

Flow Characteristics in Convective Lid-Driven Trapezoidal Cavity with Wavy Bottom Surface

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Abstract: Hybrid nanofluids, two or more nanoparticles in base fluid, has been an increasing interest in modern years due to improvement in heat transfer performance. In the present work, an analysis is carried out to study the flow characteristics and heat transfer performance on convection in a lid-driven trapezoidal cavity with sinusoidal wavy bottom surface filled with hybrid nanofluid composed of equal quantities of Cu and Al₂O₃ nanoparticles dispersed in water base fluid. The wavy bottom wall is heated while the top side of the cavity is cooled isothermally and the left and right walls are insulated. All the walls of the cavity are kept as no-slip walls except the upper one which is a moving wall driven by a uniform velocity U_0 along the x-axis. The relevant governing equations have been solved using finite element method of Galerkin weighted residual approach. The parametric study on the implication of Richardson number Ri and solid volume fraction of nanoparticles ϕ on the flow structure and heat transfer characteristics are performed in details while the Reynolds number and Prandtl number considered fixed. The numerical results indicated that the Richardson number have significant effects on the flow and heat transfer performance. On the other hand, the results also show significant effects of increasing the volume fraction of hybrid nanofluid. Moreover, it is also noticed that combination of two different nanoparticles suspension have a better performance of heat transfer. Results are presented in terms of streamlines and isotherms of the hybrid nanofluid for different values of governing parameters.

Keywords: Hybrid Nanofluid, Finite Element Method, CFD, Cavity, Heat Transfer, Convection.

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I. Introduction

Now-a-days many potential approaches are using nanofluid in heat transfer improvement for enhancing the efficiency of thermal systems such as; heat exchangers, thermal storage, solar collectors, photovoltaic/thermal system, biomedical devices, nuclear reactors, engine/vehicle, cooling of electronic components, transformers etc. The basic study of this field was done by Sarkar et al [1] previously. Convection in fluid flow in lid-driven cavities or enclosures occurs because of two competing mechanisms. Researchers [2-4] are interested in conjugate heat transfer by free convection inside cavities since it has large applications in environment and industry purpose. In order to compare side by side developments in the extent applications and increasingly developing studies about nanofluids, review papers have been conducted. Many researchers conducted a numerical investigation of natural convection in an enclosure consisting of two isothermal horizontal wavy walls and two adiabatic vertical straight walls.

Generally, "Hybrid" nanofluid can be obtained by suspending more than one type of nanoparticles in base fluid. Hybrid nanofluid, is a gradually mounting study field parallel to the incessant developments of common nanofluids. Compromised properties between the advantages and disadvantages of the properties of individual nanoparticles are a declared task of hybridization. Besides, nanoparticles suppliers exhibit noticeable differences in prices of different nanoparticles types. For example, the price of copper nanoparticles is about 12 times greater than that of alumina nanoparticles. The "hybrid" nanoparticles should be limited to those prepared as a single composite component in a base fluid for which their synthesis requires extra attention [5-6]. As well as many experiments shows that laminar natural convection in an inclined cavity with a heated undulated wall, i.e., smoothwave-like pattern. Their results concluded that the hot wall undulation affects the flow and heat transfer rate in the enclosure in which the local Nusselt number distribution results in a decrease of heat transfer rate as compared with the square enclosure.

The effect of missing the base fluid in the immediate vicinity of the nanoparticles caused by the Brownian motion of the nanoparticles was analyzed. Anyone can mix nanoparticle with base fluid to prepare hybrid nanofluid. The experiments of Ho *et al.* [7,16] conducted an experiment about the mixture of particles of micro-encapsulated phase-change material and alumina nanoparticles in base fluid water. The experimental data of the density and mass fraction and those predicted from the mixture theory. An experiment of hybrid nanofluid

based on silver-silica-oil [8]. The authors found more deviated value of the thermal conductivity with the Maxwell relation [9] at higher solid volume fractions. The unsteady conjugate free convection in a semi-circular enclosure with a solid shell filled with water-based hybrid mixture of Al_2O_3 and Cu nanoparticles has been studied numerically [10]. It was found that increasing effects of hybrid nanofluid with rising thermal conductivity of wall and Rayleigh number.

Free mode of convection in a trapezoidal wavy enclosure filled with such hybrid nanofluid depending on the existing mathematical formulae needs additional attention to disclose its performance in this significant field. The objective of this paper is an effort to contribute in starting the groundwork of this research area.

II. Physical model

The diagram of the studied configuration enclosure consists of a two-dimensional lid-driven trapezoidal enclosure of side length H . Hybrid nanofluid Al_2O_3 -Cu/water is filled in the enclosure area. The bottom wavy wall is assigned to temperature T_h while the top wall of the enclosure is cooled at a constant temperature T_c . Here $T_h > T_c$ condition is maintained in all circumstances. The bottom wall is initially straight and later added a wavy wall creating an undulation parameter $n = 0, 1, 2, 3$. The left and the right walls are thermally insulated. Furthermore, the top wall is assumed to slide in its upper plane at a constant speed U_0 .

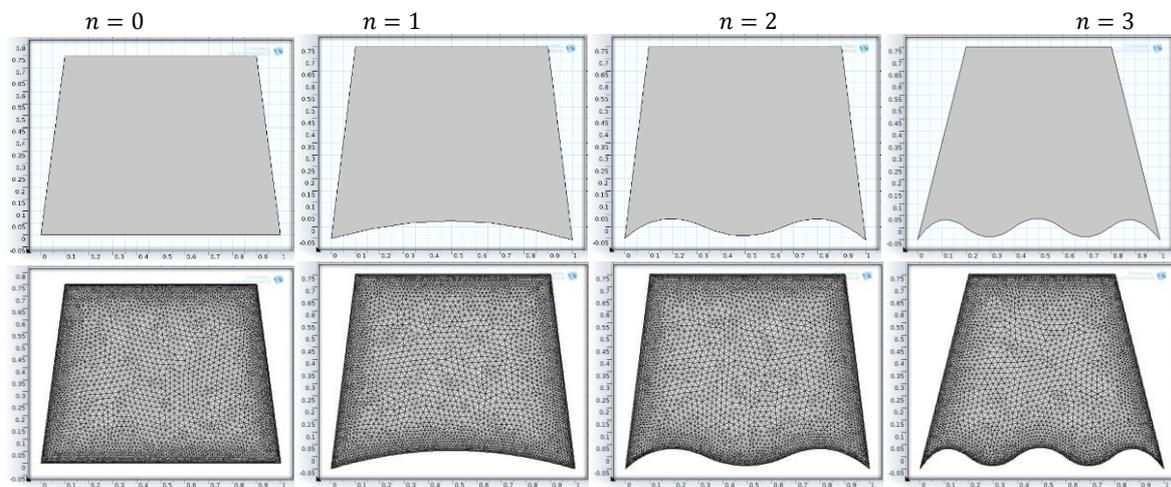


Figure. 1 Geometrical model of the interested domain

It is assumed that the fluid and nanoparticles are in thermal equilibrium and there is no slip between them. The hybrid nanofluid are considered as laminar and incompressible in the analysis. It is also assumed that the gravitational acceleration g acts to the vertical downward surface. Thermophysical properties of the used nano-particles and the base fluid are given in Table 1, assumed constant except for the density variation, which is maintained on Boussinesq approximation.

Table 1

Physical Properties	Fluid Phase (H ₂ O)	Nano-particle 1 (Cu)	Nano-particle 2 (Al ₂ O ₃)
C_p (J/kgK)	4179	385	765
ρ (kg/m ³)	997.1	8933	3970
k (W/mK)	0.6	401	40
$\beta \times 10^{-5}$ (1/K)	21	1.67×10^{-5}	0.85×10^{-5}
σ (μ S/cm)	0.05	5.96×10^{-7}	1×10^{-10}

III. Mathematical model

The two-dimensional steady state conditions have performed in numerical simulation. The non-dimensional governing partial differential equations for laminar flow and thermal energy equations using hybrid nanofluid are given below:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\rho_f}{\rho_{hnf}} \frac{\mu_{hnf}}{\mu_f} \frac{1}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\rho_f}{\rho_{hnf}} \frac{\mu_{hnf}}{\mu_f} \frac{1}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho\beta)_{hnf}}{(\rho\beta)_f} Ri\theta \quad (3)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{hnf}}{\alpha_f} \frac{1}{RePr} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)$$

where, $Pr = \frac{\mu_f C_p}{k_f}$, $Ri = \frac{g \beta_f \Delta T H}{\nu_0}$ and $Re = \frac{v_0 H \rho_f}{\mu_f}$ be the Prandtl number, Richardson number and Reynolds number respectively [12].

The above equations are non-dimensional by using the following dimensionless quantities

$$X = \frac{x}{H}, Y = \frac{y}{H}, U = \frac{uH}{\alpha_f}, V = \frac{vH}{\alpha_f}, \theta = \frac{T-T_c}{T_h-T_c}, P = \frac{pH^2}{\rho_f \alpha_f^2} \quad (5)$$

The boundary conditions imposed on the flow are taken as:

On the left and right wall: $U = V = 0, \frac{\partial \theta}{\partial Y} = 0$

On the top wall: $U = U_o, V = 0, \theta = 0$

On the wavy bottom wall: $U = V = 0, \theta = 1$

At the fluid solid wall interfaces: $\left(\frac{\partial \theta}{\partial X} \right)_{fluid} = K \left(\frac{\partial \theta}{\partial X} \right)_{solid}$, where $K = \frac{K_{hnf}}{K_f}$

The constructive equations of the effective properties of hybrid nanofluids have been chosen:

The thermal diffusivity $\alpha_{hnf} = k_{hnf} / (\rho C_p)_{hnf}$

The density, $\rho_{hnf} = (1-\phi)\rho_f + \phi_1\rho_1 + \phi_2\rho_2$

The heat capacitance,

$$(\rho C_p)_{hnf} = (1-\phi)(\rho C_p)_f + \phi_1(\rho C_p)_1 + \phi_2(\rho C_p)_2$$

The thermal expansion coefficient,

$$(\rho \beta)_{hnf} = (1-\phi)(\rho \beta)_f + \phi_1(\rho \beta)_1 + \phi_2(\rho \beta)_2$$

The specific heat,

$$C_{phnf} = \frac{(1-\phi)(\rho C_p)_f + \phi_1(\rho C_p)_1 + \phi_2(\rho C_p)_2}{(1-\phi)\rho_f + \phi_1\rho_1 + \phi_2\rho_2}$$

The viscosity of Brinkman [16] model $\mu_{hnf} = \frac{\mu_f}{(1-\phi_1-\phi_2)^{2.5}}$

IV. Computational methodology

The momentum and energy equations have been solved that is a set of partial differential equations by using the Galerkin weighted residual finite element method [14]. In this simulation, the interested domain is discretized into finite elements using non-uniform, unstructured, free triangular meshes. Applying Galerkin Weighted Residual method, the nonlinear governing partial differential equations are transferred into a system of integral equations. The basic unknowns for the above differential equations are the velocity components U, V , the temperature θ and the pressure P . The six-node triangular elements are used in this work and all are associated with velocities as well as temperature. Three corner nodes are correlated with pressure. The nonlinear algebraic equations so obtained are modified by imposition of appropriate boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using tri-diagonal matrix algorithm. The convergence criterion for the solution procedure is defined as $|\psi^{n+1} - \psi^n| \leq 10^{-4}$, where n is the number of iteration and ψ is a function from U, V and θ .

V. Result and Discussion

The implications of different values of the Richardson number (Ri) and solid volume fraction of nanoparticles (ϕ), in the enclosure will be emphasized. The results are represented in terms of streamlines and isotherm patterns. The variations of average Nusselt numbers are also highlighted. The solid fluid thermal conductivity ratio $K = 7.0$ have considered throughout the simulation.

The ratio that measures the effect of natural to forced convection modes is defined as Richardson number. Figure. 2 (a) and (b) shows the information about the sensitivity of the stream line and isotherms pattern due to the variations of Richardson number for $Re=100, Pr=6.2, \phi = 0.05$ and $K=7$. From the streamlines contours it is found that for forced convection ($Ri=1$) there exist one primary clockwise circulation cell with two secondary egg shape cores in the cavity. This is due to the dominance of forced convection exerted by the movement of the inclined walls of the cavity. At $Ri=1.0$ (combined convection regime) it can be seen that shape of the primary cell become large in size. It is also noticed that at this step the secondary two core reduces

to one single core near the heated circular cylinder. On the other hand, when $Ri=5.0$ and $Ri=10$ the clockwise vortex spread in wavy form with egg shape core near the heated moving wall. In this case the strength of the natural convection dominant the flow regime. This is reasonable because increasing the parameter Ri assists buoyancy flow, hence the natural convection mode.

The useful influence of Ri on temperature field is plotted in Figure 2 (b). From the figure it is noticed that for $Ri=1$ the isotherms line departs from the middle section and try to crowd on the left moving lid due to the influence of forced convection. In this case, less energy is noticed to carry away from the sliding left wall to the cavity and, subsequently the conduction heat transfer regime has become the dominant mode of energy transfer in the cavity with vertical isotherms near the heated wall. With further increase of $Ri=1$ steeper temperature gradient near the heated wall are evident and the isotherms get the parabolic shape near the inclined cold wall. In the considered regime, the buoyancy effect balanced the effect of sliding top wall and hence, the total heat transfer in the cavity is controlled by the combined mechanisms of forced and free convection. Escalating the convective parameter Ri up to 10, a thin boundary layer is developed near the top corner of the inclined wall. From the figure it is also ascertain that increasing in the buoyancy force causes the isotherms to deform due to the dominating influence of the convective flow.

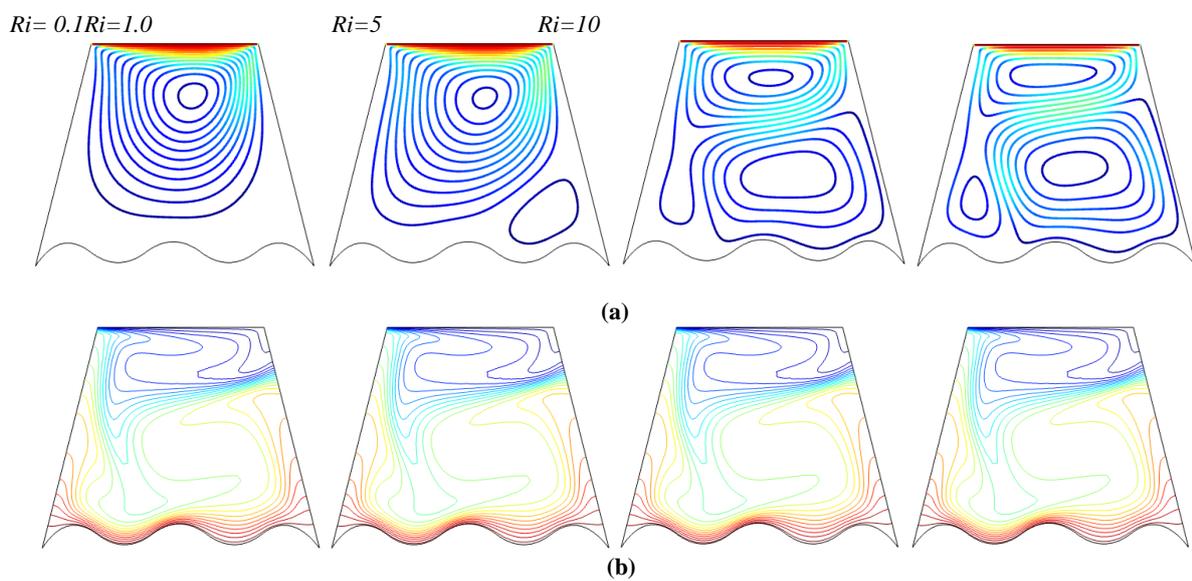
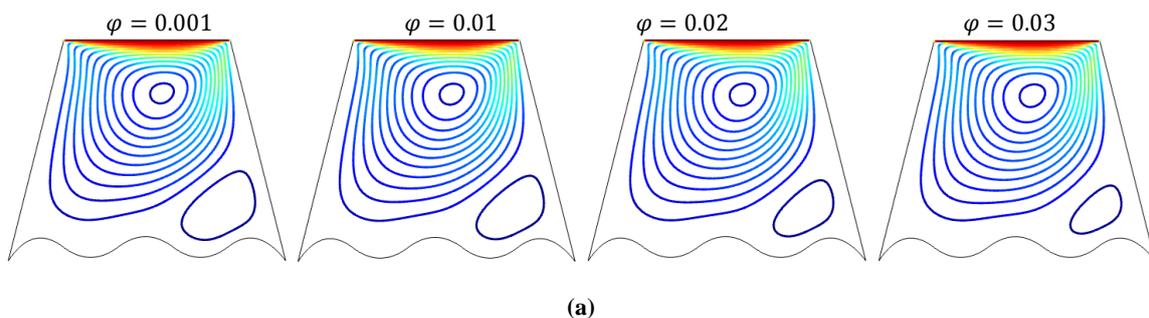


Figure. 2(a) Stream lines and (b) Isotherms for different values of Ri at $Re = 100$, $Pr = 6.2$, $\phi = 0.05$, $K = 7$

As narrated before the volume fraction of hybrid nanofluid comprises of equal quantities of two different Cu and Al_2O_3 nanoparticles. From the Figure 3(a), When the nanoparticles volume fraction is increased from $\phi = 0.0$ to $\phi = 0.05$ for $Re = 100$, $Ri = 1$, $Pr = 6.2$, $K = 7$, the vertical double eye behavior of streamlines associated with pure water ($\phi = 0$) is transformed into single eye behavior with the increase in the nanoparticle volume fraction $\phi \geq 0.03$ and the intensification of the fluid flow behavior decrease when the fluid particle volume fraction drop out. This is due to the enhancement of viscosity of the hybrid nanofluid. Since with the augment of ϕ , the thermal conductivity of the nanofluid increases, hence the buoyancy flow, these interns improved the heat transfer rate.



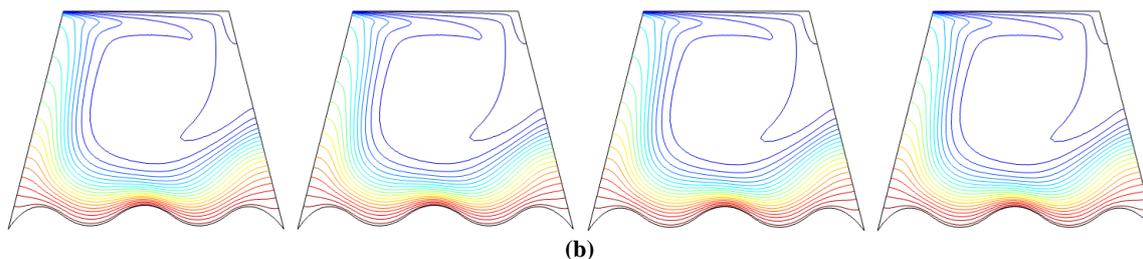


Figure 3(a) Stream lines and (b) Isotherms for different values of ϕ at $Ri=1.0, Re =100, Pr=6.2, K = 7$

Though, it is also obvious from figure 3(b), that the temperature gradients are generally not affected on escalating the solid volume fraction of nanoparticle with any fraction except with the increase of ϕ the isotherms close to the heated wall begin to be more scattered. The enrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. We can conclude that the mixing advantage of Copper and Alumina nanoparticles may blow up the convective heat transfer. Hence, this result encourages us to examine other combination of different types of nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

From the Figure 3(a), it is also obvious that with the increase of the value of Ri heat transfer rate increase. Because with the augmentation of Ri buoyancy effect enhances, so large amount of heat is transferred from the heated wall to the enclosure. It is also perceived that the rate of heat transfer is much higher for hybrid nanofluid compared to the base fluid. This is a decent discovery to use hybrid nanofluid.

VI. Conclusion

Hybrid nanofluid flow in convective lid-driven sinusoidal trapezoidal enclosure is investigated numerically. Varying the parameters Ri and ϕ have leads to the conclusion that the corrugated lid-driven cavity could be considered as an effective heat transfer apparatus for the bottom wavy surface amplitude. The mixed convection parameter Ri affects extensively on the flow structure and heat transfer mechanism inside the hybrid nanofluid filled enclosure. The increment of Richardson number promoted heat transfer by convection. Compared with regular nanofluid, a hybrid nanofluid dispersed in water base fluid has very modest enhancement on the Average Nusselt Number. The effect of mounting volume fraction of the hybrid nanofluid becomes significant in the situation in the case of small buoyancy force.

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