A Review of Static and Vibration Analysis of Functionally Graded Carbon Nano Tubes

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Abstract: Functionally Graded Materials (FGMs) are the advanced materials in the field of composites, which can resist high temperatures and are proficient in reducing the thermal stresses. In recent decades, significant investigations are reported in the predicting the response of FGM plates subjected to thermal loads. This paper presents a comprehensive review of developments, various mathematical idealizations of materials, temperature profiles, modelling techniques and solutions methods that are adopted for the thermal analysis of FGM plates. An attempt has been made to classify the various analytical and numerical methods used for the vibration and buckling analyses of FGM plates under one dimensional or three-dimensional variation of temperature with constant /linear /nonlinear temperatures profiles across the thickness. An effort has been made to focus the discussion on the various research studies carried out till recently for the thermal analysis of FGM plates. Finally, some important conclusions and the suggestions for future directions of research in this area are presented.

Keywords: FGM, FEM, CNT, Static analysis, Vibration analysis.

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I. Introduction

Nanotechnology is a multi-disciplinary field that deals with a variety of materials produced at the nanometer scale through different physical, chemical, and biological routes. Nanomaterials possess novel properties that are typically not observed in their bulk counterparts. Hence, they may overcome the many limitations of existing products with respect to cost, functionality, fabrication strategies, and overall performance[1].

The structures of micro- and nanotubes resemble those of tiny straws, affording them many advantages over other nanoparticles. In particular, carbon nanotubes (CNTs) in particular, have several interesting properties related to their structure, morphology, functionality, stability, ease of modification, and suitability in hybrid materials[2]. Carbon nanotubes have generated huge research interest from many areas of science and engineering. This is mainly due to their remarkable physical and chemical properties, such as high strength, high stiffness and high aspect ratio but very low density. As such, CNTs are often a material of choice for the fabrication of devices with unprecedented features[3], [4].

CNTs are constructed as hollow cylindrical tubes consisting of carbon (graphite) with a high aspect ratio (∼1000). Depending on the number of graphite layers, CNTs can be classified as single-walled nanotubes (SWNTs), double-walled nanotubes (DWNTs), and multi-walled nanotubes (MWNTs). Numerous studies have shown that CNTs have excellent mechanical, electrical and thermal properties[5]. Given these remarkable properties, CNTs can be considered as an excellent reinforcement for polymer composites that may significantly improve the mechanical, electrical and thermal properties of the resulting nanocomposites[4].

The nature of the carbon atoms in CNTs produces amazing properties that are suitable for a variety of applications in the electronics, photonics, renewable energy, drug delivery and the biomedical sector[6]. Therefore, the production of CNTs has been increasing drastically over the last few years. The global market for CNT primary grades was $158.6 million in 2014. This market was projected to reach $167.9 million in 2015 and $670.6 million in 2019, with a CAGR of 33.4% from 2014 to 2019.11 Technological development and innovative processing methods have helped improve the skills involved in the synthesis and functionalization of CNTs.

II. Static Analysis

Shen H.S. et al. [6], [7], [7]–[9] presented an investigation on the nonlinear bending of simply supported, functionally graded nano-composite plates reinforced by single-walled carbon nanotubes (SWCNTs)
subjected to a transverse uniform or sinusoidal load in thermal environments. They assumed material properties of SWCNTs to be temperature-dependent and obtained them from molecular dynamics simulations. The material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) were assumed to be graded in the thickness direction, and were estimated through a micromechanical model. The governing equations are based on a higher order shear deformation plate theory with a von Kármán-type of kinematic nonlinearity and include thermal effects. A two-step perturbation technique was employed to determine the load-deflection and load-bending moment curves. The numerical illustrations concern the nonlinear bending response of FG-CNTRC plates under different sets of thermal environmental conditions, from which results for uniformly distributed CNTRC plates are obtained as comparators[10]–[12]. The results show that the load-bending moment curves of the plate can be significantly increased as a result of a functionally graded reinforcement. They also confirm that the characteristics of nonlinear bending are significantly influenced by temperature rise, the character of in-plane boundary conditions, the transverse shear deformation, the plate aspect ratio as well as the nanotube volume fraction.Liao-Liang Ke[13]investigates the nonlinear free vibration of functionally graded nano-composite beams reinforced by single-walled carbon nanotubes (SWCNTs) based on Timoshenko beam theory and von Kármán geometric nonlinearity. The material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) are assumed to be graded in the thickness direction and estimated though the rule of mixture. The Ritz method was employed to derive the governing eigenvalue equation which is then solved by a direct iterative method to obtain the nonlinear vibration frequencies of FG-CNTRC beams with different end supports. A detailed parametric study is conducted to study the influences of nanotube volume fraction, vibration amplitude, slenderness ratio and end supports on the nonlinear free vibration characteristics of FG-CNTRC beams. The results for uniformly distributed carbon nanotube-reinforced composite (UD-CNTRC) beams are also provided for comparison. Numerical results were presented in both tabular and graphical forms to investigate the effects of nanotube volume fraction, vibration amplitude, slenderness ratio, end supports and CNT distribution on the nonlinear free vibration characteristics of FG-CNTRC beams.

Shen Shenhui-Shen[8]presented Thermal buckling and post-buckling behaviour for functionally graded nanocomposite plates reinforced by single-walled carbon nanotubes (SWCNTs) subjected to in-plane temperature variation. The material properties of SWCNTs are assumed to be temperature-dependent and are obtained from molecular dynamics simulations. The material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. Based on the multi-scale approach, numerical illustrations are carried out for perfect and imperfect, geometrically mid-plane symmetric FG-CNTRC plates and uniformly distributed CNTRC plates under different values of the nanotube volume fractions. The results show that the buckling temperature as well as thermal post-buckling strength of the plate can be increased as a result of a functionally graded reinforcement. It is found that in some cases the CNTRC plate with intermediate nanotube volume fraction does not have intermediate buckling temperature and initial thermal post-buckling strength.

Shen and Wang [9]in their paper deals with the large amplitude vibration of nanocomposite plates reinforced by single-walled carbon nanotubes (SWCNTs) resting on an elastic foundation in thermal environments. The SWCNTs are assumed aligned, straight and a uniform layout. Two kinds of carbon nanotube-reinforced composite (CNTRC) plates, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The material properties of FG-CNTRC plates are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. The motion equations are based on a higher order shear deformation plate theory that includes plate-foundation interaction. The thermal effects are also included and the material properties of CNTRCs are assumed to be temperature-dependent. The equations of motion are solved by an improved perturbation technique to determine nonlinear frequencies of CNTRC plates. Numerical results reveal that the natural frequencies as well as the nonlinear to linear frequency ratios are increased by increasing the CNT volume fraction. The results also show that the natural frequencies are reduced but the nonlinear to linear frequency ratios are increased by increasing the temperature rise or by decreasing the foundation stiffness. The results confirm that a functionally graded reinforcement has a significant effect on the nonlinear vibration characteristics of CNTRC plates.

Shen et.al[14]presented post-buckling analysis for nano-composite cylindrical shells reinforced by single-walled carbon nanotubes (SWCNTs) subjected to axial compression in thermal environments. Two kinds of carbon nanotube-reinforced composite (CNTRC) shells, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. The governing equations are based on a higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity. The thermal effects are also included and the material properties of CNTRCs are assumed to be temperature-dependent. A singular perturbation technique is employed to determine the buckling loads and post-buckling equilibrium paths. The numerical illustrations concern the post-buckling behaviour of axially-loaded, perfect and imperfect, FG-CNTRC cylindrical shells under different sets of thermal environmental conditions. The results...
for UD-CNTRC shell, which is a special case in the present study, are compared with those of the FG-CNTRC shell. The results show that the linear functionally graded reinforcements can increase the buckling load as well as post-buckling strength of the shell under axial compression. The results reveal that the CNT volume fraction has a significant effect on the buckling load and post-buckling behaviour of CNTRC shells. Again they presented[11] Thermal post-buckling analysis for nano-composite cylindrical shells reinforced by single walled carbon nanotubes (SWCNTs) subjected to a uniform temperature rise. The SWCNTs are assumed to be aligned and straight with a uniform layout. Two kinds of carbon nanotube-reinforced composite (CNTRC) shells, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. The governing equations are based on a higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity. The thermal effects are also included and the material properties of CNTRCs are assumed to be temperature-dependent. Based on the multi-scale approach, numerical illustrations are carried out for perfect and imperfect, FG- and UDCNTRC shells under different values of the nanotube volume fractions. The results show that the buckling temperature as well as thermal post-buckling strength of the shell can be increased as a result of a functionally graded reinforcement. It is found that in most cases the CNTRC shell with intermediate nanotube volume fraction does not have intermediate buckling temperature and initial thermal post-buckling strength.

Liew, K.M.[15]–[17], [17]–[21]et.al showed bending and free vibration analyses of thin-to-moderately thick composite plates reinforced by single-walled carbon nanotubes using the finite element method based on the first order shear deformation plate theory. Four types of distributions of the uniaxially aligned reinforcement material are considered, that is, uniform and three kinds of functionally graded distributions of carbon nanotubes along the thickness direction of plates. The effective material properties of the nano-composite plates are estimated according to the rule of mixture. Detailed parametric studies have been carried out to reveal the influences of the volume fractions of carbon nanotubes and the edge-to-thickness ratios on the bending responses, natural frequencies and mode shapes of the plates. In addition, the effects of different boundary conditions are also examined. Numerical examples are computed by an in-house finite element code and the results show good agreement with the solutions obtained by the FE commercial package ANSYS.

B. SobhaniAragh, A.H. NasrollahBarati, H. Hedayati[22] presented, natural frequencies characteristics of a continuously graded carbon nanotube-reinforced (CGCNTR) cylindrical panels based on the Eshelby–Mori–Tanaka approach. The volume fractions of oriented, straight single-walled carbon nanotubes (SWCNTs) are assumed to be graded in the thickness direction. In this research work, an equivalent continuum model based on the Eshelby–Mori–Tanaka approach is employed to estimate the effective constitutive law of the elastic isotropic medium (matrix) with oriented, straight carbon nanotubes (CNTs). The CGCNTR shell is assumed to be simply supported at one pair of opposite edges and arbitrary boundary conditions at the other edges such that trigonometric functions expansion can be used to satisfy the boundary conditions precisely at simply supported edges. The 2-D generalized differential quadrature method (GDQM) as an efficient and accurate numerical tool is used to discretize the governing equations and to implement the boundary conditions. The novelty of the present work is to exploit Eshelby–Mori–Tanaka approach in order to reveal the impacts of the volume fractions of oriented CNTs, different CNTs distributions, various mid radius-to-thickness ratio, shell angle, length-to-mean radius ratio and different combinations of free, simply supported and clamped boundary conditions on the vibrational characteristics of CGCNTR cylindrical panels. The interesting and new results show that continuously graded oriented CNTs volume fractions can be utilized for the management of vibrational behaviour of structures so that the frequency parameters of structures made of such material can be considerably improved than that of the nano composites reinforced with uniformly distributed CNTs.

H. Hedayati, B. SobhaniAragh[23] researched on 3-D elasticity solution for free vibration analysis of continuously graded CNT-reinforced (CGCNTR) annular sectorial plates resting on Pasternak elastic foundation. The elastic foundation is considered as a Pasternak model with adding a shear layer to the Winkler model. The volume fractions of randomly oriented agglomerated single-walled carbon nanotubes (SWCNTs) are assumed to be graded in the thickness direction. An embedded CNT in a polymer matrix and its surrounding inter-phase is replaced with an equivalent fiber for predicting the mechanical properties of the carbon nanotube/polymer composite. To determine the effect of CNT agglomeration on the elastic properties of CNT-reinforced composites, a two-parameter micromechanics model of agglomeration is employed. In this research work, an equivalent continuum model based on the Eshelby–Mori–Tanaka approach is employed to estimate the effective constitutive law of the elastic isotropic medium (matrix) with oriented straight CNTs. The 2-D generalized differential quadrature method (GDQM) as an efficient and accurate numerical tool is used to discretize the equations of motion and to implement the various boundary conditions. The fast rate of convergence of the method is shown and the results are compared against existing results in literature. Then, the influence of graded agglomerated CNTs, elastic foundation stiffness parameters and various geometrical
parameters on the vibration characteristics of the annular sectorial plates resting on Pasternak elastic foundation reinforced by randomly oriented agglomerated CNTs is investigated.

B. SobhaniAragh et al. [22]–[24] In their papers investigated the mechanical buckling of a functionally graded nanocomposite rectangular plate reinforced by aligned and straight single-walled carbon nanotubes (SWCNTs) subjected to uniaxial and biaxial in-plane loadings. The material properties of the nanocomposite plate are assumed to be graded in the thickness direction and vary continuously and smoothly according to two types of the symmetric carbon nanotubes volume fraction profiles. The material properties of SWCNT are determined according to molecular dynamics (MDs), and then the effective material properties at a point are estimated by either the Eshelby–Mori–Tanaka approach or the extended rule of mixture. The equilibrium and stability equations are derived using the Mindlin plate theory considering the first-order shear deformation (FSDT) effect and variational approach. The results for nanocomposite plate with uniformly distributed CNTs, which is a special case in the present study, are compared with those of the symmetric profiles of the CNTs volume fraction. A numerical study is performed to investigate the influences of the different types of compressive in-plane loadings, CNTs volume fractions, various types of CNTs volume fraction profiles, geometrical parameters and different types of estimation of effective material properties on the critical mechanical buckling load of functionally graded nanocomposite plates.

Hui-Shen Shen, Y. Xiang [9] presented post-buckling analysis for nanocomposite cylindrical shells reinforced by single-walled carbon nanotubes (SWCNTs) subjected to combined axial and radial mechanical loads in thermal environment. Two types of carbon nanotube-reinforced composite (CNTRC) shells, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, were considered. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. The governing equations were based on a higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity. The thermal effects were also included and the material properties of CNTRCs that were assumed to be temperature-dependent. A boundary layer theory and associated singular perturbation technique employed to determine the buckling loads and post-buckling equilibrium paths. The numerical illustrations concern the post-buckling behaviour of perfect and imperfect, FG-CNTRC cylindrical shells under combined action of external pressure and axial compression for different values of load-proportional parameters. The results for UD-CNTRC shell, which is a special case in the present study, are compared with those of the FG-CNTRC shell.

J. Torabi, Y. Kiani, M.R. Eslami [25]–[30] presented their study of buckling analysis of a functionally graded conical shell integrated with piezoelectric layers that is subjected to combined action of thermal and electrical loads. They assumed material properties of functionally graded conical shells to be vary continuously through the thickness direction based on a power law form. The governing equations, including the equilibrium and stability equations, were obtained based on the classical shell theory and the Sanders nonlinear kinematics relations. The case of uniform temperature distribution through the shell domain is considered. The pre-buckling forces are obtained considering the membrane solutions of linear equilibrium equations. Minimum potential energy criterion is employed to establish the stability equations. The single-mode Galerkin method is used to obtain the critical buckling temperature difference. The results are compared with the known data in the open literature. Finally, some numerical results are presented to study the effects of applied actuator voltage, shell geometry, and power law index of FGM on thermal buckling behaviour of the conical shell.

A. Alibeigloo, K.M. Liew [31]–[34] examined bending behaviour of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) rectangular plate with simply supported edges subjected to thermo-mechanical loads based on three-dimensional theory of elasticity. By using Fourier series expansion along the in plane directions and state space technique across the thickness direction for the entities exact solution for bending characteristic of plate is derived. Accuracy of the presents approach is validated by comparing the numerical results with the available published results in the literature. Investigation on the static behaviour of the plates is further carried out by considering the effects of volume fraction of carbon nanotube, uniform distribution and functionally graded distribution of carbon nanotube, aspect ratio and length to thickness ratio.

Hui-Shen Shen [12] presented A post-buckling analysis for a functionally graded composite cylindrical shell reinforced by single-walled carbon nanotubes (SWCNTs) subjected to torsion in thermal environments. The multi-scale model for functionally graded carbon nanotube-reinforced composite (FG-CNTRC) shells under torsion is proposed. A singular perturbation technique along with a two-step perturbation approach is employed to determine the buckling load and post-buckling equilibrium path. The numerical illustrations concern the torsional buckling and post-buckling behaviour of perfect and imperfect, FG-CNTRC cylindrical shells under different sets of thermal environmental conditions. The results for uniformly distributed CNTRC shell, which is a special case in the present study, are compared with those of the FG-CNTRC shell. The results show that the linear functionally graded reinforcements can increase the buckling torque as well as post-buckling strength of the shell under torsion when the reinforcement has a symmetrical distribution. The results
reveal that the carbon nanotube volume fraction has a significant effect on the buckling load and post-buckling behaviour of CNTRC shells under torsion.

The free vibration behaviour of quadrilateral laminated thin-to-moderately thick plates with carbon nanotube reinforced composite (CNTRC) layers is studied by P. Malekzadeh n, A.R. Zarei[35]. The governing equations are based on the first-order shear deformation theory (FSDT). The solution procedure is based on transforming the governing differential equations from an arbitrary straight-sided physical domain to a regular computational one, and discretization of the spatial derivatives by employing the differential quadrature method (DQM) as an efficient and accurate numerical tool. Four different profiles of single walled carbon nanotubes (SWCNTs) distribution through the thickness of layers are considered, which are uniformly distributed (UD) and three others are functionally graded (FG) distributions. The fast rate of convergence of the presented approach is numerically demonstrated and to show its high accuracy, wherever possible comparison studies with the available results in the open literature are performed. Then, the effects of volume fraction of carbon nanotubes (CNTs), geometrical shape parameters, thickness-to-length and aspect ratios, different kinds of CNTs distribution along the layers thickness and different boundary conditions on the natural frequencies of laminated plates are studied.

K.M. Liew et al. [17] studied large deflection geometrically nonlinear behaviour of carbon nanotube-reinforced functionally graded (CNTR-FG) cylindrical panels under uniform point transverse mechanical loading. The analysis is carried out using the kp-Ritz method with kernel particle function is employed to construct the shape functions for the two-dimensional displacement approximations. Based on the first-order shear deformation shell theory, nonlinear governing equations are developed with geometric nonlinearity taking the form of von Kármán strains. It is assumed that carbon nanotubes are uniaxially aligned in the axial direction and are functionally graded in thickness direction of the cylindrical panels. The effective material properties of resulting CNTR-FG panels are estimated by employing an equivalent continuum model based on the Eshelby–Mori–Tanaka approach. A stabilized conforming nodal integration scheme is employed to evaluate the system bending stiffness and the membrane as well as shear terms are calculated by the direct nodal integration method to eliminate shear locking, for a very thin cylindrical panel. Several numerical example problems are examined to reveal the influences of volume fraction of carbon nanotubes, span angle, edge-to-radius ratio and thickness on nonlinear responses of the CNTR-FG panels. Moreover, effects of different boundary conditions and distribution type of carbon nanotubes are also investigated. Further they analysed flexural strength and free vibration of carbon nanotube reinforced composite cylindrical panels. Four types of distributions of uniaxial aligned reinforcements are considered, i.e. uniform and three kinds of functionally graded distributions of carbon nanotubes along thickness direction of the panels. Material properties of nanocomposite panels are estimated by employing an equivalent continuum model based on the Eshelby–Mori–Tanaka approach. The governing equations are developed based on the first-order shear deformation shell theory. Detailed parametric studies have been carried out to reveal the influences of volume fraction of carbon nanotubes, edge-to-radius ratio and thickness on flexural strength and free vibration responses of the panels. In addition, effects of different boundary conditions and types of distributions of carbon nanotubes are examined. They also presented a post-buckling analysis of carbon nanotube-reinforced functionally graded (CNTR-FG) cylindrical panels under axial compression. Based on kernel particle approximations for the field variables, the Ritz method is employed to obtain the discretized governing equations. The cylindrical panels are reinforced by single-walled carbon nanotubes (SWCNTs) which are assumed to be graded through the thickness direction with different types of distributions. The effective material properties of CNTR-FG cylindrical panels are estimated through a micromechanical model based on the extended rule of mixture. To eliminate shear locking for a very thin cylindrical panel, the system’s bending stiffness is evaluated by a stabilized conforming nodal integration scheme and the membrane as well as shear terms are calculated by the direct nodal integration method. In the present study, the arc-length method combined with the modified Newton–Raphson method is used to trace the post-buckling path. Detailed parametric studies are carried out to investigate effects of various parameters on post-buckling behaviours of CNTR-FG cylindrical panels and results for uniformly distributed (UD) CNTR-FG cylindrical panel are provided for comparison. While studying Buckling behaviour of functionally graded carbon nano tube (FG-CNT) reinforced composite thick skew plates, they provided the element-free IMLS-Ritz method is used to obtain the buckling solutions to this problem. The first order shear deformation theory (FSDT) is employed for formulation of the energy functional to incorporate the effects of transverse shear deformation and rotary inertia. Using the IMLS approximation in the field variables and minimizing the energy functional via the Ritz procedure, a discretized eigen value equation of the problem is derived. The buckling solution can be obtained through solving this eigen value problem. The numerical stability of the IMLS-Ritz method is validated through convergence studies. The accuracy of the IMLS Ritz results is examined by comparing with the known solutions. Close agreement is found from the comparison study. Besides, parametric studies are conducted for various types of CNTs distributions, CNT ratios, aspect ratios, plate geometries and thickness-to-height ratios under different boundary conditions.
A Review on FG CNT Vibrational Static Analysis

Nuttawit Wattanasakulpong, Arisara Chaikittiratana [36] in 2015 investigates static and dynamic behaviour of carbon nanotube-reinforced composite plates resting on the Pasternak elastic foundation including shear layer and Winkler springs. The plates are reinforced by single-walled carbon nanotubes with four types of distributions of uni-axially aligned reinforcement material. Exact solutions obtained from closed-form formulation based on generalized shear deformation plate theory which can be adapted to various plate theories for bending, buckling and vibration analyses of such plates are presented. An accuracy of the present solutions is validated numerically by comparisons with some available results in the literature. Various significant parameters of carbon nanotube volume fraction, spring constant factors, plate thickness and aspect ratios, etc. are taken into investigation. According to the numerical results, it is revealed that the deflection of the plates is found to decrease as the increase of spring constant factors; while, the buckling load and natural frequency increase as the increment of the factors for every type of plate. R. Ansari et al. [35, 37, 38] In this paper, the nonlinear forced vibration behaviour of composite plates reinforced by carbon nanotubes is investigated by a numerical approach. The reinforcement is considered to be functionally graded (FG) in the thickness direction according to a micro mechanical model. The first-order shear deformation theory and von Kármán-type kinematic relations are employed. The governing equations and the corresponding boundary conditions are derived with the use of Hamilton's principle. The generalized differential quadrature (GDQ) method is utilized to achieve a discretized set of nonlinear governing equations. A Galerkin-based scheme is then applied to obtain a time-varying set of ordinary differential equations of Duffing-type. Subsequently, a time periodic discretization is done and the frequency response of plates is determined via the pseudo-arc length continuation method. Selected numerical results are given for the effects of different parameters on the nonlinear forced vibration characteristics of uniformly distributed carbon nanotube-and FG carbon nanotube-reinforced composite plates. It is found that with the increase of CNT volume fraction, the flexural stiffness of plate increases; and hence its natural frequency gets larger. Moreover, it is observed that the distribution type of CNT significantly affects the vibrational behaviour of plate. The results also show that when the mid-plane of plate is CNT-rich, the natural frequency takes its minimum value and the hardening-type response of plate is intensified. Yin Fan, Hai Wang [39] in their paper investigates the large amplitude vibration behaviour of a matrix cracked laminated beam which contains carbon nanotube reinforced composite (CNTRC) layers resting on an elastic foundation in thermal environments. The motion equations are based on a refined shear-lag model and Euler–Bernoulli beam theory and solved by means of a two-step perturbation approach. The beam-foundation interaction and thermal effects are also included. The material properties of both fiber reinforced composite layers and CNTRC layers are assumed to be temperature-dependent. The effects of the crack density, CNT volume fraction, temperature variation, foundation stiffness, as well as the beam end conditions on the vibration characteristics of hybrid laminated beams with multiple matrix cracks are discussed in detail. Numerical results reveal that the crack density plays an important role in the linear vibration of the hybrid laminated beam, but the effect of crack density is less pronounced on the nonlinear to linear frequency ratios of the same beam.

Snap-through phenomenon due to a uniform lateral pressure in a thermally post-buckled sandwich beam is analyzed in this research by M. Mirzaei a, Y. Kiani [25]. It is assumed that material properties of the core and face sheets are temperature dependent. Face sheets are reinforced with carbon nanotube whose distribution may be uniform or functionally graded. Thermomechanical properties of the face sheets are obtained using a refined rule of mixtures approach. To capture the large deflections, geometrical nonlinearity in von-Kármán sense is taken into account. Chebyshev polynomial based Ritz method is implemented into the virtual displacement principle to construct the matrix representation of the equilibrium equations. A successive displacement control strategy is used to trace the temperature dependent post-buckling equilibrium path. Due to the possibility of snap-through phenomenon, cylindrical arch-length technique is used to trace the equilibrium path of a pressurized thermally post-buckled sandwich beam beyond the limit loads. It is shown that, upper limit load of the beam increases as the temperature gradient increases. Furthermore, volume fraction of CNTs affects the snap-through load and snap-through intensity of the beam, meanwhile, the influence of graded profile of CNTs on snap-through features is almost negligible.

Static stresses analysis of carbon nano-tube reinforced composite (CNTRC) cylinder made of poly vinylidene fluoride (PVDF) is investigated in study by S. Amir et al. [37] Non-axisymmetric thermo-mechanical loads are applied on cylinder in presence of uniform longitudinal magnetic field and radial electric field. The surrounded elastic medium is modeled by Pasternak foundation because of its advantages to the Winkler type. Distribution of radial, circumferential and effective stresses, temperature field and electric displacements in CNTRC cylinder are determined based on Mori–Tanaka theory. The detailed parametric study is conducted, focusing on the remarkable effects of magnetic field intensity, elastic medium, angle orientation and volume fraction of carbon nano-tubes (CNTs) on distribution of effective stress. Results demonstrated that fatigue life of CNTRC cylinder will be significantly dependent on magnetic intensity, angle orientation and volume fraction.
of CNTs. Results of this research can be used for optimum design of thick-walled cylinders under multi-
physical fields.

In this research by M. Mirzaeia, Y. Kiani[26], linear thermal buckling of a composite conical shell made from a
polymeric matrix and reinforced with carbon nanotube fibres is investigated. Distribution of reinforcements
across the shell thickness is assumed to be uniform or functionally graded. Thermomechanical properties of the
constituents are temperature dependent. Under the assumption of first order shear deformation shell theory,
Donnell kinematic assumptions and von Kármán type of geometrical nonlinearity, the complete set of
equilibrium equations and boundary conditions of the shell are obtained. A linear membrane analysis is carried
out to obtain the pre-buckling thermal stresses of the shell. Adjacent equilibrium criterion is implemented to
establish the stability equations associated with the buckling state. The resulting equations are discreted by
means of trigonometric expansion through the circumferential direction and discrete singular convolution
method through the shell length. The established eigenvalue problem is solved iteratively to obtain the critical
buckling temperature and critical mode number. Parametric studies are presented to explore the influences of
semi-vertex angle, volume fraction of CNTs, distribution pattern of CNTs and boundary conditions. It is shown
that, conical shells with intermediate carbon nanotube volume fraction do not have, necessarily, intermediate
critical buckling temperature.

Hui-Shen Shen, Y. Xiang [27] presents an investigation on the thermal post-buckling behaviour of
nanocomposite cylindrical panels resting on elastic foundations and subjected to a uniform temperature rise.
The cylindrical panels are made of carbon nanotube reinforced composite (CNTRC) material with the carbon
nanotube reinforcement being distributed along the thickness of the panels either uniformly (UD) or
functionally graded (FG). A micromechanical model together with molecular dynamics simulation results is
employed to obtain the material properties of the FG-CNTRC panels. The governing equations for the
cylindrical panels are based on a higher-order shear deformation theory with a von Kármán-type of kinematic
nonlinearity. The panel–foundation interaction and thermal effects are also included. The material properties of
CNTRCs are assumed to be temperature-dependent and an iterative scheme is developed to obtain numerical
results. The results reveal that the nanotube volume fraction, foundation stiffness, and the panel curvature ratio
have a significant effect on the thermal post-buckling behaviour of CNTRC cylindrical panels. It is found that in
most cases the CNTRC panel with intermediate nanotube volume fraction does not necessarily have intermediate thermal post-buckling strength.

B. A. Selim a, L. W. Zhang b, K. M. Liew [40] presents the free vibration behaviour of carbon nanotube
(CNT) reinforced functionally graded composite plates in a thermal environment based on Reddy’s higher-order
shear deformation theory (HSDT). The element-free kp-Ritz method is used in this study. Four different types
of CNT distributions are considered. The literature reveals that there is a research gap in investigating the
mechanical behaviours of CNT reinforced functionally graded composite plates using Reddy’s HSDT in
association with any of the mesh-free methods. To the authors’ knowledge, this paper is the first to use this
approach to investigate the vibration behaviour of CNT reinforced functionally graded composite plates in a
thermal environment. The rule of mixture is used to estimate the resulting effective material properties. To
verify the reliability of the present model, the obtained numerical results based on a conversion study have been
compared with those found in the literature with evident agreement. Moreover, parametric studies have been
conducted on the effects of CNT distribution, boundary conditions, plate aspect ratio, plate thickness-to-width
ratio and CNT volume fraction on the non-dimensional natural frequencies. Furthermore, the effects of plate
aspect ratio and plate thickness-to-width ratio on the sequence of the first six mode shapes have been investigated.

Yin Fan, Hai Wang investigates the thermal post-buckling and a large amplitude vibration of thermally
post buckled hybrid laminated plate resting on a Pasternak elastic foundation. The plate is composed of
conventional fiber reinforced composite (FRC) layers and carbon nanotube reinforced composite (CNTRC)
layers. The CNTRC layer consists of reinforcing carbon nanotubes either uniformly distributed (UD) or
functionally graded (FG) along the thickness direction. Transverse matrix cracking is introduced only in the
FRC layers and modelled by a refined self-consistent method. The motion equations of the plate are based on a
higher order shear deformation plate theory with a von Karman-type of kinematic nonlinearity and solved by a
two-step perturbation technique. The interaction between plate and elastic foundation is also included. The
material properties of both CNTRC and FRC layers are assumed to be temperature-dependent and are estimated
by micro-mechanical models, respectively. A parametric study is conducted to investigate the effects of matrix
cracks, functionally graded distribution of CNT, volume fractions of CNT and foundation stiffness on the
thermal post-buckling and thermally post buckled vibration behaviours of cross-ply hybrid laminated plates.

Necla Togun, Süleyman M. Bagdatli [41] presents a nonlinear vibration analysis of the tensioned nano
beams with simple simple and clamped clamped boundary conditions. The size dependent Euler Bernoulli beam
model is applied to tensioned nano beam. Governing differential equation of motion of the system is obtain by
using modified couple stress theory and Hamilton's principle. The small size effect can be obtained by a material length scale parameter. The nonlinear equations of motion including stretching of the neutral axis are derived. Damping and forcing effects are considered in the analysis. The closed form approximate solution of nonlinear equations is solved by using the multiple scale method, a perturbation technique. The frequency-response curves of the system are constructed. Moreover, the effect of different system parameters on the vibration of the system are determined and presented numerically and graphically. The size effect is significant for very thin beams whose height is at the nanoscale. The vibration frequency predicted by the modified couple stress theory is larger than that by the classical beam theory. Comparison studies are also performed to verify the present formulation and solutions.

Superlative properties of nanocomposite have motivated considerable research efforts in recent years. Nanocomposite plates of quadrilateral shapes are important structural components used in a variety of engineering structures. R. Ansari, A. Shahabodini, M. FaghihShojaei[38] aims to develop a variational formulation to describe the vibrational behaviour of functionally graded (FG) nanocomposite straight-sided quadrilateral plates reinforced by carbon nanotubes (CNTs) in thermal environments. Various profiles of single-walled carbon nanotubes (SWCNTs) distribution along the thickness are taken into consideration. The mathematical formulation is developed in the variational form based on the first order shear deformation plate theory (FSDPT) with consideration of thermal effects. Discretization process of the energy functional is done on a computational domain using a mapping-differential quadrature (DQ) methodology. Discrete form of the governing equations is directly derived from a weak formulation which does not involve any transformation and discretization of the high order derivatives appeared in the equations of the strong form. Numerical results are given and compared with the ones reported in the literature to evaluate the convergence behaviour and accuracy of the proposed solution. Subsequently, the influences of temperature on natural frequencies of the nanocomposite quadrilateral plates with different geometric parameters, CNT distributions in thickness direction and boundary conditions are investigated.

Thermoelastic analysis by A. Alibeigloo[34] is of composite cylindrical panel reinforced by single walled carbon nanotube (SWCNT) with simply supported edges by using three-dimensional theory of elasticity. Thermoelastic constant of carbon nanotube (CNT) as well as polymer matrix are assumed to be temperature independent. The volume fractions of oriented, straight SWCNTs are assumed to be uniformly distributed (UD) and or graded in the thickness direction according to four kinds of CNT distributions. The effective material properties of the nanocomposite cylindrical panel are based on rule of mixture. At first temperature distribution in three dimensions is obtained by solving heat conduction differential equation with variable coefficient. By applying Fourier series expansion to the stress and displacement fields along the axial and circumferential direction and state space technique along the radial direction thermoelastic analysis is carried out. Moreover, effects of volume fraction of carbon nanotube, uniform distribution and functionally graded distribution of CNT, mid radius to thickness ratio, length to mid radius ratio, thermal and mechanical surface boundary conditions on bending behaviour of FG-CNTRC cylindrical panel are also examined.

The main objective is to present the buckling and vibration analysis of thermally prestressed functionally graded carbon-nanotube-reinforced composite (FG-CNTRC) annular sector plates resting on the elastic foundation via the variational differential quadrature (VDQ) method by Reza Ansari, Jalal Torabi, MostafaFaghihShojaei[35], [42]. The material properties of nanocomposite plate are considered to continuously vary across the thickness and are estimated according to the modified rule of mixture. The governing equations are derived on the basis of first order shear deformation theory. Applying two-dimensional generalized differential quadrature (GDQ) method, the energy functional of the structure is discretized. Then, based on Hamilton's principle and the VDQ method, the reduced forms of mass and stiffness matrices are obtained. After verifying the accuracy of the present method, comprehensive numerical results are presented to examine the effects of important parameters on the stability and vibrational behaviour of the nanotube-reinforced composite annular sector plates. The results indicate that functionally graded distributions of CNTs in the thickness direction and the increase of elastic foundation coefficients can improve the stability of the structure.

Y. Kiani research deals with the post-buckling problem of carbon nanotube reinforced composite plates subjected to uniform temperature rise loading. Distribution of carbon nanotubes as reinforcements may be uniform or functionally graded. To account for the large deformations of the plate, von- Kármán type of geometrical nonlinearity is included into the formulation. The virtual displacements principle associated with the conventional Ritz formulation whose shape functions are selected as the Chebyshev polynomials is used to obtain the matrix representation of the nonlinear equilibrium equations. The solution method is general and may be used for arbitrary combination of boundary conditions. The post-buckling equilibrium path which is governed by a nonlinear eigenvalue problem is traced using a displacement control strategy. Results of this study are compared with the available data in the open literature for the cases of isotropic homogeneous plates and cross-ply laminated plates. Afterwards numerical results are given for FG-CNTRC plates. It is shown that,
FG-X pattern results in higher buckling temperature and also decreases the post-buckling deflection of the plate. Furthermore, this type of composites are eager to exhibit the secondary instability which is designated with a snap-through phenomenon in the post-buckling equilibrium path.

L.W. Zhang investigated the effect of in-plane forces on the vibration behaviour of carbon nanotube (CNT) reinforced composite skew plates. The analysis is performed by implementing the first-order shear deformation theory (FSDT) with the element-free/mesh-free Improved Moving Least Square-Ritz (IMLS-Ritz) method for solution to the problem. Two varieties of carbon nanotube-reinforced composite skew plates, namely uniformly distributed and functionally graded reinforcement are considered. A micromechanical model is employed to estimate the material properties of CNT-reinforced composite plates. Comparison studies are implemented to validate the accuracy of the proposed method. The frequency parameters and mode shapes for the skew plates are presented. A detailed parametric study is carried out to reveal many complicated effect on the frequency parameters of the plate. These effects include in-plane stress ratio, boundary conditions, CNT-volume fraction and geometric size. The obtained results will be references of future research and corresponding engineering project.

A numerical meshless discretization technique is developed by RezaAnsari, JalalTorabi, Amir HoseinShakouri[42]within the framework of variational formulation to present the linear free vibration analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) elliptical plates. The effective material properties of nanocomposite plate are continuously varied across the thickness direction and are evaluated based on the extended rule of mixture. The governing equations are derived on the basis of the first order shear deformation theory. To this end, the matrix form of Hamilton’s principle is first presented. Then, based on the moving least-squares (MLS) approximation and background cells approach, the meshless differential and integral operators are constructed to perform the discretization process. After conducting the comparison and convergence study, various numerical results are reported to explore the effects of concerned parameters on the natural frequencies of composite elliptical plates reinforced with carbon nanotubes (CNTs). Results reveal that functionally grading of CNTs through the thickness direction can considerably improve the vibrational characteristics of FG-CNTRC elliptical plates.

A.H. Sofiyev a, *, Z. Zerin b, N. Kuruoglu[43] in study investigated, the thermoelastic buckling of functionally graded material (FGM) conical shells under nonlinear temperature rise across the thickness in the framework of the shear deformation theory (SDT). To the derivation of basic equations is used modified Donnell-type shell theory. The Galerkin method is used to obtain the formula for non-linear buckling temperature difference of freely supported FGM truncated conical shell in the framework of the SDT. By changing the properties of FGMs and volume fraction index, the effect of transverse shear deformations on the non-linear buckling temperature difference is evaluated by comparing with the results of the classical shell theory (CST). Meanwhile, the effect of geometric parameters on the non-linear buckling temperature difference of the shear deformable FGM conical shells is discussed in detail.

III. Conclusion

The exceptional mechanical and physical properties demonstrated for carbon nanotubes, combined with their low density, make this new form of carbon an excellent candidate for composite reinforcement. Before these extraordinary properties observed at the nanoscale are realized in a macroscopic composite, considerable basic research is necessary. Full understanding of the thermo-mechanical behaviour of nanotube-based composites, requires knowledge of the elastic and fracture properties of carbon nanotubes as well as of interactions at the nanotube/matrix interface. Although this requirement is no different from that in conventional fiber composites [3], the scale of the reinforcement phase diameter has changed from micrometre (e.g. glass and carbon fibers) to nano-meter. The change in reinforcement scale poses new challenges in the development of processing techniques for these composites as well as the development of characterization techniques and methodologies to measure the elastic and fracture behaviour of carbon nanotubes and their composites. The nanometer scale of the reinforcement presents additional challenges in mechanics research since we now must account for interactions at the atomic-scale. Preliminary research in nanotube-based composites has indicated that there is potential in carbon nanotubes for reinforcement, but, most importantly, it has illustrated the significant challenges that must be overcome before the potential is realized. Critical to the use of nanotubes as a structural material, there is a need for development of nanotube production techniques at the scale needed for producing macroscopic composites that are cost-effective. Fundamental work in processing, characterization, and analysis/modelling is crucial before the structural and functional properties of this new class of nanocomposite can be optimized.

References

DOI: 10.9790/1684-1406023646 www.iosrjournals.org 44 | Page
A Review on FG CNT Vibrational static Analysis


