Analysis of Friction Losses in Pipe with Analytical Method

Divyesh A. Patel¹ Vimal N. Chaudhari² Deep R. Patel³

¹(Lecturer in Mechanical Department in Goverment Polytechnic Waghai)
²(Lecturer in Mechanical Department in Goverment Polytechnic Waghai)
³(Lecturer in Mechanical Department in Goverment Polytechnic Waghai)

Corresponding Author: Divyesh A. Patel

Abstract: The frictional loss is complex phenomenon even in single-phase or multi-phase fluid flow. A lot of modifications have been carried out from many years and various formulas have been developed to calculate head loss based on the experimental data. Darcy-Weisbach and Colebrook and white are still in used for predicting the losses in the commercial pipeline system. But Colebrook and white approximation is implicit in nature, requires iteration process to develop the accurate friction factor value. Three different pipe materials stainless steel, Cast iron and PVC for all the pipe materials have been used to predict the results for Reynolds number up to 50000.

Keywords: Analytical method, Colebrook and White equation, Friction losses, Flow pattern Reynold number.

Date of Submission: 04-04-2019 Date of acceptance: 19-04-2019

I. Introduction

There are many problems associated with pipe flow whether the fluid is in rest or in motion. Frictional losses are one which is commonly encountered in transportation of fluid from one place to another through the pipes. When a fluid moves, energy is dissipated due to friction; turbulence dissipates even more energy for high Reynolds number flows.

Friction loss refers to that portion of pressure lost by fluids while moving through a pipe, hose or other limited space. Because of viscosity, there is friction within the fluid as well as friction of the fluid against the pipe. This friction converts some of the pressure energy of the flowing fluid into heat and raises the temperature of the fluid and piping. This phenomenon can be critical in the operation of some equipment. To move a given volume of liquid through a pipe requires a certain amount of energy. An energy or pressure difference must exist to cause the liquid to move. A portion of that energy is lost due to the resistance to flow. This resistance to flow is called head loss due to friction. Friction between the pipe wall and the fluid tries to slow down the fluid unless to get an assistance from gravity or naturally occurring pressure, generally have to install pumps or compressors to counter the friction. Friction from the walls of the pipe on the liquid is the head loss, caused by friction. Friction losses are very dependent upon the viscosity of the liquid and the amount of turbulence in the flow. Friction loss is the part of the total head loss that occurs as the fluid flows through straight pipes. The head loss for fluid flow is directly proportional to the length of pipe, the square of the fluid velocity, and a term for fluid friction called the friction factor. The head loss is inversely proportional to the diameter of the pipe. Hence the friction loss must be calculated in order to properly size pipe, elbows, valves, and other fitting along the piping system. When a fluid flows through a pipe the internal roughness of the pipe wall can create local eddy currents within the fluid adding a resistance to flow of the fluid. Pipes with smooth walls such as glass, copper, brass and polyethylene have only a small effect on the frictional resistance. Pipes with less smooth walls such as concrete, cast iron and steel will create larger eddy currents which will sometimes have a significant effect on the frictional resistance.

1.1 Analytical analysis conducted in this study are following

The flow of water through pipe has been studied by many researchers, and various theoretical and empirical solutions have been suggested from time to time. Darcy and Weisbach equation is widely used in the industry for finding the losses related to the flow of fluid through pipes. Attempt is made here through analytical analysis to find out the friction factor and wall shear stress related to the pipe flow because influence of these two parameters are really need to be understand for estimating the frictional losses. Friction factor greatly depends upon the types flow and the roughness of the pipe material. In laminar flow, the value of friction factor depends only upon the Reynolds number while in turbulent flow, Reynolds number and roughness of the pipe material both affects the values of the friction factor. Wall shear stress depends upon the roughness of the pipe
material. For smooth pipe, values of wall shear stress will be less while for rough pipes, values of wall shear stress will be higher. Increasing the Reynolds number increases the values of wall shear stress.

Values of friction factor and wall shear stress are calculated in the analytical work with the help of various equations suggested by the scientists for friction factor value of cast iron pipe using \( \text{D}= 0.0127 \text{ m} \) diameters for Reynolds number up to 50000. Water has been considered as working fluid for the analysis purpose.

Fluid flowing in pipes has two primary flow patterns. It can be either laminar when all of the fluid particles flow in parallel lines at even velocities and it can be turbulent when the fluid particles have a random motion interposed on an average flow in the general direction of flow. There is also a critical zone when the flow can be either laminar or turbulent or a mixture. It has been proved experimentally by Osborne Reynolds that the nature of flow depends on the mean flow velocity (v), the pipe diameter (D), the density (ρ) and the fluid viscosity Fluid Viscosity (μ). A dimensionless variable called the Reynolds number which is simply a ratio of the fluid dynamic forces and the fluid viscous forces, is used to determine what flow pattern will occur. The equation for the Reynolds Number is

\[
\text{Re} = \frac{\rho v D}{\mu}
\]  

(1.1)

Flow in pipes is considered laminar if the relevant Reynolds number is less than 2000, and it is turbulent if the Reynolds number is greater than 4000. Between these two values there is the critical zone in which the flow can be either laminar or turbulent or the flow can change between the patterns. It is important to know the type of flow in the pipe when assessing friction losses when determining the relevant friction factors.

Absolute Pipe Roughness is a measure of pipe wall irregularities of commercial pipes; the absolute roughness has dimensions of length and is usually expressed in millimeter or feet. Relative Roughness of a pipe wall can be defined as the ratio of absolute roughness to the pipe normal diameter. Relative roughness is often used for pressure drop calculations for pipes and is an important parameter for determining friction factor based on Reynolds number for flow in a pipe. (7) Absolute roughness is usually defined for a material and can be measured experimentally. Typical roughness values are mentioned in below table for different pipe materials.

<table>
<thead>
<tr>
<th>Type of pipe</th>
<th>Roughness (ε), mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVC, Plastic tubing</td>
<td>0.0015</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>0.045</td>
</tr>
<tr>
<td>Rusted steel</td>
<td>0.1 to 1.0</td>
</tr>
<tr>
<td>Galvanised steel</td>
<td>0.15</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.26</td>
</tr>
</tbody>
</table>

1.2 Closure

For the purpose of this analytical analysis, following assumptions are made for calculations,

1. The flow is steady and uniform throughout the whole length of the pipe.
2. The velocity gradient is exists only in the direction perpendicular to the flow direction, and there is no change of velocity along the length of the pipe.
3. The pressure gradient is exists only in the flow direction i.e. the direction parallel to the flow direction, and there is no change of pressure across the pipe.
4. This particular study is performed considering only the horizontal cases, Vertical & inclined cases are not incorporated in the work.

1.3 Darcy-Weisbach equation

Pressure loss in steady pipe flow is calculated using the Darcy-Weisbach equation. This equation includes the Darcy friction factor. The exact solution of the Darcy friction factor in turbulent got by looking at the Moody diagram or by solving it from the Colebrook equation or solving from some explicit approximations like chen, barr, swamee & jain etc. (8)

The head loss \( h_f \) due to friction undergone by a fluid motion in a pipe is usually calculated through the Darcy-Weisbach relation as;

\[
h_f = f \frac{L V^2}{D 2g}
\]  

(1.2)

\( h_f \) = Head loss
\( L \) = Characteristics length of the pipe
\( V \) = Velocity of the flow of liquid
\( f \) = Darcy friction factor
\( D \) = Diameter of the pipe.
\( g \) = Acceleration due to the gravity.
1.3.1 Calculation of Friction factor
Before the pipe losses can be estimated, the friction factor must be calculated. The friction factor will be dependent on the pipe size, inner roughness of the pipe, flow velocity and fluid viscosity. The flow condition, whether ‘turbulent’ or not, will be the method used to calculate the friction factor. There are many equations used for calculation of friction factor. Some of them namely, Chen approximation, Barr approximation, Swamee and Jain approximation, Zigrang and Sylvester approximation and Romeo, Royo and Monzon approximation are used in this particular study.

1.3.2 Hagen-Poiseuille equation
Hagen-Poiseuille proposed an equation which is widely used for calculation of friction factor.

\[
h_f = \frac{32 \mu L V}{\rho g D^2}
\]  

(1.3)

We can just equate the Hagen-Poiseuille and the Darcy-Weisbach Equations:

\[
\frac{32 \mu L V}{\rho g D^2} = f \frac{L V^2}{D} \frac{2g}{2g}
\]

Hence, for laminar flow we have:

\[
f = \frac{64}{R_e} \quad (1.4)
\]

The friction factor is linearly dependent on \( R \),

Where, \( R \), the Reynolds number. Whereas, in turbulent flow (\( R \geq 2000 \)), the friction factor, \( f \) depends upon the Reynolds number (\( R \)) and on the relative roughness of the pipe, \( \varepsilon/D \). The general behavior of turbulent pipe flow in the presence of surface roughness is well established. When \( \varepsilon \) is very small compared to the pipe diameter \( D \) i.e. \( \varepsilon/D \rightarrow 0 \) When \( \varepsilon \) is very small compared to the pipe diameter \( D \) i.e. \( \varepsilon/D \rightarrow 0 \), \( f \) depends only on \( R \). When \( \varepsilon/D \) is of a significant value, at low \( R \), the flow can be considered as in smooth regime (there is no effect of roughness). As \( R \) increases, the flow becomes transitionally rough, called as transition regime in which the friction factor rises above the smooth value and is a function of both \( \varepsilon \) and \( R \) and as \( R \) increases more and more, the flow eventually reaches a fully rough regime in which \( f \) is independent of \( R \).

In a smooth pipe flow, the viscous sub layer completely submerges the effect of roughness on the flow. In this case, the friction factor \( f \) is a function of \( R \) and is independent of the effect of roughness on the flow. \( (8) \)

1.3.3 Blasius Equation
Blasius determined an equation from experiments on ‘smooth’ pipes. The Blasius equation is the most simple equation for solving the Darcy friction factor because the Blasius equation has no term for pipe roughness, is valid only to smooth pipes. It is widely used for calculation of friction factor of turbulent flow. The Blasius equation is valid only for smooth pipe \( (6) \).

\[
f = \frac{0.316}{R_e^{0.25}} \quad (1.5)
\]

1.3.4 Prandtl equation
Prandtl equation was widely used for turbulent regime in smooth pipes which is implicit in friction factor. Prandtl derived a formula from the logarithmic velocity profiles and experimental data on smooth pipes. \( (6) \)

\[
\frac{1}{\sqrt{f}} = 2.0 \log_{10} \left( \frac{R \cdot \sqrt{f}}{2.51} \right) = 2.1 \log_{10} \left( R \cdot \sqrt{f} \right)
\]

(1.6)

1.3.5 Nikuradse equation
The development of approximate equations for the calculation of friction factor began with Nikuradse’s turbulent pipe flow investigations in 1932 and 1933. Nikuradse had verified the Prandtl’s mixing length theory and proposed the following universal resistance equation for fully developed turbulent flow in smooth pipe; \( (6) \)

\[
\frac{1}{\sqrt{f}} = 2.0 \log(R \sqrt{f}) - 0.8
\]

(1.7)
1.3.6 Von Karman equation

The following form of the equation is first derived by Von Karman (Schlichting, 1979) and later supported by Nikuradse’s experiments. Von Karman’s relation was widely used for turbulent regime in rough pipes. (6)

\[ \frac{1}{\sqrt{f}} = 2 \log \left( \frac{D}{\varepsilon} \right) + 1.74 \]  

(1.8)

1.3.7 Analytical solution with Colebrook and White equation

Colebrook and White (1937) proposed the equation for transition regime in which the friction factor varies with both R and ε/D. The equation universally adopted for calculating the friction factor.

\[ \frac{1}{\sqrt{f}} = -2 \log \left( \frac{\varepsilon}{3.7} + \frac{2.51}{R\sqrt{f}} \right) \]  

(1.9)

Equation (1.9) covers not only the transition region but also the fully developed smooth and rough pipes. By putting ε→ 0, Eq. (1.9) reduces to Eq. (1.7) for smooth pipes and as R→∞; Eq. (1.9) becomes Eq. (1.8) for rough pipes. Moody (1944) presented a friction diagram for commercial pipe friction factors based on the Colebrook–White equation, which has been extensively used for practical applications. Because of Moody’s work and the demonstrated applicability of Colebrook-White equation over a wide range of Reynolds numbers and relative roughness value ε/D, Eq. (1.9) has become the accepted standard for calculating the friction factors. It suffers; however, from being an implicit equation in f and thus requires an iterative solution. (6)

Newton Raphson is one of the iterative calculation method presented here for solving the Colebrook equation (1.9)

Given a function f defined over the real x, and its derivative f’, begin with a first guess x₀ for a root of the function f. provided the function is reasonably well-behaved a better approximation x₁ is

\[ x_1 = x_0 - \frac{f(x)}{f'(x)} \]  

(1.10)

The process is repeated as,

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]  

(1.11)

Here, Colebrook equation is,

\[ \frac{1}{\sqrt{f}} = -2 \log \left( \frac{\varepsilon}{3.7} + \frac{2.51}{R\sqrt{f}} \right) \]  

Consider,

\[ F = \frac{1}{\sqrt{f}} + 2 \log \left( \frac{\varepsilon}{3.7} + \frac{2.51}{R\sqrt{f}} \right) \]  

(1.12)

Taking first order derivation,

\[ \frac{dF}{df} = -\frac{1}{2f} \cdot f^{-1.5} \left\{ 1 + \frac{2 \times 2.51}{\ln \left( \frac{\varepsilon}{3.7 \times D} + \frac{2.51}{R\sqrt{f}} \right)} \right\} \]  

(1.13)

As per Newton-Raphson method,

\[ f_1 = f_0 - \frac{F}{\left( \frac{dF}{df} \right)} \]  

The process is repeated as,

\[ f_{n+1} = f_n - \frac{F}{\left( \frac{dF}{df} \right)} \]  

(1.14)

For find the friction factor value of cast iron pipe in which water is flowing.

Surface roughness, ε = 0.00026 m           Inside diameter, D= 0.0127 m           Density of water, ρ = 1000 Kg/m³
Velocity, V= 0.3159242 m/s           Viscosity of water, μ = 0.001003 Ns/m³

Reynolds number \((Re) = \frac{\rho \cdot V \cdot D}{\mu} = 4000.23661\)
Putting all the parameter in equation 1.12 and 1.13 we get,

\[ F = \frac{1}{\sqrt{f}} + 2 \cdot \log \left( \frac{0.00553 + \frac{2.51}{4000.23661 \sqrt{f}}}{0.00553 + \frac{2.51}{4000.23661 \sqrt{f_0}}} \right) \]  

(1.15)

And,

\[ \frac{dF}{df} = -\frac{1}{2} \cdot f^{-1.5} \left[ 1 + \frac{5.02}{\ln \left( \frac{0.00553 + \frac{2.51}{4000.23661 \sqrt{f_0}}}{0.00553 + \frac{2.51}{4000.23661 \sqrt{f_0}}} \right)} \right] \]  

(1.16)

Taking \( n=0 \) and putting 1.15 and 1.16 equation in 1.14,

\[ f_1 = f_0 - \frac{1}{\sqrt{f_0} + 2 \cdot \log \left( \frac{0.00553 + \frac{2.51}{4000.23661 \sqrt{f_0}}}{0.00553 + \frac{2.51}{4000.23661 \sqrt{f_0}}} \right)} \]  

(1.17)

Taking first initial value of friction factor \( f_0 = 0.050 \) from Moody chart

\[ f_1 = 0.050 - \frac{1}{\sqrt{0.050} + 2 \cdot \log \left( \frac{0.00553 + \frac{2.51}{4000.23661 \sqrt{0.050}}}{0.00553 + \frac{2.51}{4000.23661 \sqrt{0.050}}} \right)} \]  

\[ \frac{dF}{df} = -\frac{1}{2} \cdot 0.050^{-1.5} \left[ 1 + \frac{5.02}{\ln \left( \frac{0.00553 + \frac{2.51}{4000.23661 \sqrt{0.050}}}{0.00553 + \frac{2.51}{4000.23661 \sqrt{0.050}}} \right)} \right] \]

So, \( f_1 = 0.057021 \)

Now, by putting again the \( f_1 = 0.057021 \) in equation in equation 1.14 for \( n=1 \)

We get, \( f_2 = 0.057327 \)

Now, by putting again the \( f_2 = 0.057327 \) in equation in equation 1.14 for \( n=2 \)

We get, \( f_3 = 0.057307 \)

Now, by putting again the \( f_3 = 0.057307 \) in equation in equation 1.14 for \( n=3 \)

We get, \( f_4 = 0.057309 \)

Now, by putting again the \( f_4 = 0.057309 \) in equation in equation 1.14 for \( n=4 \)

We get, \( f_5 = 0.057309 \)

Now, by putting again the \( f_5 = 0.057309 \) in equation in equation 1.14 for \( n=5 \)

We get, \( f_6 = 0.057309 \)

After evaluating the friction factor value for 6 times we get accurate value which is 0.057309. This iteration process requires more computational time to calculate the accurate values of friction factor.

II. Conclusion

Colebrook equation is applicable over a very wide range of Reynolds number and relative roughness values; this equation becomes the accepted standard of accuracy for calculated hydraulic friction factor. Colebrook equation suffers from being implicit in nature because friction factor \( (f) \) is present in both side of the equation and thus requires an iterative solution where convergence to 0.01% typically requires less than 7
iterations. Implicit Colebrook equation cannot be rearranged to derive friction factor directly in one step. Iterative calculus can cause a problem in simulation of flow in a pipe system in which it may be necessary to evaluate friction factor hundreds or thousands of times. This is the main reason for attempting to develop a relationship that is a reasonable approximation for the Colebrook equation but which is explicit in friction factor. Following are some existing explicit approximation of the implicit Colebrook equation; Most of the available approximations of the Colebrook equation are very accurate, except Moody’s approximation (6).

References

[8]. http://www.pipeflow.co.uk