Abstract : By the design method of finite time sliding mode reaching law, study the longitudinal and lateral control for vehicle following and lane changing in intelligent transportation system. Based on coupled longitudinal and lateral model of vehicle, by using front and rear wheel steering technology, the coupled control laws for vehicle following and lane changing were designed. The stability of control systems was analyzed by using Lyapunov function method. The simulation was done applying Matlab toolbox, and the simulation results indicate that the longitudinal and lateral control performance was good by using the control laws designed in this paper.

Keywords - Intelligent transportation system, Lane changing, Vehicle following, Cooperative control, sliding mode

I. Introduction

In recent years, with the development of network communication and control technology, intelligent vehicles have gradually become the research focus in the field of vehicle engineering in the world [1,2]. The research on automatic control of intelligent vehicle mainly includes longitudinal control and lateral control. Vehicle following [3,4] is the longitudinal control measures of the vehicle. Its purpose is to automatically adjust the speed of the controlled vehicle to keep the distance between it and the vehicle in front within the set range. Vehicle lane change [5-7] belongs to the research field of intelligent vehicle lateral control, which refers to the process of controlling a vehicle into another lane along the expected track from one lane. The lane changing process of vehicle is also influenced by the longitudinal motion behavior of vehicle. Refs.[8,9] studied the longitudinal control law of vehicle during the lane change process. In this paper, adopted the kinematics model, and the dynamic behavior of vehicle is not considered. In ref.[10], considering the following behavior of vehicles, designed the longitudinal and lateral coupling control laws for lane change of vehicle. In this paper, the vehicle dynamics model based on front wheel steering is adopted. In ref. [11], the four-wheel steering dynamic model is used to study the vehicle lane change, but assumed that the longitudinal speed is a constant. Ref.[12] studied the trajectory tracking control of vehicle for lane change. In this paper, according to the longitudinal and lateral movement constraints of the vehicle in the lane changing process, by the sliding model control method, designed the lane changing trajectory based on the five-order multinomial and cancelled the assumption that the longitudinal speed is constant, but adopted the linear form of sliding model approaching law, and time for the system state reaching the equilibrium point is infinite.

In this paper, by the design method of finite time sliding mode reaching law, the coupled control of vehicle following and lane change is studied. From the relationship between the global coordinate system and body coordinate system, establish the vehicle longitudinal and lateral tracking error model, based on the front and rear wheel steering dynamics model of vehicle, the sliding mode control law is designed.

II. Vehicle Dynamics Model

The vehicle dynamics model adopted in this paper is derived from the ideal model proposed by Ackermann and ignores the influence of road slope and roll motion. According to the automobile theory, the vehicle longitudinal and lateral dynamics model can be expressed as

$$\dot{x}_i = \frac{1}{\delta m_i} (F_i - m_i g f_R + m_i \ddot{y}_i - C_i \dot{x}_i^2)$$

$$\dot{\gamma}_i = \frac{2(C_i + C_{i\gamma})}{m_i \dot{x}_i} \gamma_i - \left[ \dot{x}_i + \frac{2(C_i f_i f_i - C_i l_i l_i)}{m \dot{x}_i} \right] \dot{\gamma}_i + \frac{2C_i}{m} \delta_f + \frac{2C_i}{m} \delta_r$$

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\[
\ddot{y}_i = -\frac{2(C_i f l_i^2 y_i + C_i r l_i^2)}{I_{iz}\dot{x}_i} \ddot{x}_i - \frac{2(C_i f l_i^2 y_i - C_i r l_i^2)}{I_{iz}\dot{x}_i} \dot{y}_i + \frac{2C_i f l_i^2 y_i}{I_{iz}} \dot{\delta}_f - \frac{2C_i r l_i^2}{I_{iz}} \dot{\delta}_r
\]

where, \(m_i\) is the mass of vehicle \(M_i\), \(\delta\) is the correction coefficient of rotating mass, \(I_{iz}\) is the total vehicle inertia about vertical axis, \(F_i\) is the driving or braking force on the wheel of vehicle \(M_i\), \(f_r\) is the rolling resistance coefficient for road, \(C_{iA}\) is air resistance coefficient of vehicle \(M_i\), \(C_{if}\) and \(C_{ir}\) are the front and rear tire cornering stiffness, respectively, \(l_{if}\) and \(l_{ir}\) are respectively the distance from center of mass to the front axle and the centroid to the rear axle, \(\delta_{if}\) and \(\delta_{ir}\) are the front and rear wheel steering angle, respectively.

Equations (1), (2) and (3) can be simplified as follows:

\[
\begin{align*}
\dot{x}_i &= a_{i1} \dot{y}_i + a_{i2} \dot{x}_i + u_i \\
\dot{y}_i &= b_{i1} \dot{y}_i + b_{i2} \dot{x}_i + v_i \\
\ddot{y}_i &= c_{i1} \dot{y}_i + c_{i2} \dot{x}_i + v_1
\end{align*}
\]  

where

\[
egin{align*}
u_i &= \frac{F_i}{\delta m_i} - \frac{g f_r}{\delta} + \frac{1}{\delta} \ a_{i1} = - \frac{C_{iA}}{\delta m_i} \\
b_{i1} &= - \frac{2(C_i f l_i^2 y_i + C_i r l_i^2)}{m_i \dot{x}_i}, b_{i2} = - \dot{x}_i - \frac{2(C_i f l_i^2 y_i - C_i r l_i^2)}{m_i \dot{x}_i} \\
c_{i1} &= - \frac{2(C_i f l_i^2 y_i + C_i r l_i^2)}{I_{iz} \dot{x}_i}, c_{i2} = - \frac{2(C_i f l_i^2 y_i - C_i r l_i^2)}{I_{iz} \dot{x}_i} \\
v_1 &= \frac{2C_i f l_i^2 y_i}{m_i} \delta_{if} + \frac{2C_i r l_i^2}{m_i} \delta_{ir}, v_2 = \frac{2C_i f l_i^2 y_i}{I_{iz}} \dot{\delta}_f - \frac{2C_i r l_i^2}{I_{iz}} \dot{\delta}_r
\end{align*}
\]

III. Design of Control Law

3.1 Tracking error model

Assume that the road has two lanes, Starting lane and Target lane, respectively, with lane spacing d. The global cartesian coordinate system is established with a point on Starting lane as the origin as showed in Fig.1. The X-axis is along the Starting lane center line, and the driving direction is positive. The Y-axis is perpendicular to the X-axis, and the direction from Starting lane to Target lane is positive. Suppose \(M_0\) represents the leading vehicle of the vehicle platoon, \(M_i\) represents the \(i\)-th following vehicle, \(i = 1, 2, \cdots, n\), where \(n\) is the number of following vehicles. It is assumed that the displacement, velocity and acceleration information of the leading vehicle can be transmitted to each following vehicle behind. The displacement, velocity and acceleration information of the \(i\)-1st vehicle can be transmitted to the \(i\)-th vehicle.

Fig.1 Tracking error diagram for lane changing and vehicle following

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The position error along the X-axis between $M_i$ and $M_{i-1}$ is defined as:

$$e_{i,i-1}(t) = X_i(t) - X_{i-1}(t) + L_i$$  \hspace{1cm} (9)

Where $L_i$ represents the expected distance between the $i$-th vehicle and the $i-1$st vehicle. The position error between $M_i$ and $M_0$ is defined as:

$$e_{i,0}(t) = X_i(t) - X_0(t) + \sum_{j=1}^{i} L_j$$  \hspace{1cm} (10)

The tracking error of the vehicle $M_i$, along the X-axis is defined as:

$$e_{X}(t) = \dot{X}_i(t) - \dot{X}_i(t) + (1 - \lambda) \ e_{i,0}(t)$$  \hspace{1cm} (11)

Where, $0 < \lambda < 1$, is a fixed parameter. In the process of the vehicle following, vehicle $M_i$ should keep driving along the lane centerline, that is, the expected displacement along the Y-axis is 0. When lane change is required, the tracking error of vehicle $M_i$, along the Y-axis is defined as

$$e_{Y}(t) = Y_i(t) - Y_{id}(t)$$  \hspace{1cm} (12)

Where $Y_{id}(t)$ represents the lateral expected displacement for lane changing of vehicle $M_i$.

Assuming that $\psi_{id}$ is the desired yaw angle of the vehicle, and define the tracking error of yaw angle:

$$e_{w}(t) = \psi_i(t) - \psi_{id}(t)$$  \hspace{1cm} (13)

Where $\psi_i$ is the yaw angle of the controlled vehicle $M_i$.

In the body coordinate system, assuming longitudinal displacement of the vehicle is $x$, the lateral displacement of the vehicle is $y$, and the angle between the longitudinal axis of the vehicle $M$ and the X-axis of the global coordinate system is $\psi$. In the global coordinate system, the speed of the vehicle along the coordinate axis is

$$\begin{align*}
\dot{X}_i &= \dot{x}_i \cos \psi_i - \dot{y}_i \sin \psi_i \\
\dot{Y}_i &= \dot{x}_i \sin \psi_i + \dot{y}_i \cos \psi_i
\end{align*}$$  \hspace{1cm} (14)

From equation (14), the acceleration along the coordinate axis is:

$$\begin{align*}
\ddot{X}_i &= \ddot{x}_i \cos \psi_i - \ddot{y}_i \sin \psi_i + w_{1i} \\
\ddot{Y}_i &= \ddot{x}_i \sin \psi_i + \ddot{y}_i \cos \psi_i + w_{2i}
\end{align*}$$  \hspace{1cm} (15)

where, $w_{1i} = -\dot{x}_i \dot{y}_i \sin \psi_i - \dot{y}_i \dot{y}_i \cos \psi_i$, $w_{2i} = \dot{x}_i \dot{x}_i \cos \psi_i - \dot{y}_i \dot{y}_i \sin \psi_i$.

According to (11), (12) and (13), we have

$$\begin{align*}
\dot{e}_X &= \lambda (\dot{x}_i \cos \psi_i - \dot{y}_i \sin \psi_i - \dot{X}_{i-1}) + (1 - \lambda) (\dot{x}_i \cos \psi_i - \dot{y}_i \sin \psi_i - \dot{X}_0) \\
\dot{e}_Y &= \dot{x}_i \sin \psi_i + \dot{y}_i \cos \psi_i - \dot{Y}_i \\
\dot{e}_w &= \dot{\psi}_i - \dot{\psi}_{id}
\end{align*}$$  \hspace{1cm} (16)

From (15), we get

$$\begin{align*}
\dot{e}_X &= \lambda (\ddot{x}_i \cos \psi_i - \ddot{y}_i \sin \psi_i + w_{1i} - \ddot{X}_{i-1}) + (1 - \lambda) (\ddot{x}_i \cos \psi_i - \ddot{y}_i \sin \psi_i + w_{1i} - \ddot{X}_0) \\
\dot{e}_Y &= \ddot{x}_i \sin \psi_i + \ddot{y}_i \cos \psi_i + w_{2i} - \ddot{Y}_i \\
\dot{e}_w &= \ddot{\psi}_i - \ddot{\psi}_{id}
\end{align*}$$  \hspace{1cm} (17)

### 3.2 Control law

Adopting the sliding mode control method, define the switching function as

$$\begin{align*}
S_X &= \dot{e}_X + \rho_{1i} \dot{e}_X \\
S_Y &= \dot{e}_Y + \rho_{2i} \dot{e}_Y \\
S_w &= \dot{e}_w + \rho_{3i} \dot{e}_w
\end{align*}$$  \hspace{1cm} (18)

Where, $\rho_{ij} > 0$, $\varphi_{ij} > 0$, $k_{ij}$ and $l_{ij}$ are positive odd Numbers, and $l_{ij} > k_{ij}$, $j = 1, 2, 3$.

Differentiate formula (18), with (17), we get
Define the Lyapunov function $V_i$ here, $V_i = \sum_{j=1}^{3} \frac{1}{2} S_{ij}^2$.

3.3 Stability analysis

Define the Lyapunov function

$$V_i = \begin{bmatrix} \frac{S_{ix}^2}{2} \\ \frac{S_{iy}^2}{2} \\ \frac{S_{iz}^2}{2} \end{bmatrix}$$

Take the derivative of equation (25), connect equations (4), (19) and (20), we get
for $\eta_j > 0$, $\varphi_j > 0$, $k_j$, and $l_j$ are positive odd numbers, and $l_j > k_j$, $j = 1, 2, 3$, so

$$
\begin{align*}
- \eta_{i1}S_{ix}^2 - \varphi_{i1} S_{ix}^{k_{y1}/l_{y1}} + 1 & \leq 0 \\
- \eta_{i2}S_{iy}^2 - \varphi_{i2} S_{iy}^{k_{y2}/l_{y2}} + 1 & \leq 0 \\
- \eta_{i3}S_{iw}^2 - \varphi_{i3} S_{iw}^{k_{y3}/l_{y3}} + 1 & \leq 0
\end{align*}
$$

From (26), then we have, when $t \to \infty$, $S_{ix} \to 0$, $S_{iy} \to 0$, $S_{iw} \to 0$. Furthermore, from the differential equation

$$
\begin{align*}
\dot{S}_{ix} + \eta_{i1}S_{ix}^2 + \varphi_{i1} S_{ix}^{k_{y1}/l_{y1}} & = 0 \\
\dot{S}_{iy} + \eta_{i2}S_{iy}^2 + \varphi_{i2} S_{iy}^{k_{y2}/l_{y2}} & = 0 \\
\dot{S}_{iw} + \eta_{i3}S_{iw}^2 + \varphi_{i3} S_{iw}^{k_{y3}/l_{y3}} & = 0
\end{align*}
$$

We get, the system state reaches the sliding mode in a finite time and has a fast convergence rate.

In sliding mode motion, $S_i = 0$, according to the switching function (18), it can be obtained that

$$
\begin{align*}
\dot{e}_{ix} + \rho_{i1} e_{ix} & = 0 \\
\dot{e}_{iy} + \rho_{i2} e_{iy} & = 0 \\
\dot{e}_{iw} + \rho_{i3} e_{iw} & = 0
\end{align*}
$$

According to equation (27), when system state moves along the sliding mode, the system has a fast convergence rate.

### IV. Simulation Results

The platoon consists of four vehicles, one leading vehicle and three following vehicles. The control law adopts the form as (23). Set the correction coefficient of rotating mass $\delta$ as 1.1, the coefficient of rolling resistance $f_k$ as 0.02, and the coefficient of vehicle air resistance $C_A$ as 0.4. for other parameters of the following vehicle as shown in table 1.

<table>
<thead>
<tr>
<th>Param</th>
<th>$m$ (kg)</th>
<th>$I_z$ (kgm$^2$)</th>
<th>$l_f$ (m)</th>
<th>$l_r$ (m)</th>
<th>$C_f$ (kN/rad)</th>
<th>$C_r$ (kN/rad)</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>2100</td>
<td>3150</td>
<td>1.35</td>
<td>1.28</td>
<td>72</td>
<td>82</td>
</tr>
<tr>
<td>2</td>
<td>1600</td>
<td>2600</td>
<td>1.50</td>
<td>1.32</td>
<td>53</td>
<td>62</td>
</tr>
<tr>
<td>3</td>
<td>1800</td>
<td>2950</td>
<td>1.30</td>
<td>1.42</td>
<td>73</td>
<td>80</td>
</tr>
</tbody>
</table>

Suppose that the initial longitudinal velocity of the three following vehicles in the platoon are: $\dot{x}_1(0) = 20$ m/s, $\dot{x}_2(0) = 21$ m/s, and $\dot{x}_3(0) = 22$ m/s; The initial yaw angles are $\gamma_{f1}(0) = 0.2$ rad, $\gamma_{f2}(0) = 0.1$ rad, and $\gamma_{f3}(0) = 0.05$ rad. The initial values of lateral velocity and yaw rate are all 0.

Set expected spacing of vehicles $L_i = 12$, $i = 1, 2, 3$; The initial displacement of the lead vehicle and the following vehicle along the X-axis are: $X_0(0) = 50$ m, $X_1(0) = 39$ m, $X_2(0) = 27.5$ m, and $X_3(0) = 15.7$ m; The initial displacement along the Y-axis are $Y_1(0) = 1$ m, $Y_2(0) = 0.5$ m, and $Y_3(0) = -0.3$ m.

The simulation time set to 10s, the initial velocity of the lead vehicle along the X-axis was set as $\dot{X}_0(0) = 20$ m/s, and the acceleration was changed as follows:
Let the vehicle $M_i$ change lanes in the process of following the front vehicle, the lane centerline spacing is $d = 3\text{m}$, the lane change starts at $t_{on} = 4\text{s}$ and ends at $t_{off} = 7.5\text{s}$. Then the expected displacement along the Y-axis can be expressed as:

$$Y_{id}(t) = \begin{cases} 0, & 0 \leq t < 7.5 \\ f_i(t), & 4 \leq t \leq 7.5 \\ 3, & 7.5 < t \leq 7.5 \end{cases}$$

(29)

Where, $f_i(t)$ represents the expected quintic polynomial trajectory for lane change of the $i$ vehicle, $i = 1, 2, 3$.

In the simulation, the expected values of vehicle displacement at the beginning of lane change are $Y_1(4) = -0.0149\text{m}$, $Y_2(4) = -0.0147\text{m}$, and $Y_3(4) = -0.0199\text{m}$; expected values of the vehicle velocity are $\dot{Y}_1(4) = 0.1261\text{m/s}$, $\dot{Y}_2(4) = 0.1244\text{m/s}$, and $\dot{Y}_3(4) = -0.0204\text{m/s}$; expected values of the acceleration are $\ddot{Y}_1(4) = -0.0126$, $\ddot{Y}_2(4) = -0.0219$, and $\ddot{Y}_3(4) = -0.0126$.

At the end time of the lane change, the expected vehicle state are $Y_{id}(7.5) = 3$, $\dot{Y}_{id}(7.5) = 0$, and $\ddot{Y}_{id}(7.5) = 0$, then the expected lane change trajectory can be obtained according to equation as

$$f_i(t) = \begin{cases} 0, & 0 \leq t < 3 \\ 3, & 3 \leq t < 4 \\ 19 - 4t, & 4 \leq t < 5.5 \\ -3, & 5.5 \leq t < 6.5 \\ 0, & 6.5 \leq t \leq 10 \end{cases}$$

(28)

Where, $4 \leq t \leq 7.5$.

According to the vehicle's initial motion state and its expected value, it can be known that the errors of initial position along the X-axis are $e_x(0) = 1\text{m}$, $e_y(0) = 0.5\text{m}$, and $e_z(0) = 0.2\text{m}$; the tracking errors along the Y-axis are $e_{y1}(0) = 1\text{m}$, $e_{y2}(0) = 0.5\text{m}$, and $e_{y3}(0) = -0.3\text{m}$; Errors of yaw angle are $e_{\psi x}(0) = 0.2\text{rad}$, $e_{\psi y}(0) = 0.1$, and $e_{\psi z}(0) = 0.05$.

Figure 2 to figure 4 are simulation results of tracking control for following and lane change of vehicle. In Fig. 2, sub-figures (a), (b) and (c) represent respectively the speed, displacement and spacing errors of three following vehicle and lead vehicle along the X-axis. In Fig. 3, sub-graphs (a), (b) and (c) represent respectively the indeed and expected values of vehicle velocity, displacement and tracking error along the Y-axis. Subgraphs (a), (b) and (c) in Fig 4 represent respectively the vehicle yaw rate, actual and expected values of yaw angle and their tracking errors. As can be seen from the figures, the vehicle spacing error along the X-axis, trajectory tracking error along the Y-axis and yaw angle error all approach 0 with the control method presented in the paper, it indicates that the control system has asymptotic stability.
In this paper, by finite time sliding mode reaching law, study the longitudinal and lateral coupled control of vehicle in intelligent transportation system. Due to applying the front and rear wheels steering dynamics model, the control law designed can guarantee the lateral position error, yaw angle error, and longitudinal spacing error asymptotically tends to zero. Since using the nonlinear reaching law of sliding model, system state reaches the sliding mode in a finite time.

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References


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