On the Applicability of Multiseasonal Streamflow Generation Models for Intermittent Rivers

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Abstract: Monthly and annually streamflow generation models have been used for Monte Carlo simulations, in order to design, optimization and risk analysis of multipurpose reservoirs systems. Auto-regressive models, in general, preserve the statistical parameters of the historical time series when they are applied in humid watersheds. However, they are not able to reproduce the persistence encountered in the historical series of intermittent rivers from semi-arid areas. In this study is presented one application of several streamflow generation models for semi-arid area of the Northeast Brazil. Frag1 model and Frag2 model showed the best results. The Matalas model also proved to be quite efficient in reproducing streamflow. For water resources management in semi-arid regions, hydrologic droughts risk assessment is of huge importance for water resources policy makers.

Keywords: water resources, optimization, semi-arid regions, streamflow generation

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I. Introduction

The brazilian semi-arid region has an area of about 1 million km². It is characterized by strong temporal and spatial rainfall variability (400 to 1800 mm/year) and a high evaporation rate (above 2000 mm/year), associated with geological restrictive conditions (crystalline basement of reduced hydrological potential), that causes river intermittency. The construction of artificial dams along the major rivers of the region throughout the last century, was indispensable for water supply, especially during drought periods.

Droughts in Northeast Brazil, which tend to intensify due to climate change (Martins et al., 2013), have repeatedly brought famine, mass migration and social conflicts in this region. Its prediction, monitoring and management, however, remain a central research theme. In water resources management in semiarid regions such as the Northeast of Brazil, it is fundamental to have tools to aid decision making (Freitas, 2010).

Several studies have indicated the influence of numerous atmospheric phenomena on rainfall in Northeast Brazil (Hastenrath, 1984; Freitas and Billib, 1997; Andreoli and Kayano, 2007; Moscati and Gan, 2007; Rusteberg and Freitas, 2018). Climatological studies have indicated the existence of a strong relationship between sea surface temperature distribution (SST - sea surface temperature) along the tropical Atlantic basin temperature and the semiarid northeastern Brazil precipitation, as well as a decadal trend associated with changes in the meridional position of the ITCZ - Intertropical Convergence Zone (Moura and Schukla, 1981; Rao and Hada, 1990; Billib and Freitas, 1996). These phenomena are indicative to be related to climate variability and extreme droughts and floods in the region.

For the design and operation of multi-purpose and multi-use surface reservoirs systems (water supply, irrigation, energy production, etc.) deterministic rainfall-runoff models and stochastic streamflow generation models are usually employed, depending on data availability (Freitas, 1996).

For generating synthetic streamflow several models with different time intervals have been reported in the literature. In general, these models can be grouped into two categories: direct generation models and disaggregation's models. In the first class belong models, which generate flow simultaneously for different time intervals (Fernandez and Salas, 1986; Sum, 1987; Bartolini et al., 1988; Claps et al., 1993). By disaggregation's models discharges are generated initially for a longer period of time, e.g. a year, and then broken into smaller time intervals, such as monthly, weekly, daily, etc.

According to Dracup et al. (1980) four basic considerations must be evaluated for the definition of droughts, that are: 1) what is the greatest interest in the analysis, i.e. what is the nature of the deficit of water to be investigated (meteorological, hydrological or agricultural); 2) what is the time series discretization used in the analysis (annual, semiannual, monthly, etc.); 3) what is the threshold level of separation between flood and drought events; and 4) the choice of the regionalization and standardization methods to be adopted.

A hydrological drought can be defined as a one or more sequenced years, when the average annual flow remains below the long term mean annual flow, considering all the existing series (Dracup et al., 1980). A

drought event may thus be characterized by three parameters, namely the duration D in years; the accumulated deficit or severity S and the magnitude M, which represents the average cumulative deficit below the mean annual flow.

When applying stochastic models for generating streamflow time series it is necessary to observe not only the characteristics of the streamflow time series, but also the use for which they are intended. One of the most important aspects in the analysis of water resources in semiarid regions are the impacts of extreme events, in particular prolonged droughts, on water resources systems. For this it is essential to generate long time series of synthetic streamflows. Askew et al. (1971), Stedinger and Taylor (1982), as well as Dracup and Kendall (1992) discussed the inability of traditional models, based on Markov chain, to reproduce the frequency distribution of extreme drought events that occurred in the historical series.

The SAGE software consists of models (Figure 1), adapted from models cited in the literature in order to reproduce the typical characteristics of intermittent rivers of the semiarid regions. The software, in its current version, was written in Visual Basic language, in such a way that it, through a user-friendly interface, could be used as a support tool in watershed committees in the generation of synthetic flow series necessary for the simulation of reservoir operation on grant (authorization) studies, among others.

II. **Streamflow Generation Models**

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Figure 1: The SAGE Software for Streamflow Generation

A first annual model to be described is called Thomas-Fiering model or AR(1) model, i.e. Auto-Regressive of order 1, which is based on a stochastic process (Mass et al., 1962). The second model is the Gamma Autoregressive or GAR(1) model, proposed by Fernandez and Salas (1990). Both models are particular cases of the ARMA model (Box and Jenkins, 1976).

Annual Model 2.1.

2.1.1. AR(1) Model The original AR(1) model, also known as Thomas-Fiering Model, can be described by the following equation: $Q_i = \mu + \rho (Q_{i-1} - \mu) + t_i \sigma (1 - \rho^2)^{1/2}$

(1)

with

 $Q_{i = \text{streamflow in year i;}}$

 $\mu_{=\text{population mean}}$;

 σ = standard deviation of the population;

 $\rho_{\rm = lag-one \ correlation \ coefficient \ of \ the \ population;}$

 t_i = random variable N(0,1).

2.1.2. GAR(1) Model

A 1st order Gamma Autoregressive Model or GAR(1) model can be described by an additive process as follows: $Q_i = \phi Q_{i-1} + \varepsilon_i$ (2)

with

 Q_{i} = streamflow in the time interval i;

 ϕ = autocorrelation coefficient;

 \mathcal{E}_i = independent random variable.

For a random number generation with gamma distribution the following scheme has been used:

$$\varepsilon = \lambda (1 - \phi) + \eta \tag{3}$$

01 (1)

where

$$\begin{cases} \eta = 0 \to M = 0\\ \eta = \sum_{j=1}^{M} Y_{j}(\phi)^{U_{j}} \to M > 0 \end{cases}$$

with

M = Poisson random variable with mean equal to
$$-\beta \ln(\phi)$$

 U_{j}^{j} = uniform random variable (0,1)
 Y_{j}^{j} = random variable with exponential distribution with mean equal to $1/\alpha$

2.1.3 ARR (Alternating Renewal Reward) Model

The annual model Alternating Renewal Reward by Dracup and Kendall (1992) is based on the characteristics (duration, severity and magnitude) of droughts and floods periods found in the historical series. It makes use of the geometric distribution to simulate the droughts and floods duration and the gamma distribution with two parameters for reproduction of the severity.

A basic assumption on modeling process of annual flows through the ARR model is that drought events come from different populations, ie, the deficit Yi (deficit in year i) is uniformly distributed and independent, dependent, however, on duration. For the annual flow generation two stages are performed: 1) drought flood modeling process, 2) flow modeling within drought or flood periods. The model can therefore be found in one of two possible stages. If, for example, the system is assumed a priori to be flood, then DH_1 years of flood are generated. The following step adopted is to assume the system is to be in drought condition, and

 DL_1 years of drought are generated, and so forth. DH_n and DL_n are intended to have independent and uniformly distributed.

Thus, the problem consists only in identifying the probability distribution functions for the two stage model. For the flood/drought modeling process, geometric distributions have been used and for the modeling of flood and drought severity, two-parameter gamma distribution have been applied. A limiting factor for the adjusting of duration (length) and severity distributions is the small sample. During a about 80 years data period, for example, there are approximately only 15 drought periods. To overcome this deficiency two procedures have been employed: decimation and standardization. In the analysis of time series with time interval less than one year, that is, if there is periodicity, the original variable can be replaced by a standardization procedure, in order to remove this periodicity.

The decimation procedure (Bloomfield, 1976) was used to obtain n (number of months) streamflow series, through the use of standardized flow values of the straight years, for each drought (flood), to each of the

n series. The drought and flood periods severities are thus evaluated. This procedure simulates a regionalization through n flows sets of n different positions of a homogeneous region, subject to the same climatic conditions. To simulate flood and drought duration geometric distributions was used, as follows:

$$f_x(x) = pq^{x-1} \tag{4}$$

For the severity two-parameter gamma distributions for each duration (time length), was employed, as Kendall and Dracup (1992):

$$f(Y_D) = \frac{\lambda e^{-\lambda Y_D} (\lambda Y_D)^{r-1}}{\Gamma(r)}$$
(5)

with

 Γ = gamma function; r = shape parameter; λ = scale parameter.

2.2. Monthly Model

2.2.1. PAR(1) Model

The monthly Thomas-Fiering model or PAR (1) can be represented by the following equation:

$$Q_{i,j} = \overline{Q} + \frac{s_j}{s_{j-1}} r_j (Q_{i,j-1} - \overline{Q}_{j-1}) + t'_j s_j (1 - r_j^2)^{1/2} , \qquad (6)$$

with

 $Q_{i,j-1} e^{i,j} Q_{i,j} = \text{flow in month j-1 and j of year i}$ $\overline{Q}_{j-1} e^{i,j} \overline{Q}_{j} = \text{average flow of months j-1 and j}$ $s_{j-1} e^{i,j} = \text{standard deviation of months j-1 e j}$ $r_{j} = \text{correlation coefficient of the month j}$ $\hat{r}_{j} = \text{random variable of an asymmetric distribution}$

Unsatisfactory results arising from applications of the conventional Thomas-Fiering model in semi-arid regions, with many zero flows, brought proposals for modifications of this model presented by Clarke (1973) and Filho (1978). These procedures take into account the independence between the occurrence or non-occurrence of flow in each month of the year.

Initially probabilities of occurrences of flow for each month have been determined, given by:

$$P_j = \frac{n'}{n}$$

with

n' = number of non-zero values of flow in a given month j with n observations; n = number of years in the historical time series

For each month a random number with uniform distribution Vj (0,1) is then generated and compared to a value of probability of flow.

$$V_{j} > P_{j} \Longrightarrow Q_{i,j} = 0$$
$$V_{j} \le P_{j} \Longrightarrow Q_{i,j} > 0$$

when $Q_{i,j} > 0 e^{Q_{i,j-1}} = 0$, then $Q_{i,j}$ is determined as follows: $Q_{i,j} = \overline{Q_j} + s_j t_j$

(8)

(7)

where

 t_j = random number with a normal distribution N (0,1).

Another modification of this model was proposed by Matalas (1967). A lognormal probability distribution is often used. Not only due to characteristics of asymmetric flows monthly, but also due to difficulty of obtaining a representative value estimator population skewness, since the flow time series are usually short. As the logarithm of zero is not defined, which would make it impossible to apply directly the log transformation to historical series of intermittent rivers, Matalas (1967) presented a way to estimate the parameters of the series in the logarithmic domain without changing the series itself.

2.2.2. Two-tier Model

However, most models do not capture the distribution and persistence of the annual flow. The monthly generated flows (by monthly models) when they are combined, normally differ from synthetic annual flows (generated by annually models), particularly when the model is specified in terms of logarithms of streamflow, or some other transformation effected. In such cases, either the monthly or annual flows need to be adjusted to maintain such consistency. A relevant question is how to adjust the seasonal flows generated without substantially distorting their marginal distributions. In this study various adjustment methods have been tested and analyzed.

By the application of stochastic flow models in water resources systems is important, therefore, that not only the statistical parameters of monthly flows, but also the annual streamflows, are reproduced. In general, negative flows have been generated and the distortions in the marginal distribution have been also observed. The use of an adjustment procedure is of paramount importance, especially in evaluating the ability of the generation model to reproduce the extreme drought periods and estimate with reasonable accuracy the vicinity of historical annual stream flows.

2.2.3. Method of Fragments

The Method of Fragments from Svanidze (1980) is based on the disaggregation of annual flows generated by some annually model (in this case, the ARR model) into monthly flows (or shorter time interval). The model is characterized by estimating, for each month j and each year i of the historical flow series, the so-called fragments, given by:

$$f_{i,j} = \frac{Q_{i,j}}{\sum_{j=1}^{n} Q_{i,j}}$$
(9)

where:

n = number of months (n = 12); $Q_{i,j}$ = streamflow in month j of year i.

The fragments $f_{i,j}$ correspond to the percentage of annual streamflow (the denominator of the equation above) in year i. Following the historic annual flow values are placed in ascending order and separated into classes. The limits of the class intervals are formed by the mean values of successive flows. The total number of classes is equal to the number of years within the flow series measurement. The first class has zero as the lower limit and the last class upper limit of the last class is infinite. The annual streamflows generated are then distributed according to the class intervals and fragmented into monthly values.

2.2.4. The Disaggregation Model

The disaggregation model proposed by Valencia and Schaake (1973) uses an annual flow generation model, and then disaggregate these annual streamflow values into monthly, weekly or daily streamflow values. This model is based on the following equation:

$$Q_i = AM_i + BV_i \tag{10}$$

where

 Q_i = matrix flow A = parameter vector [12x1]

$$\overline{M} = \frac{1}{12} \sum_{j=1}^{12} \overline{Q}_j$$

 M_i = column matrix flow in year i subtracted from μ estimated by B = parameter matriz [12x12]

 $V_{j} =$ random component vector

The parameter vector A is estimated by $A = E(Q_iM_i)E^{-1}(M_iM_i)$, where E() is the expected value and E-1() is the inverse matrix of the expected value. The parameter array B is in turn determined from the following expression, which can be obtained by spectral decomposition or principal component analysis:

$$BB^{T} = E(Q_{i}Q^{T}_{i}) - E^{-1}(M_{i}M_{i})E(Q_{i}M_{i})E^{T}(Q_{i}M_{i})$$
(11)

III. Streamflow Generation Models

The various models have been consolidated into a computer package called SAGE (<u>S</u>tochastische <u>AbflussGE</u>nerierungsmodell), composed of the following synthetic flow generation models (Freitas, 1995; Freitas, 2010):

- PAR-model (Thomas/Fiering) with modification by Clarke (1973)
- PAR-model (Thomas/Fiering) with modification by Matalas (1967)
- Two-tier model (PAR(1)/AR(1)) with log-gama distribution)
- Two-tier model (PAR(1)/AR(1) with log-normal distribution)
- Two-tier model PAR(1)/GAR(1) by Fernandez and Salas
- Fragment method for AR(1) with log-gama distribution
- Fragment method for AR(1) with log-normal distribution
- Fragment method for GAR(1)
- Disaggregation model / AR(1) by Valencia and Schaake (1973)
- ARRF model (Alternating Reward Renewal Model / Fragment method)

In order to verify the applicability of these models to the Brazilian semiarid intermittent rivers, they have been applied to four representative northeastern Brazil rivers (Figure 2), these basins with areas ranging from 410 to 5695 km² (Table 1).



Figure 2: Location of analyzed basins.

To analyze the performance of flow generation models in semiarid regions three basic criteria are needed, namely: (1) analysis of the statistical parameters of the generated series, (2) analysis of the result of the reservoir operation simulation and (3) analysis of the characteristics (duration, severity and magnitude) of drought and flood generated periods (Freitas, 1995).

Nr.	Station	River	Basin area (km2)	Annual average flow (m3/s)	Period				
1	Faz. Cajazeiras	Acaraú	1550	7.45	1963-82				
2	Sitio Poço Dantas	Bastiões	3700	3.88	1968-81				
3	Sitio Novos	São Gonçalo	410	3.05	1963-75				
4	Limeira	Capibaribe	5695	6.55	1957-75				

Table 1: Characteristics of analyzed basins

For each basin were generated 100 series with 50-years of extension, by each model. In Figure 3 are shown the historical value, the median value (100 series generated), mean and standard deviation, respectively. The Frag1 and Frag2 models were that best reproduced the analyzed statistical parameters.



Figure 3: Statistical parameters of the generated and historical series (mean value and standard deviation)

Table 2 shows the characteristics of historic and generated series for the Faz. Cajazeiras station, on Acaraú river. In Tables 3-5 are the statistics of the characteristic parameters of hydrological droughts and floods periods (duration, severity and magnitude) for annually models.

	S	station.		
	historic	generated serie		
Statistical paremeters	serie	PAR(1) GA	R(1)	
mean (m ³ /s)	91.384	4 98.882	115.720	
standard deviation (m3/s)	104.223	3 96.545	122.283	
variation coefficient	1.14	0.976	1.057	
asymmetry coefficient	1.57	9 1.923	1.607	
lag-1 correlation coefficient	0.26	5 0.244	0.270	

Table 2: Comparisons of the statistical characteristics of the historical and generated series for Faz. Cajazeiras

Table 3:	Comparison	of the	statistics	of the	hydro	logical	drought	duration	of the	historical	and	generated	series
				fo	r Faz.	Cajaze	eiras stati	ion.					

his	storic	generat	ted serie	
Statistical paremeters se	erie	PAR(1)	GAR(1)	
mean (year)	3.133		3.053	3.141
standard deviation (year)	2.232		2.445	2.603
variation coefficient	0.712		0.801	0.829
asymmetry coefficient	1.184		1.692	1.909
lag-1 correlation coefficient	0.451		-0.004	0.018

Table 4:	Comparison of the statistics of the hydrological drought severity of the historical and generated series
	for Faz. Cajazeiras station.

for f uz. Cujuzenus station.								
	historic generated serie							
Statistical paremeters	serie	PAR(1)	GAR(1)					
mean (m ³ /s*year)	204.761		181.444	244.755				
stand. deviation (m3/s*year)	172.733		171.353	239.075				
variation coefficient	0.844		0.944	0.977				
asymmetry coefficient	1.231		1.714	1.935				
lag-1 correlation coefficient	0.393		-0.025	0.011				

Table 5: Comparison of the statistics of the hydrological drought magnitude of the historical and generated
series for Faz. Cajazeiras station.

~								
	historic generated serie							
Statistical paremeters	serie	PAR(1)	GAR(1)					
mean (m ³ /s)	59.581		55.336	71.772				
standard deviation (m ³ /s)	22.164		24.068	29.513				
variation coefficient	0.372		0.435	0.411				
asymmetry coefficient	-0.839		-0.252	-0.494				
lag-1 correlation coefficient	0.183		-0.033	-0.058				

Table 6 shows the average adjustment errors (bias and rmse) for Faz. Cajazeiras station.

	<u> </u>			/	2	
model	mean		standard dev	iation	lag-1 correlation coef.	
	bias	rmse	bias	bias rmse		rmse
TF/Clarke	0.2876	0.2321	0.2060	0.1674	6.6039	4.1213
TF/Matalas	0.1965	0.1953	0.1639	0.1591	6.4141	1.7117
Two-tier1	0.3237	0.2652	0.2810	0.2007	7.1211	3.9973
Two-tier2	0.3031	0.2808	0.2369	0.2922	8.7538	2.2963
Two-tier3	0.3325	0.2704	0.2691	0.3102	8.5416	5.7129
Frag1	0.0966	0.2136	0.0838	0.1673	0.1276	1.9774
Frag2	0.1381	0.1825	0.1387	0.1028	0.8318	2.6558
Frag3	0.2076	0.1801	0.1111	0.1304	1.1984	1.9313
Disag.	0.4867	0.4691	0.3299	0.3379	7.3794	7.1152

IV. Conclusion

In this work one application of the SAGE software was presented, as well as the description of the models used to flow generate in semiarid region. When applied to the various basins of the semi-arid the Frag1 and Frag2 models showed better results, thereby proving to be a useful tool in the design and optimization of reservoir systems in semi-arid regions. The Matalas model also proved to be quite efficient in reproducing flow in semi-arid northeastern Brazil. Freitas (2016) and Rusteberg and Freitas (2018) applied the SAGE system successfully.

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