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Review on the Curves and Surfaces Modeling Technology

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Abstract: *With the continuous development of computer technology, curve and surface modelling technology is more and more widely used in modern industrial product design and manufacturing process. However, due to errors in design and measurement, the smoothness of curves often fails to meet the requirements of design and manufacture. Curve smoothness not only affects the appearance of the product, but also directly affects the degree of difficulty in manufacturing process and the mechanical properties of the product. It is an important sign to evaluate the quality of the product. The research status of curve and surface modelling technology, interpolation, approximation and fitting of curves, fairing criteria and fairing methods of curves are discussed.*
Keyword- *Curve and surface modeling; Curve fairing; interpolation; approximation; fitting*

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I. Introduction

With the development of computer technology, especially microcomputers and their drawing technology, Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) have been widely used in machinery, electronics, aerospace, construction and other fields. application. The development and application level of CAD/CAM technology has gradually become one of the important indicators to measure the level of modernization of a country.

Since CAD/CAM technology originated in the aviation industry, due to the complex shape of the aircraft and the large number of free-form curved surfaces, CAD/CAM technology is closely linked to the free-form surface modeling technology from the beginning. The curved surface modeling technique was established by Coons, Bezier and other masters in the 1960s[1]. After years of research and development, the curve and surface modeling technology has formed a geometric theory system with the NURBS curve and surface parameterized feature design and implicit algebraic curve and surface representation as the main body, with interpolation, approximation and fitting as the skeleton.

II. Research status of curve and surface modeling technology

2.1 Curve surface modeling technology classification

The continuous expansion of CAD/CAM technology applications has led to an increasing demand for curved surface modeling. Because the traditional curve and surface modeling method is too simple, it is not convenient for the user to use, and it is difficult to effectively modify the curved surface, some newer and more flexible curved surface modeling methods have appeared, as follows:
(1) Function curve surface modeling technology
(2) Parametric curve surface modeling technology
(3) Implicit curve and surface modeling technology
(4) Curved surface modeling technology based on partial differential equation
(5) Curved surface deformation method modeling technology
(6) Subdivision curve surface modeling technology
(7) Curved surface modeling technology based on physical model
(8) Curved surface modeling technology of shape mixing
(9) Other modeling techniques

2.2 Development history of curve and surface modeling technology

As the basic geometric element of the CAGD system, the main form of expression has experienced the following development stages in the past few decades:

In 1963, Ferguson first proposed a vector function method that represented curves and surfaces as parameters, and introduced the parameters cubic and bicubic surface patches[2][3].The parametric form of curve and surface becomes the standard form of shape mathematical description from then on. Before that, the description of the curve has always been in the form of explicit scalar function $y = y(x)$ or implicit function equation $F(x,y)=0$, and the description of the surface is in the standard form of $z=z(x,y)$ or $F(x,y,z)=0$.

In 1964, Coons[4] proposed a general surface description method that defines a surface by giving four boundaries around a closed curve.Although such a surface is simple in form, there is an inconvenience in shape control and splicing.In 1967, Coons[5] further promoted this idea. But both have shape control and connectivity issues.

In 1972, Bezier[6] proposed a new method for controlling the design curve of a polygon.Curved surfaces generated by Bezier technology have a number of good properties, such as convex hull, affine transformation invariance, variability reduction, and endpoint interpolation. The method is not only simple and easy to use, but also solves the overall shape control problem beautifully, and advances the design of the curved surface to a big step, laying a solid foundation for the further development of the curved surface modeling. but there are still connection problems and partial modification problems.

In 1972, Boor [7] and Cox [8] each independently proposed a standard algorithm for B-spline calculation, which made B-spline begin to be widely used.The B-spline method almost inherits all the advantages of the Bezier method, overcomes its shortcomings, and successfully solves the local control problem and the splicing problem on the basis of continuous parameters. With the technology of node insertion technology, segmentation technology and ascending order technology, the B-spline method has entered a more practical stage, and theoretically proved that Bezier curve and surface is a special case of B-spline curve and surface.

In 1974, Gordon and Riesenfeld applied B-spline theory to shape description and proposed a B-spline curve and surface method[9-10]. This method almost inherits all the advantages of the Bezier method, overcomes the shortcomings of the Bezier method, successfully solves the local control problem, and easily solves the stitching problem on the basis of parameter continuity, thus making the shape of the free curve surface. The description problem has been better solved.

With the development of production, the B-spline method shows obvious deficiency: it cannot accurately represent the conic section line and the elementary analytic surface, which results in the unique geometric definition of the product, so that the curve and surface have no unified mathematical description, which is easy to cause confusion in production management.

In 1975, Versprille[11]first proposed a rational B-spline method, proposed the NURBS method, and studied its properties. This method can not only express free-form surfaces, but also accurately represent conic curves, quadric surfaces and rotating surfaces. Since then, until the late 1980s, the NURBS method has finally become the most popular mathematical method in modern curved surface modeling mainly due to the achievements of Piegl[12-13] and Tiller[14-15]. In 1991, the International Standardization Organization (ISO) promulgated an international standard for Standard for Exchange of Product model data(STEP), using the NURBS method as the only mathematical method for defining the geometry of industrial products, making the NURBS method the most important foundation for the development of curved surface modeling technology.

III. Interpolation, approximation and fitting of curves

3.1 Interpolation

What is interpolation? Interpolation, in short, is to construct approximate function $\varphi(x)$ of $f(x)$ with the function values of some points of given unknown function $f(x)$, which is called interpolation function $\varphi(x)$.

3.2Approximation

What is approximation? Approximation, simply speaking, constructing a curve to make it most close to a set of given type points in a certain sense, we call it curve approximation, the constructed curve is called approximation curve.

3.3 Fitting

Interpolation and approximation are collectively referred to as fitting[16].

Fitting does not have a complete definition and mathematical expression like the interpolation and approximation mentioned above. Fitting means that in the design process, the generated curves can meet some design requirements by interpolation or approximation, such as "smooth" and "smooth" curves. For curves, smoothing refers to their continuity or more precise requirements on tangent vectors.

IV. Curve smoothing

4.1 Fairing meaning

What is "fairing"? "Fairing", as the name suggests, is smooth and pleasing to the eye. "Smoothness" and what we call "smoothness" in our daily life are two different concepts, which cannot be confused. "Smoothness" usually refers to the parametric continuity or geometric continuity of a curve. It is a mathematical term. And "smoothing" has not only the requirement of continuity in mathematics, but also the requirement of

function (such as aesthetics, mechanics, NC machining). Because it involves the aesthetics of geometric shape and is influenced to a great extent by people's subjective factors, in general, "smoothness" is still a vague concept without an accurate definition and unified standard. Now, can we not smooth the curve? The answer is No. If there is no inherent rule for smoothing, how can we judge the smoothness of curves? Thus, the key to the problem is not whether fairness has objectivity, but how to coordinate the relationship between the objectivity and uncertainty of fairness, which is the problem to be solved by the definition and criterion of fairness.
Two problems need to be solved for smoothing curves:
(1) what kind of curve is smooth, that is, fairing criteria;
(2) For unfaired curves, which mathematical treatment should be adopted to satisfy or improve their fairness, that is, fairing treatment method.

4.2Curve fairing criteria
To smooth curves, we need to give specific fairing criteria first. Here are some fairing criteria which are often used in fairing processing.
(1) For plane curves
fairing criteria1(Farin proposed)[17]:
For a curve, if its corresponding curvature curve is continuous, has appropriate symbols (if the concavity and convexity of the curve are known), and is as close as possible to a piecewise monotone function with as few monotone segments as possible, then the curve is considered monotone.
fairing criteria2(Su Buqing,Liu Dingyuan proposed)[18]:
(a) The two order parameter is continuous (*C2* continuous).
(b) There are no additional inflection points.
(c) The curvature changes are more uniform.
fairing criteria3(Shi Fazhong proposed) [19]:
(a) The two order geometric continuum (refers to the position, tangent direction and curvature vector continuous, as *G2*).
(b) There are no singular points and unnecessary inflection points.
(c) The curvature changes more evenly.
(d) Strain energy is small.
(2) For space curve

The following criteria are proposed in document[20]:
(i) Two order smoothness
(a) The two order vector of a curve is continuous, and the curvature is continuous.
(b) The curve (quadratic) of the low order spline may have a jump in the curvature of the node, which requires the jump degree and the minimum possible.

$$\sum\left|k\left(t_i^+\right)-k\left(t_i^-\right)\right|<\varepsilon \tag{1}$$

(ii) There is no excess inflection point
(a)There should be *G* inflection points in the curve, while there are more than *G* inflection points in fitting (interpolation and approximation).
(b) There should be an inflection point where there should be no turning point.
(iii) The curvature changes more evenly
(iv) There is no redundant variable deflection point (point whose deflection is zero, usually related to the point whose deflection is variable), that is to say, the following situation is not allowed:
(a) There should be *H* deflection points, and there are redundant *H* deflection points when fitting (interpolation and approximation).
(b) There should be no turning point where there should be no turning point.
(v) Uniform variation of torsion
(a) The torsion may be discontinuous at the node, and the leaping and small enough should be made at this time.

$$\sum\left|\tau\left(t_i^+\right)-\tau\left(t_i^-\right)\right|<\varepsilon \tag{2}$$

(b) The variation of torsion is uniform, and there is no continuous sign change.

4.3Curve smoothing method
At present, according to the number of type points (or control points) modified each time, curve fairing algorithms can be divided into global fairing and local fairing. The global fairing algorithm is represented by least squares method and energy method. This kind of fairing algorithm can reduce the curvature of the curve as a whole and make the curvature of the curve change more uniformly, but the shape of the faired curve changes greatly compared with the original curve, and the direction of deformation cannot be controlled. In the local

fairing algorithm, the point selection modification method and curvature method are used. Rate method is representative. This kind of smoothing method has better smoothing effect in a small local range, but it cannot eliminate the concave defects.

(1) Global fairing

(a) least square method

In curve reconstruction, least squares method is the most important method. Its basic principle is to consider the deviation of approximate function $\varphi(x)$ from $\delta_i = \varphi(x_i) - y_i \quad (i = 0,1,\ldots,m)$ on given value point $(x_i, y_i)(i = 0,1,\ldots,m)$ as a whole, and to make it the smallest according to a certain measure standard.

1. Minimize $\max_{0 \le i \le m} |\varphi(x_i) - y_i|$

2. Minimize $\sum_{i=0}^{m} |\varphi(x_i) - y_i|$

3. Minimize $\sum_{i=0}^{m} |\varphi(x_i) - y_i|^2$

(b) energy method

In 1969, Hosaka[21] proposed a curve smoothing method based on the energy extremum principle, called energy method. The basic idea of this method is to use the cumulative chord length cubic spline as the reconstruction curve and the total energy of the spline as the objective function. Its fairing process is to solve the extremum problem of the objective function. The mechanical model of this method is very intuitive. Assuming that the sequence of shape points before and after smoothing a curve is $M_i\,(i = 0,1,\ldots,n)$ and $N_i\,(i = 0,1,\ldots,n)$, the formula for calculating strain energy is as follows:

$$E = \frac{1}{2}\alpha\int k^2 dx + \frac{1}{2}\sum_{i=0}^{n}\beta_i \left\| N_i - M_i \right\|^2 \qquad (3)$$

In (3), α ——Stiffness coefficient of the curve;

β_i ——Elastic coefficient;

k ——Curve curvature.

(2) Local fairing

Two commonly used global fairing algorithms are introduced. Both least squares method and energy method are suitable for the case of relatively large number of non-fairing points. However, when there are few non-fairing points (such as only a little non-fairing), the above two methods will undoubtedly result in a large amount of computation and speed of operation. Slow down. Several common local smoothing algorithms are introduced below.

(a) Point revision method

The smoothing process of the point selection method is: Step1: find out the "bad points" one by one; Step2: modify the "bad points". These two processes are discussed below.

Step1:Discrimination of "bad points"

There are two methods to distinguish "bad points": (i) Users decide "bad points" by themselves according to their observations, which is called interactive method. (ii) According to the corresponding fairing criteria, a criterion for judging "bad points" is established to accurately identify all "bad points", which is called automatic method.

$\boldsymbol{Q}_i\,(i = 0,1,\ldots,n)$ is assigned to the set value, assuming that $\boldsymbol{Q}(t)$ is the curve of interpolation and $\{\boldsymbol{Q}_i\}$, and k_i is the relative curvature at the point of type value. For automatic methods, in general, we use the following "bad points" criteria:

1) The k_i discontinuous type value points are called the 1 kind of bad points.

2) In the sign sequence $\{sign(k_i)\}$ of curvature, the point $\boldsymbol{Q}_i$ of continuous sign change is called two kinds of bad points even if the condition $k_{i-1}k_i < 0$ and $k_i k_{i+1} < 0$ hold.

3) The points of continuous sign change in the first order difference symbol sequence $\{sign(\Delta k_i)\}$ of k_i, even if condition $\Delta k_{i-1}\Delta k_i < 0$ and $\Delta k_i \Delta k_{i+1} < 0$ hold, are called three kinds of bad points.

Step2: Modification of "bad points"

After identifying the "bad points", we need to modify them. At present, the commonly used "bad point" modification methods include kejellander method[22], node deletion and insertion method[23], roundness method and base spline method. The advantage of this method is that the criterion is simple, local modification, fast calculation speed and fairing effect is very good in the case of fewer "bad points". The disadvantage of this method is that when there are many "bad points" in succession, the criterion of "bad points" depends on the curvature of adjacent points to calculate, so the criterion will appear certain. The degree of misjudgment is not very good.

(b) Curvature method

The objective function of energy smoothing is to calculate the square and integral of curvature. The objective of smoothing is to reduce the strain energy of the curve as much as possible, that is, to reduce the curvature of the curve, so that the curve area is smooth and the shape change is large.

Chen Liang[24] developed a curve smoothing method based on the curvature method of wavelet decomposition, extracted the part with lower frequency after wavelet decomposition as a new curvature map, and finally reconstructed the smoothed product contour curve. The basic idea of these methods is to calculate the curvature map of the curve, then smooth the curvature map of the curve, not directly smooth the curve. Then, according to the curvature map after smoothing and combining with the original value points, the original curve can be inversely calculated, so that the fairing curve of the original curve can be obtained.

V. Conclusion

The development of curve and surface modeling technology provides powerful support for modern manufacturing industry. It extends from traditional curve interpolation, approximation and fitting to various smoothing techniques, and presents the phenomenon of cross-fusion of multi-disciplinary and multi-field. The industry has also put new technology and theory into industry application very quickly, which shows the importance of curve and surface modeling technology for modern industry. Throughout the development history of curve and surface modeling technology, finding a better curve fairing technology is the key to promote the whole CAGD research.

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