3D Analysis for Conical Tanks under Seismic Loads

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Abstract: In this study, conical ground and elevated tanks' seismic response has been investigated. This study has been motivated with the deficiency of data in design codes concerning conical tanks seismic analysis. Seismic analysis by American Water Works Association for welded steel tanks for water storage (AWWA D-100) or Egyptian code for loads (EC201-2012) is based on an approximate method by using the mechanical analog of cylinder tanks by substituting the conical tank with an equivalent cylinder tank. 3D Finite element models have been modeled using added mass approach in order to represent the fluid-structure interaction. A previously established mechanical analog for conical tanks under horizontal seismic force has been used in the finite element model. The 3D finite element model is verified with El Damatty and Sweedan (2006) work and with Maheri et al. (1988) experimental work. A wide range of conical tanks has been studied to investigate the seismic response of conical tanks through evaluating maximum base shear, maximum overturning moment and maximum moment at the bottom of wall using response spectrum method. Those obtained values using finite element model, AWWA-D100 and ECP201-2012 have been compared to each other. This study reveals that approximate methods underestimate the seismic response of conical tanks, hence some correction factors have been suggested to be used if AWWA-D100 or ECP201-2012 procedures to be used. It also has been concluded that for the same tank volume and the same liquid height to radius at vessel base ratio (h/Rₜₜ), a vertical inclination angle of tank's walls (θ) of 15° gives the minimum moment at the bottom of wall.

Keywords: added mass approach; conical tanks; fluid-structure interaction; seismic analysis; seismic response

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I. Introduction

Liquid-containing tanks are used in water distribution systems and in industries, also water supply is essential for controlling fires, so tanks should stay effective to prevent disastrous damage [3]. Due to inclination of conical tanks' walls, the seismic response of a conical tank is different from the behavior of a cylindrical tank. Research work associated with seismic response of cylindrical vessels is somewhat intensive [5]. According to most of current building codes [2,4], if the container has a truncated reversed cone bottom, an equivalent cylinder tank may be considered. Although the stability of conical tanks under hydrostatic pressure has been investigated intensively by Vandepitte et al. (1982) and El Damatty et al. (1997), enumerated studies have investigated seismic response of conical tanks. The first numerical research on the seismic response of conical tanks has been studied by El Damatty et al. (1997). An experiment studies have been conducted by Sweedan and El Damatty (2002) to investigate the dynamic characteristics of conical vessels. El Damatty and Sweedan (2006) have established an equivalent mechanical analog and have presented charts showing different fluid parameters required to to study the seismic response of conical tanks under horizontal ground excitations [5].

The main objective of this study is to create 3D finite element models for a wide range of conical tanks by using the fluid parameters developed by El Damatty and Sweedan (2006) [5] to investigate the fluid-structure interaction and seismic response of those tanks with response spectrum analysis. Results obtained have been analyzed to study the effect of inclination of walls in conical tanks, also finite element models' results have been compared with the corresponding values obtained from American Water Works Association for welded steel tanks for water storage(AWWA-D100) and Egyptian code for loads(ECP201-2012) procedures.

II. Modeling the fluid-structure configuration

2.1 Simplified models

Simplified mechanical analogs have been developed by Housner (1963) and followed by many researchers and are being used in many design codes. For ground tanks, the fluid is represented through two masses, impulsive mass m, that represents the portion of fluid that accelerates rigidly with tank walls and attached to tank wall, and convective mass m, that represent the sloshing of the fluid and attached to tank wall.
walls by spring with stiffness $K_c$ as shown in fig.(1). Also a two-mass model has been developed by Housner (1963) for elevated tanks where the system has been considered as a two separate single degree of freedom systems and represented by $m_2$ that represent the convective portion of the fluid and $m_1$ that represent the impulsive portion of the fluid and tank mass added to tank self-weight and some portion of supporting structure weight as shown in fig.(2).

![Ground tanks mechanical analog](image1.png)

**Figure (1):** Ground tanks mechanical analog

![Elevated tanks two-mass model](image2.png)

**Figure (2):** Elevated tanks two-mass model

### 2.2 Finite element method

There are many methods to model fluid-structure interaction with finite element problems such as added mass approach (Westergaard 1931; Barton and Parker 1987; Dogangun 1996a), the eulerian approach (Zienkiewicz and Bettes 1978), the lagrangian approach (Wilson and Khalvatı 1983; Olson and Bathe 1983; Dogangun et al. 1996b, 1997; Dogangun and Livaoglu 2004) or the eulerian-lagrangian approach (Donea et al. 1982). Special programs are required to use all these methods except added mass approach [3,6]. In the added mass approach the fluid masses is added to the structure’s mass at certain heights, the general equation of motion can be written as the following:

\[
M\ddot{U} + C\dot{U} + KU = -Mg \tag{1}
\]

Where, $M$ is the total mass for both structure and fluid, $C$ the damping of the structure, $K$ the stiffness of the structure, $U$ the displacement, $\dot{U}$ the relative velocity, and $\ddot{U}$ the relative acceleration.
III. Conical tank fluid parameters

In order to use added mass approach in the finite element models (FEM), representing the fluid effect is based on some parameters, those parameters are flexible mass ratio \( \frac{m_f}{m_t} \) that represents the fundamental impulsive mode as this portion vibrates with the deformation of vessel’s walls, flexible mass height ratio \( \frac{h_f}{h_w} \) that represent the effective location of \( m_f \), convective mass ratio \( \frac{m_s}{m_t} \) that represent the portion of the fluid that moves freely in a sloshing phase, convective mass height ratio \( \frac{h_s}{h_w} \) that represent the effective location of \( m_s \), and convective mode link stiffness \( k_s \) represent the link stiffness that connects the convective mass \( m_s \) to the tank walls. El Damatty and Sweedan (2006) have presented charts to estimate these parameters depending on the geometry characteristics of the tank that can be used to represent the fluid-structure interaction during seismic excitation. The model comprises rigid, flexible, and sloshing components. It is noted that the residual rigid mass \( m_o \) is equal to \( (m_r - m_f) \) represent the portion of the fluid that vibrates with the base of the vessel, also sloshing frequency \( f_s \) is used to get convective mode link stiffness \( k_s \). Figs. (3A) through (3C)[5] show charts that may be used to extract the required fluid parameters for the 3D finite element models used in this paper.

Where \( m_t \) is fluid flexible mass, \( m_r \) is total fluid mass, \( h_f \) is flexible mass height, \( h_w \) is fluid height, \( m_s \) is fluid convective mass, \( h_s \) is convective mass height, \( k_s \) is the stiffness of the convective mode, \( m_o \) is the residual rigid mass, \( m_i \) is the mass for all impulsive modes, and \( f_s \) is convective mode frequency.

![Variation of mechanical analog masses](image)

Figure (3A): Fluid masses ratio used in the FEM (El Damatty and Sweedan (2006))
IV. Design codes procedures

In this study, AWWA-D100 (2005), and ECP201 (2012) are studied and compared with the finite element model as discussed before. To the best of the author’s knowledge, there is no guidelines for seismic design of conical tanks and most of current design codes use an equivalent cylinder tank to determine the seismic response of conical tanks [12,14]. The equivalent cylinder tank in AWWA-D100 (2005) differs from the one specified in ECP201 (2012) as explained in appendix A and appendix B.
V. Finite element model

In this study, SAP2000 (V 15.10) structural analysis program [8] has been used in the seismic analysis of the studied tanks. Ground tanks have been assumed to be fully anchored to the foundation; therefore boundary condition at the base nodes has been assumed to be fixed in both elevated and ground tanks and other nodes to be free as shown in fig.(4). Columns and beams have been modelled using the frame element, while walls of the vessel have been modelled using the shell element.

The added mass approach has been used to model fluid-tank interaction in the models. For the impulsive mass, different techniques may be used for adding it to vessel’s walls, first technique is to distribute the impulsive mass equally over the nodes at height from the bottom of vessel to the height of \( h_f + (h_i - h_s)/2 \), for the second technique, the hydrodynamic pressure distribution on the vessel’s wall is estimated and the mass distribution will be in accordance with that hydrodynamic pressure, lastly for the third technique, a previous study by Algreane, G. A. I. et al. [7] has concluded that the impulsive mass may be simply distributed equally over nodes of the vessel’s walls at a height of \( h_i \). In this study the third technique has been used and the flexible mass has been distributed equally over nodes of the vessel’s walls at a height of \( h_i \) from vessel base in both X and Y global directions. The residual rigid mass \( m_o \) has been added equally to the nodes of the base of the vessel in both X and Y global directions.

For convective mass, it has been lumped at the centre and at a height of \( h_s \) from vessel base in both X and Y global directions, this mass has been connected to the nodes of the vessel at the same level using link element of stiffness of \( k_s/2 \) in order to represent the convective mode precisely [10]. Figs. (4) to (5B) illustrate the assignment of impulsive and convective masses. Also it should be noted that the stiffness of the link element doesn’t depend on tank walls and that the convective mode is always higher than the tank’s modes and has a very minor effect on seismic action of the tank, therefore it is not expected that this mode will cause seismic action concentration on the tank.

The effect of hydrodynamic pressure acting on the base of the vessels has not been considered in this study.

Response spectrum analysis has been used to estimate seismic response using RITZ modal analysis. A total of six modes have been considered in models to capture convective and impulsive modes, modal analysis results have shown a sufficient mass participation ratio with ratio higher than 90%, including more modes wouldn’t have made a notable difference, also design codes as explained in appendices includes only convective mode and the \( 1^{st} \) impulsive mode. SRSS method has been used for both modal and directional combination. Damping values for the tanks have been taken as 5% for the impulsive modes and 0.5% for the convective mode, different damping ratios have been considered in the models with a default value of 5% with using overrides for the first two modes of 0.5% as convective modes in X and Y directions are always the first two modes. Response spectrum function used in this study is shown in Fig.(6).

It should be noted that the modelling procedure used in this study is imperfect to tank vessel dimension where the corresponding effective heights that represents hydrodynamic pressure effect of liquid exceeds the vessel height. This condition is usually associated with values of \( \Theta_V \) larger than 30° which may be unpractical angle, although if a tank of \( \Theta_V \) larger than 30° is to be studied, the modelling procedure in this study may be used with some modifications to represent the significant effect of the vertical resultant of the hydrodynamic pressure on vessel’s walls.
Figure (5A): Section plan at impulsive mass level (hi)

Figure (5B): Section plan at convective mass level (hs)
VI. Model verification

The finite element modelling method used in this study has been verified in order to make sure that finite element models represent the actual seismic response of the conical tanks. Model verification has been done on three steps, the first step is to verify the discussed mechanical analog parameters of the conical tank, the second step is to verify finite element model results compared to manual procedure explained by El Damatty, Ashraf & Sweedan, Amr. (2006), and the third step is to compare the finite element model results with an experiment conducted by Maheri MR, Severn RT [1].

6.1 Conical tank mechanical analog parameters verification

The mechanical analog parameters of the conical tank depends on the geometry characteristic of the tank, in order to make sure of the validity of that analog, a conical tank with vessel walls height to base radius ratio \((\frac{h_w}{R_b}) = 2\) has been selected where vertical inclination angle of vessel’s walls \((\theta_v)\) is set to be zero, output parameters have been compared with ECP201-2012 parameters of the cylinder tank as displayed in Table (1). Results have shown a very good agreement between the two mechanical analogs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conical Tank with (\theta_v = 0)</th>
<th>Cylinder Tank by ECP201</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{m_{r}}{m_{t}})</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>(\frac{h_{r}}{h_{t}})</td>
<td>0.38</td>
<td>0.40</td>
</tr>
<tr>
<td>(\frac{m_{s}}{m_{t}})</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>(\frac{h_{s}}{h_{t}})</td>
<td>0.78</td>
<td>0.76</td>
</tr>
<tr>
<td>(K_{c}\frac{h}{m*g})</td>
<td>0.81</td>
<td>0.84</td>
</tr>
</tbody>
</table>

6.2 Finite element model compared to manual procedure by El Damatty

To study the validity of the finite element modelling method used in this study, three different conical tanks have been modelled using the method stated previously. Results have been compared to the manual procedure explained by El Damatty, Ashraf & Sweedan, Amr [5]. Results are displayed in Table (2). Results have shown a very good agreement between the two methods with difference not exceeding 8%. A detailed discussion of this procedure is presented in appendix c.

<table>
<thead>
<tr>
<th>Tank geometry</th>
<th>Response Spectrum Type</th>
<th>Finite Element Model</th>
<th>Equivalent mechanical analog (manual procedure)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_v)</td>
<td>(h_v) (m)</td>
<td>(R_v) (m)</td>
<td>(h_w/R_b)</td>
<td>Base shear (KN)</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>EC8-RS</td>
</tr>
</tbody>
</table>
### 6.3 Finite element model compared to experiment conducted by Maheri MR, Severn RT

To further verify the finite element modelling method used in this study, a finite element model has been compared with an experiment conducted by Maheri MR, Severn RT [1] on cylinder steel tanks. This cylinder tank has been selected due to the rare of availability of detailed experiment studies conducted on pure conical tanks, also a cylinder tank is a conical tank with $\Theta_v = 0$, thus the same modelling procedure for conical tanks has been used.

Maheri MR, Severn RT have measured the base shear force in a water tank subjected to Parkfield 1966 earthquake record with peak ground acceleration $= 0.25g$ as shown in Fig.(8). Tank’s dimensions are shown in fig.(7). The maximum base shear measured in the experiment is 570 N while the finite element model has shown a very good agreement with maximum base shear of 540 N as shown in Table (3) and fig.(9) [1].

<table>
<thead>
<tr>
<th>Default Type(2) 0.2g</th>
<th>EC8-RS Default Type(1) 0.2g</th>
<th>15</th>
<th>6</th>
<th>2</th>
<th>3</th>
<th>30</th>
<th>9</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1322</td>
<td>298</td>
<td>1222</td>
<td>0.67%</td>
<td>8.0%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>14060</td>
<td>1805</td>
<td>13234</td>
<td>7.9%</td>
<td>6.2%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 7: Water tank studied by Maheri MR, Severn RT](image)

![Figure 8: Parkfield 1966 earthquake record](image)
Table 3: Comparison between experiments conducted by Maheri MR, Severn RT and finite element model

<table>
<thead>
<tr>
<th>Finite Element Model</th>
<th>Experiment</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base shear (N)</td>
<td>Base shear (N)</td>
<td>Base shear</td>
</tr>
<tr>
<td>540</td>
<td>570</td>
<td>5.2%</td>
</tr>
<tr>
<td>0.03</td>
<td>0.034</td>
<td></td>
</tr>
<tr>
<td>0.0082</td>
<td>0.0068</td>
<td></td>
</tr>
</tbody>
</table>

VII. Effect of walls inclination in conical tanks

In order to investigate the effect of walls inclination of conical tanks, a wide range of conical tanks have been studied to include different inclination angle of walls ($\theta_v$) as follows: $5^\circ$, $10^\circ$, $15^\circ$, $20^\circ$, $25^\circ$, and $30^\circ$. For each $\theta_v$, different ($h_w/R_b$) ratios have been studied as follows: 1.5, 2, 2.5, and 3 with different liquid volume as follows 250 m$^3$, 500 m$^3$, 1000 m$^3$, and 2000 m$^3$.

Fig. (10) shows geometric characteristics of the conical tank in this study. The studied tanks contain water with density 1000 Kg/m$^3$, height of water to vessel’s height has been taken as 0.8. Each tank has been studied as ground tank, elevated tank of height 10 m, and finally elevated tank of height 20 m. For elevated tanks, staging frame system has been used for supporting the vessel’s with total number of twelve columns connected with beams each 3.33 m. The studied tanks have been assumed to be concrete tank with grade $F_{cu}=30$ Mpa. Tanks’ walls have been taken as 250 mm thickness, for staging frames, connecting beams dimensions have been taken as 600mm x 600mm, columns radius has been taken as 325 mm in case of 10 m height elevated tanks and 650 mm in case of 20 m height elevated tanks. Response spectrum function used has been defined as discussed in clause (5) and shown in Fig. (6). Figs. (11) shows sample of the studied tanks.
Results are displayed in Fig. (12) showing moment at the bottom of wall for both ground and elevated tanks in different $\Theta_v$.

**Figure (12)** Moment at the bottom wall for different vessel’s volumes from finite element models.
Results show that for the same fluid volume, as $\Theta_V$ increases the fluid height in the vessel decreases so moment below vessel’s walls will decrease till a certain value of $\Theta_V$ where the effect of vessel’s wall inclination becomes more significant due the effect of vertical resultant of the hydrodynamic pressure on vessel’s walls. Fig. (12) shows that the value of $\Theta_V = 15^\circ$ is the vessel’s walls vertical inclination angle where the moment below vessel’s walls become the least for the same volume and the same $h_v/R_v$.

**VIII. Comparison between FEM and design codes**

In order to check the adequacy of design codes approximate methods, the same finite element models studied in clause (7) have been solved using ECP201-2012 and AWWA-D100 procedure. Comparison have been done between FEM and design codes for both ground tanks and elevated tanks.

**8.1 Ground tanks**

Ground tanks results are displayed in Fig. (13), this group includes base moment differences for ground tanks between FEM and ECP201, and between FEM and AWWA-D100. These differences may be used as correction factors to be multiplied by AWWA-D100 and ECP201 base moments, these correction factors depend on $\Theta_V$ of the conical tank and conical tank’s volume. For other tank volumes, interpolation and extrapolation may be used. On the other hand, it has been noted that base shear resulted from AWWA-D100 and ECP201 procedures don’t almost need correction as the base shear resulted from ECP201 is so close to the FEM or higher and the base shear resulted from AWWA-D100 is always higher than FEM, so design codes may be considered more conservative in calculating base. Also correction factors are suggested, correction factors depend on $\Theta_V$ of the conical tank and conical tank’s volume. For other tank volumes, interpolation may be used. Fig. (13) Shows base moment correction factor for both AWWA-D100 and ECP 201 in case of ground tanks.

![Figure (13) Base moment correction factors for ground tanks](image-url)
In case of ground tanks, for ECP 201 – 2012, the equivalent cylinder tank methodology used generates a broad tank which has a height of liquid less than the one in the conical tank, it is also noted that broad tanks have equivalent impulsive heights less than narrow tanks. This leads to base moment less than the one calculated by finite element model that represent the real conical tank which means that ECP 201 – 2012 equivalent cylinder tank is less conservative and doesn’t represent the real seismic response of conical tanks. On the other hand, although equivalent masses ratio is almost the same for the equivalent cylinder tank in ECP 201 2012 and the equivalent conical tank used in the finite element model, base shear calculated by ECP 201–2012 is higher than the one calculated by finite element model, this is due to the fact that the equivalent tank used in ECP 201-2012 is broader which leads to a more rigid tank with the same total mass.

For AWWA D100, unlike ECP 201-2012, the equivalent cylinder tank methodology used generates a cylinder tank with height and radius larger than original conical tank, thus the equivalent cylinder tank water volume is higher than the original volume, although the impulsive mass ratio used is higher than the ones used in ECP 201-2012 and finite element model, the impulsive mass height is less. This leads to a base moment less than the one calculated by finite element model and higher than the one generated by ECP 201-2012 equivalent cylinder tank methodology. On the other hand, base shear generated by AWWA D100 methodology is in most of cases the highest value between all the methods investigated due to the high tank water volume used.

8.2 Elevated tanks

Elevated results are displayed in Fig. (14). This group includes base moment difference for elevated tanks between FEM and ECP201, and between FEM and AWWA-D100. These differences may be used as correction factors to be multiplied by AWWA-D100 and ECP201 base moments, these correction factors depend on Θv of the conical tank and conical tank’s volume. For other tank volumes, interpolation and extrapolation may be used. On the other hand, it has been noted that base shear resulted from AWWA-D100 and ECP201 procedures don’t almost need correction as the base shear resulted from ECP201 is so close to the FEM or higher and the base shear resulted from AWWA-D100 is always higher than FEM, so design codes may be considered more conservative in calculating base. Also correction factors are suggested, correction factors depend on Θv of the conical tank and conical tank’s volume. For other tank volumes, interpolation may be used. Fig. (14) Shows base moment correction factor for both AWWA-D100 and ECP 201 in case of ground tanks.
In case of elevated tanks, for ECP 201 – 2012, the equivalent cylinder tank methodology used generates a broad tank which has a height of liquid less than the one in the conical tank, it is also noted that broad tanks have equivalent impulsive heights less than narrow tanks. This leads to base moment less than the one calculated by finite element model that represent the real conical tank which means that ECP 201 – 2012 equivalent cylinder tank is less conservative and doesn’t represent the real seismic response of conical tanks. On the other hand, base shear calculated by ECP 201–2012 is in most cases same as the one calculated by finite element model, this is due to the fact that equivalent masses ratio are almost the same for the equivalent cylinder tank in ECP 201 2012 and the equivalent conical tank used in the finite element model, and the slight difference in base shear if any- are only due to the difference in the rigidity between the two tanks due to the geometric difference between them.

For AWWA D100, unlike ECP 201-2012, the equivalent cylinder tank methodology used generates a cylinder tank with height and radius larger than original conical tank, thus the equivalent cylinder tank volume is higher than the original volume, although the impulsive mass ratio used is higher than the ones used in ECP 201-2012 and finite element model, the impulsive mass height is less. This leads to a base moment less than the one calculated by finite element model and higher than the one generated by ECP 201-2012 equivalent cylinder tank methodology. On the other hand, base shear generated by AWWA D100 methodology is the highest value between all the methods investigated due to the high tank volume used.

IX. Summary and Conclusion

In this paper, 3D Finite element models have been modeled to investigate the seismic response of conical tank using response spectrum analysis. Conical tank mechanical analog parameters have been used to create finite element model (FEM) used in this study. A wide range of conical tanks have been studied using 3D FEM in order to check the effect of inclination of walls in conical tanks. It has been concluded that vessel’s walls vertical inclination angle ($\theta_v$) of 15$^\circ$ is the angle where the internal forces in the conical tank become the least. Also FEM results have been compared to American Water Works Association (AWWA-D100) and Egyptian code for Loads (ECP201-2012) equivalent cylinder tanks. Results have shown that both AWWA-D100 and ECP201 codes equivalent cylinder tanks don’t represent the conical tank accurately. Base shear resulted from AWWA-D100 procedure is higher than FEM and ECP201, while base shear resulted from ECP201 procedure has showed a better agreement with FEM, hence base shear in both codes may be considered conservative. Overturning moment resulted from AWWA-D100 procedure is about from 30% to 45% less than FEM in case of ground tanks, and about 10% to 40% less than FEM in case of elevated tanks depending on tank walls inclination, while the resulted overturning moment from ECP201 procedure is about 20% to 70% less than FEM in case of ground tanks, and about 20% to 45% less than FEM in case of elevated tanks depending on tank walls inclination, this high difference is due to that the equivalent tank in ECP201 has the same fluid volume with a shorter and broader cylinder tank. It has been concluded that equivalent cylinder tank doesn’t represent conical tank accurately and is not recommended in seismic analysis of conical tanks, hence some correction factors have been reached and presented in the form of graphs for ground and elevated tanks, correction factors are recommended to be used in case of using AWWA-D100 or ECP201 procedure for conical tanks.

References


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Appendix A. ECP201-2012 (ECP201) procedures [2]

ECP201 procedure is explained in the following steps:

1. Get the geometry of the cylinder equivalent tank using the liquid volume of the conical tank (V) and the radius is the radius at the top of liquid surface (R_T), therefore height of liquid (h_weq) will be driven as:

\[ 1.1 \ h_{weq} = \frac{V}{(\pi R_T^2)} \]  

(1)

Where, \( h_{weq} \) is water height in the cylinder tank, \( V \) is the liquid volume in conical tank, and \( R_T \) is the radius at top of liquid surface in conical tank.

2. Determine the parameters for that cylinder tank, parameters are impulsive and convective masses and heights, and convective mode spring stiffness using figs (A1) and (A2).

3. Add tank mass and an appropriate portion of one third of supporting structure mass in case of elevated tank- to the impulsive mass portion of the fluid.

4. Determine the convective mode`s period (T_c) using the stiffness and mass from step #2.

5. Determine the impulsive mode`s period (T_i), for ground tanks (T_i) may be estimated using factor (C_i) in the code using the following equation:

\[ 5.1 \ T_i = \left( \frac{C_i h_{weq}}{\sqrt{tu}} \right) \sqrt{\frac{\rho}{E}} \]  

(2)

Where, \( C_i \) is the impulsive mode`s period factor obtained from fig (A3), \( tu \) is the wall thickness, \( \rho \) is the fluid`s mass density, and \( E \) is the young`s modulus of the tank wall.

For elevated tanks (T_i) may be estimated using finite element model or any other method depending on the structure system of the studied tank.

6. Determine spectral acceleration using impulsive and convective modes` periods.

7. Determine base shear and overturning moment using square-root sum of squares method (SRSS)

![Figure A1](image1.png)

Figure A1 Ratios of convective and impulsive masses and convective spring stiffness (ECP201-2012)

![Figure A2](image2.png)

Figure A2 heights of impulsive and convective masses (ECP201-2012)
Appendix B. American Water Works Association (AWWA-D100) procedures [4,9]

AWWA-D100 procedure is explained in the following steps:

1. Get the geometry of the cylinder equivalent tank using the following equations:
   \[ h_{weq} = \frac{h \cos \Theta_v}{\cos \Theta_v} \]  \hspace{1cm} (3)
   
   Where, \( h \) is water height in the conical tank, and \( \Theta_v \) is the vertical inclination angle of the conical tank height.

2. Equivalent tank radius:
   \[ R_{eq} = \frac{2R_b + h_w \tan \Theta_v}{2\cos \Theta_v} \]  \hspace{1cm} (4)
   
   Where, \( R_b \) is the radius of the conical tank base.

3. Equivalent tank Volume:
   \[ V_{eq} = \pi \left( R_{eq} \right)^2 h_{weq} \]  \hspace{1cm} (5)

2. Determine the parameters for that cylinder tank using the following equations:

   When \( D/H \geq 1.333 \):
   
   2.1. Impulsive mass:
   \[ m_i = \frac{\tan \left( 0.866 \frac{2R_b}{h_{weq}} \right)}{\left( 0.866 \frac{2R_b}{h_{weq}} \right)} \times m_t \]  \hspace{1cm} (6)
   
   Where, \( m_t \) is total fluid mass.

   2.2. Impulsive mass height:
   \[ h_{i} = 0.375 h_{weq} \]  \hspace{1cm} (7)

   When \( D/H < 1.333 \):
   
   2.3. Impulsive mass:
   \[ m_i = (1.0 - 0.218 \frac{2R_b}{h_{weq}}) \times m_t \]  \hspace{1cm} (8)

   2.4. Impulsive mass height:
   \[ h_{i} = (0.5 - 0.094 \frac{2R_b}{h_{weq}}) h_{weq} \]  \hspace{1cm} (9)

   2.5. Convective mass:
   \[ m_c = (0.23 \frac{D}{H} \tan \left( 3.67 \frac{h_{weq}}{2R_b} \right)) \times m_t \]  \hspace{1cm} (10)

   2.6. Convective mass height:
   \[ h_c = (1.0 - \frac{3.67 \frac{h_{weq}}{2R_b}}{\sinh \left( \frac{3.67 \frac{h_{weq}}{2R_b}}{2R_b} \right)}) \times h_{weq} \]  \hspace{1cm} (11)

3. Add tank mass and an appropriate portion of one third of supporting structure mass in case of elevated tank to the impulsive mass portion of the fluid.

4. Determine the convective mode’s period \( (T_c) \) using the following equation:
   \[ T_c = \frac{2\pi}{3.68g \times \tanh \left( \frac{3.68 \frac{h_{weq}}{2R_b}}{2R_b} \right)} \]  \hspace{1cm} (12)

5. Determine the impulsive mode’s period \( (T_i) \) using any proper method.
6- Determine spectral acceleration using impulsive and convective modes’ periods
Determine base shear and overturning moment using square root summation of square method (SRSS).

Appendix C. Method proposed by El Damatty, Ashraf & Sweedan, Amr [5]

1- Calculate total liquid mass (mt) in the conical tank
2- Based on hw/Rb and θv values, get the followings from figs (3A) to (3C):
   a. Convective mass ratio (ms/mt)
   b. Convective mass height (hs/hw)
   c. The convective mode frequency (fs)
   d. Flexible mass ratio (mf/mt)
   e. Flexible mass height (hf/hw)
   f. Rigid mass ratio (mr/mt)
   g. Rigid mass height (hr/hw)
   h. Impulsive mode frequency ratio (f1 x h x √ρ)
   i. Effective tank mass activated in the impulsive mode (mef/mb)

3- Calculate analog parameters ms, hs, mf, hf, mr, f1, and mef based on step#2
4- Calculate stiffness of the tank using the following equation:
   K = 4π² f1² mf

5- Calculate liquid-shell system impulsive mode (fsys) that includes the effective tank mass using the following equation:
   fsys = (K / mf + mef) / 2π

6- Calculate convective mode spectral acceleration (Sas) from response spectrum curve based on convective mode frequency (fs)
7- Calculate impulsive mode spectral acceleration (Sasys) from response spectrum curve based on convective mode frequency (fsys)
8- Calculate base shear (Q) using SRSS method based on the following equations:
   Q1 = ((mr-mf) + mab) * G^-1(t)max
   Q2 = (mf + mef) * Sasys
   Q3 = ms * Sas
   Q = √(Q1² + Q2² + Q3²)

9- Calculate base moment (M) using SRSS method based on the following equations:
   M1 = ((mr-mf) * hr – (mf + mab) * hf) * G^-1(t)max
   M2 = (mf + mef) * Sasys * hf
   M3 = ms * Sas * hs
   M = √(Q1² + Q2² + Q3²)