure 5 : HSCT Configuration

Multidisciplinary Analysis and Design (MAD) center for advanced vehicles uses HSCT configuration design as a test case for the evaluation of new design optimization methodologies and techniques developed in-house [17]. The test case is described as minimizing the takeoff gross weight (TOGW) of a High Speed Civil Transport (HSCT) aircraft with range of 5500 nautical mile, designed to cruise at Mach 2.4 and ferry 250 passengers. TOGW was selected as the objective function for optimization problem since it represents a composite measure of merit for the *aircraft as a system*. TOGW is expressed as a sum of the dry weight (i.e., the weight of the aircraft including payload, but without fuel) and the fuel weight. The dry weight of the aircraft is correlated to the initial acquisition cost of the aircraft and fuel weight represents the yearly recurring costs of aircraft operations [18]. From Multidisciplinary perspective the choice of the gross weight as the objective function incorporates structural and aerodynamic considerations. The structural considerations are directly related to the aircraft empty weight, while the aerodynamic performance dictates the drag and hence the thrust required to overcome the drag which dictates the fuel weight required for the mission. The HSCT design is described by twenty five design variables and sixty eight constraints. Twenty four of these design variables describe the geometry of the aircraft and can be divided into five categories, wing planform, airfoil shape, tail area, nacelle placement and fuselage shape. One variables, mission fuel, defines the cruise mission. Details of the twenty five design variables are given in Table 1.

Table 1 HSCT Design Variables

Design Variable No	Description
1	Wing root chord
2	LE break point, x
3	LE break point, y
4	TE break point, x
5	TE break point, y
6	LE wing tip, x
7	Wing tip chord
8	Wing semi-span
9	Max t/c location
10	Airfoil t/c at root
11	Airfoil t/c at LE break
12	Airfoil t/c at tip
13	Fuselage restraint 1,x
14	Fuselage restraint 1,r
15	Fuselage restraint 2,x
16	Fuselage restraint 2,r
17	Fuselage restraint 3,x
18	Fuselage restraint 3,r
19	Fuselage restraint 4,x
20	Fuselage restraint 4,r
21	Nacelle 1, y
22	Nacelle 2, y
23	Mission fuel
24	Vertical tail area
25	Horizontal tail area

Sixty eight design constraints define geometry, system performance and aerodynamic performance and are given in Table 2.

Table 2: HSCT Constraints

Index	Constraint
1	Fuel volume \leq 50 wing volume
2	Wing root TE \leq Tail LE
3-20	Wing chord \geq 7.0 ft
21	LE break within wing semi-span
22	TE break within wing semi-span
23	Root chord t/c ratio \geq 1.5%
24	LE break chord t/c ratio \geq 1.5%
25	Tip chord t/c ratio \geq 1.5%
26-30	Fuselage restraints
31	Wing spike prevention
32	Nacelle 1 inboard of nacelle 2
33	Nacelle 2 inboard of semi-span
34	Range \geq 5500 nautical miles
35	CL at landing speed \leq 1
36-53	Section CL at landing \leq 2
54	Landing angle of attack \leq 12°
55-58	Engine scrape at landing
59	Wing tip scrape at landing
60	TE break scrape at landing
61	Rudder deflection \leq 22.5°
62	Bank angle at landing \leq 5°
63	Tail deflection at approach \leq 22.5°
64	Takeoff rotation to occur \leq Vmin
65	Engine-out limit with vertical tail
66	Balanced field length \leq 11000 ft
67-68	Mission segments: thrust available \geq thrust required

Multiple configuration of HSCT were analysed over a period of time at NASA Multidisciplinary Analysis and Design (MAD) center for advanced vehicles. The data from this analysis has been collated and is a part of NASA Multidisciplinary Design Optimization Test Suite [17]. It consists of analyses of 2490 HSCT configuration. The data from each analysis is represented in a matrix of 19 rows and 5 columns, and each of the 2,490 matrices is separated by a blank line. The breakup of 95 numbers taken row by row from each 19 x 5 matrix is as follows

- Number 1 to Number 25 are the x vector of 25 design variables which describes each HSCT aircraft configuration. The 25 Design variables are scaled to the order of 1 to 10.
- Number 26 wing bending material weight
- Number 27 is the takeoff gross weight (TOGW(x)). The objective function, TOGW(x), is not scaled.
- Number 28 to Number 95 represent the sixty eight constraints. The constraints are unscaled and are of order 100-1000 (with negative numbers indicating design infeasibility).

VI. Response Surface Model

Using the legacy data of HSCT, polynomial based surrogate models (cubic response surface) are created for the objective function and 68 constraints. For generating the response surface models the 68 constraints are scaled. The scaling procedure used for constraints is

$$\text{Scaled Constraint Value} = (\text{Maximum value} - \text{Actual Value}) / \text{Limit}$$

where *Limit* is defined as difference between maximum value and minimum value of the constraints. The surrogate model is of the form given below

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^k \beta_{iii} x_i^3 + \sum_{i=1}^k \sum_{j=i}^k \beta_{ij} x_i x_j \quad (9)$$

Latin Hypercube design (LHD) as DOE strategy was used to generate 2000 experimental design points for 25 design variables. The data points nearest to the 2000 experimental design points were selected from the HSCT data and used for generating cubic response surface model. The cubic response surface model is fitted to the data using least square method. For the objective function TOGW, the graph of predicted vs actual value is shown in Figure 6. Residual / error is calculated for the objective function for all 2490 data points. The error is normalised with variance and estimated as follows

$$\text{Normaised Residuals} = \frac{\text{actual value} - \text{predicted value}}{\text{variance } \sigma}$$

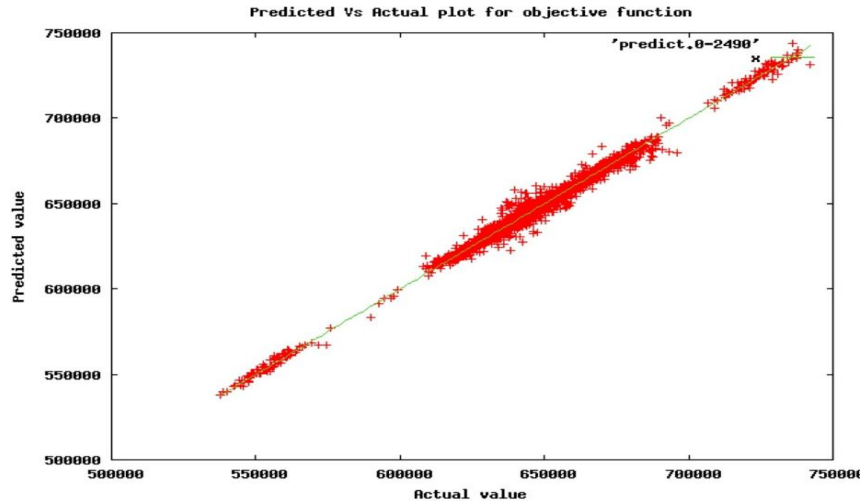


Figure 6 Plot of Objective Function

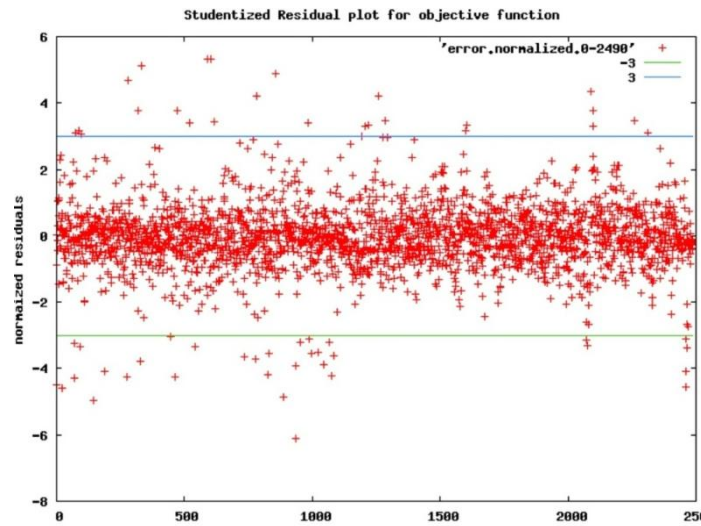


Figure 7 Normalized Residual Error Plot for Objective Functions

The normalised error for the objective function with $+3\sigma$ and -3σ limits is shown in figure 7. Model statistics R^2 and $R^2_{adjusted}$ for objective function are .991093 and 0.982265 respectively. Figure 8 and 9 indicate the spread of R^2 and $R^2_{adjusted}$ for 66 constraints.

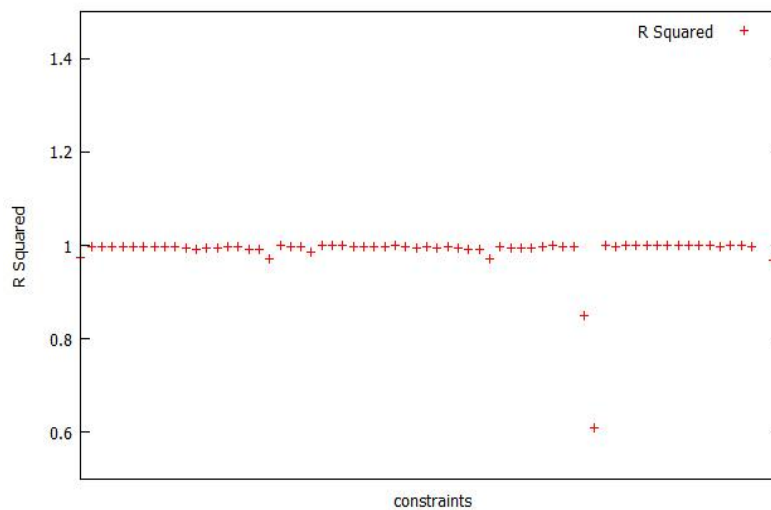


Figure 8 : R^2 for 66 constraints

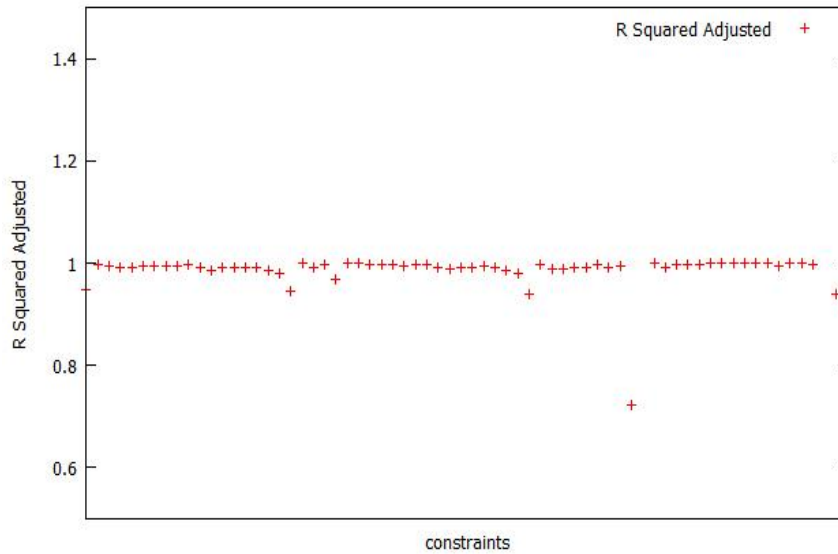


Figure 9: R² adjusted for 66 constraints

The error for each model is characterized using mean and variance σ . The Error metrics for the objective function and 66 scaled constraints are given in table 3.

Table 3: Error Metrics for the Surrogate Model

Function	Mean	Sigma	Function	Mean	Sigma
Objective	-87.030	2956.607	Constraint 34	0.001617	0.172839
Constraint 1	0.002317	0.330295	Constraint 35	0.014505	0.221591
Constraint 2	0.000351	0.009246	Constraint 36	0.010143	0.184822
Constraint 3	0.001463	0.034273	Constraint 37	-0.000420	0.137855
Constraint 4	0.001411	0.032661	Constraint 38	0.002242	0.140118
Constraint 5	0.001360	0.032563	Constraint 39	0.004293	0.123276
Constraint 6	0.001555	0.032148	Constraint 40	0.002108	0.102640
Constraint 7	-0.000217	0.034550	Constraint 41	-0.000411	0.013342
Constraint 8	-0.002202	0.039148	Constraint 42	-0.000489	0.023651
Constraint 9	-0.002477	0.037638	Constraint 43	-0.000490	0.023634
Constraint 10	-0.002208	0.041348	Constraint 44	-0.000430	0.025833
Constraint 11	0.003470	0.042901	Constraint 45	-0.000439	0.025776
Constraint 12	0.004246	0.051777	Constraint 46	-0.001345	0.031570
Constraint 13	-0.000858	0.052123	Constraint 47	0.000000	0.000000
Constraint 14	0.002668	0.057427	Constraint 48	-0.011687	0.292862
Constraint 15	0.001782	0.048474	Constraint 49	0.001291	0.034009
Constraint 16	-0.001079	0.041063	Constraint 50	0.022865	2.302002
Constraint 17	0.000149	0.047517	Constraint 51	-0.000079	0.001688
Constraint 18	0.001348	0.051881	Constraint 52	-0.000399	0.010767
Constraint 19	0.000765	0.060724	Constraint 53	0.001689	0.025668
Constraint 20	-0.000000	0.000000	Constraint 54	-0.000998	0.020468
Constraint 21	-0.000080	0.007034	Constraint 55	0.000915	0.030134
Constraint 22	-0.004000	0.025713	Constraint 56	-0.000000	0.000000
Constraint 23	-0.003393	0.069140	Constraint 57	0.000000	0.000000
Constraint 24	-0.000000	0.000001	Constraint 58	0.000000	0.000000
Constraint 25	0.000470	0.031493	Constraint 59	0.000000	0.000000
Constraint 26	0.000926	0.063048	Constraint 60	-0.000003	0.000068
Constraint 27	0.002525	0.090631	Constraint 61	-0.000049	0.004281
Constraint 28	-0.005665	0.120384	Constraint 62	0.000024	0.045508
Constraint 29	-0.015115	0.154662	Constraint 63	-0.000000	0.000000
Constraint 30	-0.017200	0.163188	Constraint 64	0.000000	0.000000
Constraint 31	-0.011297	0.167016	Constraint 65	0.001646	0.050521
Constraint 32	0.017417	0.168936	Constraint 66	-77.501	3508.771
Constraint 33	0.022139	0.193345			

7.1 Test case1

Problem Statement

Minimize

$$f_t(\bar{x}) \quad \text{where} \quad (\bar{x}) \in R^{25}$$

$$x_{imin} \leq x_i \leq x_{imax} \quad \text{where} \quad i = 1,2,3 \dots 25$$

Subject to

$$\bar{g}_t(\bar{x}) \geq 0 \quad \text{where} \quad \bar{g}_{nt} \in R^{66}$$

CSFQP was run for the above design problem. The starting point for the optimizer was an initial feasible design point. An optimal point was reached after 72 iteration. The d0 norm after 72 iteration was $7.817e - 07$. The number of functions calls for by the optimizer for the objective function 1931. The value of the weight function was 332601.83. This value of the objective function is used as reference value for comparing the other two test cases.

7.2 Test case 2

Problem Statement

Minimize

$$f_{nt}(\bar{x}) \quad \text{where} \quad (\bar{x}) \in R^{25}$$

$$x_{imin} \leq x_i \leq x_{imax} \quad \text{where} \quad i = 1,2,3 \dots 25$$

Subject to

$$\bar{g}_{nt}(\bar{x}) \geq 0 \quad \text{where} \quad \bar{g}_{nt} \in R^{66}$$

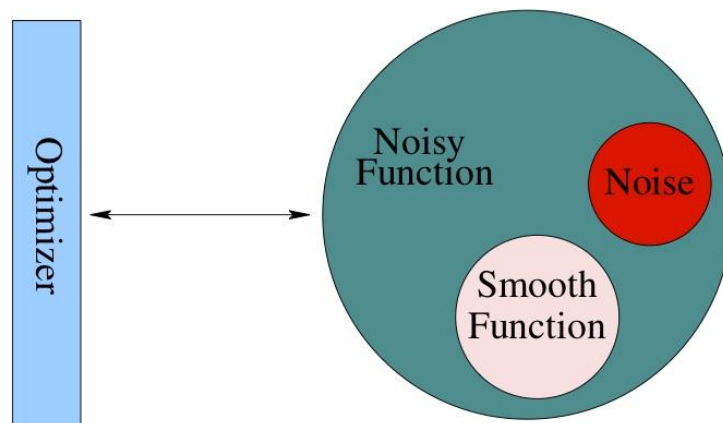


Figure 11: Test Case 2

CSFQP was run for the above design problem with initial design point. The value of objective function at the initial design point is 6.21725×10^5 . The optimizer failed to converge after 10 iteration. The value of Objective function after 10 iteration is 6.21417×10^5 . The values of the objective function has not reduced much after 10 iteration. The objective function was called 844 times. The function calls to each of 66 constraints was ranging between 840 to 849. The d0 norm and the step size at the 10 iteration were 3.6058990×10^0 and $1.4210854 \times 10^{-14}$ respectively. The optimizer failed as the step size was small. As seen here with a noisy function a gradient based optimizer fails to converge to optima due to numerical noise.

7.3 Test case 3 - Smart RSM

In test case 3 Smart RSM interfaces between optimizer and the noisy function. It generates validated quadratic RSM (with its subspace) for objective function and 66 constraints to be used by optimizer. The optimization parameters are similar to earlier test cases. The process was repeated twice with two different acceptable modelling error of 10% and 5%. The Smart RSM also interacts with two databases, Input/output and RSM database. At the starting of the optimization process there is no data in both databases during optimization process the databases get populated and values are checked to reuse existing RSM and avoid redundant function calls.

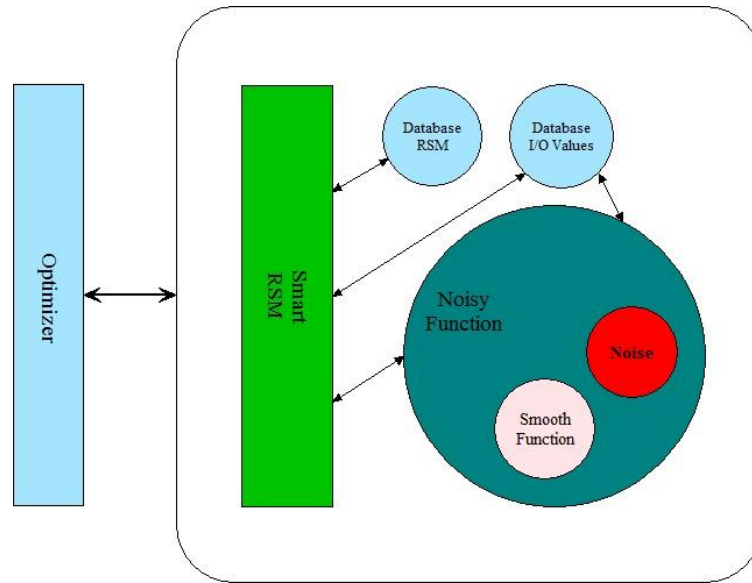


Figure 12: Test Case 3

Acceptable modelling error	10%
Number of iterations	36
d0norm	2.67651×10^{-7}
Number of functions calls for objective function	1636
Objective function $f_{nt}(\bar{x})$	374942.05
$\% \text{ error} = \frac{f_{nt}(\bar{x}) - f_t(\bar{x})}{f_{nt}(\bar{x})} \times 100$	12.7 %

Acceptable modelling error	5%
Number of iterations	41
d0norm	9.390395×10^{-7}
Number of functions calls for objective function	2426
Objective function $f_{nt}(\bar{x})$	366345.560329
$\% \text{ error} = \frac{f_{nt}(\bar{x}) - f_t(\bar{x})}{f_{nt}(\bar{x})} \times 100$	10.1%

VIII. Result

Objective function/design analysis tools of the form $f_t(\bar{x})$ are smooth and simple functions. Typically these are empirical methods or simple equation used during pre-conceptual design stage. For conceptual/detail design, high fidelity design analysis tools are preferred which are complex and invariably have numerical noise and are similar to $f_{nt}(\bar{x})$. During multidisciplinary optimization process with these design analysis tools, the optimization process either fails due to non-convergence or requires large number of iterations. Smart RSM present a way to overcome these issues.

Table 4 shows the number of iterations, calls to the objective function, and value reached after optimization for the test cases. Due to the numerical noise introduced in test case 2 the optimization process has failed after 10 iterations without convergence. It is also observed that the value of objective function value has not reduced appreciably to 6.21417×10^5 from the initial value of 6.21725×10^5 during optimization process. The number of functions calls by the optimizer for objective function was 844 till non convergence.

Table 4 Test Case Results

		Iteration	No Of Objective Function calls	Objective Function optimal value (Kg)
Test Case 1		72	1932	332061.8
Test Case 2		Failed (10)	844	621417.3
Test Case 3	10%	36	1636	374942.1
	5%	41	2426	366345.5

With the implementation of Smart RSM, Test case 3, quadratic approximations with move limits are created and used by optimizer. The number of objective function calls by the smart RSM for an acceptable modelling error of 10% and 5% is 1636 and 2426 respectively. For a acceptable modelling error of 10%, Table

6 gives the details of the function calls for objective functions and 66 constraints. Complex aircraft design has large number of constraints criteria that needs to met. In the case of HSCT there are 66 design constraints. These design constraints are estimated with use of multidisciplinary analysis. This increases the computation efforts and time. With the implementation of Smart RSM on design tools, both for objective function and constraints, there would a saving in computation time and circumventing the problems associated with numerical noise. Further, with appropriate database managements for RSM a design knowledge base could also be developed. Table 5 shows number of models generated for objective function and constraints for 10`% modelling error.

Table 5 : Number of Surrogate models generated

Function	RSM created	Function	RSM created
Objective	3	Constraint 34	9
Constraint 1	3	Constraint 35	14
Constraint 2	2	Constraint 36	11
Constraint 3	8	Constraint 37	11
Constraint 4	11	Constraint 38	9
Constraint 5	12	Constraint 39	11
Constraint 6	15	Constraint 40	13
Constraint 7	7	Constraint 41	12
Constraint 8	14	Constraint 42	7
Constraint 9	15	Constraint 43	8
Constraint 10	15	Constraint 44	3
Constraint 11	14	Constraint 45	3
Constraint 12	11	Constraint 46	15
Constraint 13	3	Constraint 47	1
Constraint 14	16	Constraint 48	3
Constraint 15	6	Constraint 49	18
Constraint 16	11	Constraint 50	0
Constraint 17	3	Constraint 51	3
Constraint 18	3	Constraint 52	3
Constraint 19	10	Constraint 53	15
Constraint 20	1	Constraint 54	20
Constraint 21	3	Constraint 55	11
Constraint 22	13	Constraint 56	1
Constraint 23	11	Constraint 57	1
Constraint 24	1	Constraint 58	1
Constraint 25	5	Constraint 59	1
Constraint 26	12	Constraint 60	1
Constraint 27	15	Constraint 61	0
Constraint 28	10	Constraint 62	21
Constraint 29	9	Constraint 63	1
Constraint 30	15	Constraint 64	1
Constraint 31	15	Constraint 65	13
Constraint 32	14	Constraint 66	0
Constraint 33	12		

Table 6: Function calls details

Function	No of calls	Function	No of calls
Objective	1636	Constraint 34	5210
Constraint 1	2118	Constraint 35	8067
Constraint 2	1117	Constraint 36	6817
Constraint 3	5177	Constraint 37	9099
Constraint 4	6468	Constraint 38	8972
Constraint 5	6549	Constraint 39	9106
Constraint 6	9030	Constraint 40	10601
Constraint 7	4030	Constraint 41	12949
Constraint 8	8795	Constraint 42	6437
Constraint 9	9147	Constraint 43	5886
Constraint 10	8764	Constraint 44	2227
Constraint 11	8672	Constraint 45	2725
Constraint 12	6234	Constraint 46	10535
Constraint 13	2374	Constraint 47	704
Constraint 14	9331	Constraint 48	1643
Constraint 15	5832	Constraint 49	13543
Constraint 16	8164	Constraint 50	947
Constraint 17	2362	Constraint 51	1620
Constraint 18	2227	Constraint 52	2227
Constraint 19	7206	Constraint 53	8963
Constraint 20	704	Constraint 54	19637

Constraint 21	2227	Constraint 55	8428
Constraint 22	10189	Constraint 56	704
Constraint 23	7195	Constraint 57	704
Constraint 24	704	Constraint 58	704
Constraint 25	3016	Constraint 59	704
Constraint 26	6610	Constraint 60	704
Constraint 27	9494	Constraint 61	946
Constraint 28	5670	Constraint 62	19843
Constraint 29	5105	Constraint 63	704
Constraint 30	8986	Constraint 64	704
Constraint 31	9786	Constraint 65	8599
Constraint 32	7846	Constraint 66	946
Constraint 33	7221		

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