# Design and Analysis of a Parallel Link Manipulator for Heavy Lifting in Warehouses

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Abstract: This paper is intended to put forth a new approach to robot design that is centered around Parallel Link Manipulator. Parallel Link Manipulator is a challenging but equally rewarding field of research. It can prove to be the remedy for the long-standing problem of low 'Lifting Weight to Robot Weight Ratio'. The very large size of actuators inserial link manipulators for heavy lifting is also a problem. Very heavy construction is required for heavy lifting purposes. In this paper we propose a new parallel link manipulator design to work as normal operating robot in warehouse and factories. We also present its forward and inverse kinematics with static structural analysis for testing the maximum safe lifting capacity of the robot. The main advantage of the proposed design is its great maximum lifting capacity to body weight ratio with ease in manipulability. **Key Words:** Lifting Capacity to body weight ratio, Parallel Link Manipulators, Forward Kinematics, Inverse

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# I. Introduction

The Science Fictions that we watched not so long ago and disparaged them as just a vague extrapolation of work done in robotics till then, does seem to have rubbed off positively on the mind of scientists. Tremendous amount of progress has been made in all the aspects of Robotics. Technologies like Artificial Intelligence and Machine learning etc. have given a shot in the arm to the Robotics Industry. Consequently, the pliability of robots in umpteen number of industries and offices has become a reality today.

Today a major untold problem of serial link manipulator is low lifting capacity with respect to body weight of the robot. Thus, very heavy construction is required for a serial link manipulator to lift heavy objects in warehouses. To get a detailed perspective on this ratio, it is worthwhile to look at the following analysis.

In Industries, a 1750 kg of Robot (IRB 660) has a weight lifting capacity of approximately 180 to 250 kg. The ratio that comes out is meagerly 0.1 to 0.15. This abysmally low ratio is a serious challenge to the applicability of robots in Heavy Industries, construction sites, etc

Through our study we have tried to address this issue. We need to design a robot system which can overcome this problem. Specifically, we need to design a robot that can be used in warehouse setting which has high maximum lifting capacity to body weight ratio. Thus, the robot should have simple, robust and light weight construction as compared to maximum load that can be lifted by robot.

Another major requirement for a robot to work in warehouse is ease of control and manipulability. This is a very important requirement in industry as most of the warehouses and factories have very low tolerance for error.

Most of the serial link manipulator also have the problem of compounding of required actuator strength when moving from end effector to base of the robot. It means that every successive actuator in the serial link manipulator should be powerful enough to lift the weight of entire robot body and object weight after the actuator till end-effector. Thus, the base actuator in a normal serial link manipulator need to be very powerful. Due to this cost of the large sizeactuatorsused in Serial Link Manipulator is a very much considerable factor.

In this paper we want to address most of these problems by presenting a new design for a robot that can be used in a warehouse or factory setting.

# II. Problem Formulation

We formulate the task of building such a robot in four main phases

- I. Design Phase
- II. Forward Kinematics Phase
- III. Inverse Kinematics Phase
- IV. Structural Analysis Phase

In these phases we aim to design and analyse the robot which fulfils the desired requirements. The main objectives that we want to achieve are

- I. A robot with considerably high lifting weight to body wight ratio as compared to a normal serial link 6 DOF manipulator.
- II. A robot which should be considerably easy if not easiest to manipulate.
- III. A robot which should be free from compounding of actuator power problem.

#### DESIGN

Today majority of the robot used in industry are Serial link Manipulators. They are most widely used because they are easy to manipulate, design and construct. But Serial link robots have a major drawback i.e. they require very heavy construction to lift heavy objects. The maximum lifting capacity to robot body weight ratio is usually very low (less than 1). On the other hand, Parallel link manipulators are very robust and rigid, but are also very complex to manipulate, design and manufacture. Usually Parallel link manipulators have very high maximum lifting capacity to body weight ratio (can be more than 1). They achieve this capability by parallelly using various link to distribute the load. Since we want to design a robot with high maximum lifting capacity to body weight ratio design a parallel link manipulator.

Another major problem is manipulability. Parallel link manipulators are famous for their complex manipulability. A very complex parallel link manipulator has high chances of losing its accuracy and manipulability. This is not a very desirable characteristic of a robot that is being designed to work in warehouses and factories where tolerance for error is very low. So, the parallel link manipulator that we want to design should also be considerably easy and fast to manipulate.

Therefore, to meet the desired requirements, we propose the following design.



Figure 1: Isometric view of proposed robot design



Figure 2: Side view of proposed robot design

The proposed design has following key features:

• The robot consists of one common "base plate" which consist of one revolute joint with grounded stationary base plate.

• The robot has one main manipulator chain which has two links. The vertical link in main chain is called as "standard link 1" while the longer horizontal link is termed as "main link". In the proposed design the main link is twice the length of standard link 1.

• The robot also has secondary supporting chain. It consists of one very large piston-cylinder link which is nearly of same size as that of main link in its maximum extended configuration. Another vertical link is known as "standard link 2". It is of exactly the same dimension of standard link 1.

• The "cylinder link" (in piston-cylinder link) which is attached to base plate has a rigid vertical extrusion towards ground. The extrusion provides robot with another contact to ground and also a very firm secondary support.

• The End-effector consists of two supporting plate and two gripper pads with friction pad for increased gripping strength. The first link plate is known as "roll plate" and is attached to main link and provides pitch motion to end-effector and also serves as base to connect to later "gripper plate". The joint between "roll plate" and "gripper plate" provides roll motion to end effector. The "gripper plate" has a very wide construction which helps robot to exercise strong grip over large boxes.

This robot is designed to work in two configurations. One is "parallelogram configuration" and the other is "extended configuration". The parallelogram configuration can be used to lift object which has height lower than height of the robot in rectangular configuration (a case in parallelogram configuration). While the extended configuration is used to lift much taller and higher objects. Both of the configurations are shown below.



Figure 3: Parallelogram Configuration



Figure 4: Extended Configuration

The secondary supporting chain acts as a support to main chain in both the configuration. The secondary chain also increases the maximum lifting capacity while not creating much difference in manipulability.

**The robot has five degree of freedom**. The degree of freedom is in X, Y, Z as translational DOF while robot also has roll and pitch rotational DOF. The robot is similar to a serial link manipulator. Due to which it is significantly easier to manipulate.

# **IV.** Forward Kinematics

TheParallel link manipulators are known to have very complex forward kinematics. And our proposed robot is not very different in this respect. But with geometrical and analytical method formula for each unknown term in this robot can be derived. Therefore, we have used geometric and calculus to find formula for every

variable in this robot. The forward kinematics of this robot is derived for two separate chains individually. The Forward kinematic equation for supporting chain converges with main manipulator chain at "standard link 2" and "main link" revolute joint. Due to this arrangement we can find inverse kinematic of this robot using analytical and geometrical methods.



Figure 5: Parameter description of robot

# For Main Chain and Manipulator

In main chain of the robot one or more frames are assigned to each link. The initial frame starts from Frame 0 i.e. ground frame and goes to tool frame via main chain. The description of these frames is given as follows: **Frame 0**: Ground Stationary Frame

Frame 1: Attached to base of "Base Plate". L0 distance from Frame 0 in +z axis.

Frame 2: At "Base Plate" and "Standard link 1" revolute joint. Attached to "Base Plate" at "Base Plate" and "Standard link 1" junction axis.

**Frame 3**: At same point as of Frame 2 but can rotate about z axis with respect to frame 2. Attached to "Standard link 1".

Frame 4: At "Standard link 1" and "main link" revolute joint. Attached to "main link".

Frame 5: At "main link" and "roll plate" revolute joint. Attached to "roll plate".

**Frame 9**: At "main link" and "Standard link 2" revolute joint. Attached to "main link" and at 2\*X distance from frame 4 along x axis.

**Frame {Tool}:** At L5 distance from Frame 4. On gripper plate. Gives revolute motion for roll DOF. Attached to "Gripper plate". Acts as final end-effector frame.

Denavit-Hartenberg Parameters for the Main chain and manipulator of the proposed robot is given in the table 1 below.

# D-H Parameters for Main Chain and Manipulator

	α (along axis)	a (along axis)	d (along axis)	θ (along axis)
Frame 0 $\rightarrow$ Frame 1	0	L0 (z axis)	0	q1 (z axis)
Frame 1 $\rightarrow$ Frame 2	+90 (x axis)	L1 (z axis)	-d2 (x axis)	0
Frame 2 $\rightarrow$ Frame 3	0	0	0	q3 (z axis)
Frame 3 $\rightarrow$ Frame 4	0	X (x axis)	0	q4 (z axis)
Frame 4 $\rightarrow$ Frame 5	0	2X + L4 (x axis)	0	q5 (z axis)
Frame 5 → Frame{Tool}	α5 (x axis)	L5 (x axis)	0	0
Frame 4 $\rightarrow$ Frame 9	0	2*x (x axis)	0	0

 Table 1: Frame parameters of Main Chain

Therefore,

 ${}^{0}[T]_{\{\text{Tool}\}} = {}^{0}[T]_{1} x {}^{1}[T]_{2} x {}^{2}[T]_{3} x {}^{3}[T]_{4} x {}^{4}[T]_{5} x {}^{5}[T]_{\{\text{Tool}\}}$ (4.1)

 $\omega(1,1)$  $\omega(1,2)$  $\omega(1,3)$  $x^{-}$  $\omega(2,2)$  $\omega(2,1)$  $\omega(2,3)$ y  ${}^{0}[T]_{\{Tool\}} =$ (4.2) $\omega(3,1) \quad \omega(3,2)$  $\omega(3,3)$ Ζ 0 0 0 The values of the above terms in matrix are:  $\omega(1,1) = \cos(q3 + q4 + q5) * \cos(q1)$  $\omega(1,2) = \sin(\alpha 5) * \sin(q 1) - \sin(q 3 + q 4 + q 5) * \cos(\alpha 5) * \cos(q 1)$  $\omega(1,3) = \cos(\alpha 5) * \sin(q 1) + \sin(q 3 + q 4 + q 5) * \sin(\alpha 5) * \cos(q 1)$  $\omega(2,1) = \cos(q3 + q4 + q5) * \sin(q1)$  $\omega(2,2) = -\sin(\alpha 5) * \cos(q1) - \sin(q3 + q4 + q5) * \cos(\alpha 5) * \sin(q1)$  $\omega(2,3) = \sin(q3 + q4 + q5) * \sin(\alpha 5) * \sin(q1) - \cos(\alpha 5) * \cos(q1)$  $\omega(3,1) = sin(q3 + q4 + q5)$  $\omega(3,2) = \cos(q3 + q4 + q5) * \cos(\alpha 5)$  $\omega(3,3) = -\cos(q3 + q4 + q5) * \sin(\alpha 5)$  $x = \cos(q1) * [(\cos(q3 + q4) * (L4 + 2 * x) - d2 + x * \cos(q3)) + L5 * \cos(q3 + q4 + q5)]$ y = sin(q1) \* [cos(q3 + q4) \* (L4 + 2 \* x) - d2 + x \* cos(q3)) + L5 \* cos(q3 + q4 + q5)] $z = L0 + L1 + sin(q_3 + q_4) * (L4 + 2 * x) + x * sin(q_3) + L5 * sin(q_3 + q_4 + q_5)$ In main chain and manipulator part of this robot there is another matrix that is very important to find inverse kinematics for supporting chain. That matrix is <sup>0</sup>[T]<sub>5-main</sub>. It is represented as:  ${}^{0}[T]_{5-main} =$  ${}^{0}[T]_{1} \times {}^{1}[T]_{2} \times {}^{2}[T]_{3} \times {}^{3}[T]_{4} \times {}^{4}[T]_{5}$ (43)Therefore.  $[\omega 5(1,1) \quad \omega 5(1,2) \quad \omega 5(1,3)$ x5 $\omega 5(2,1) \quad \omega 5(2,2) \quad \omega 5(2,3)$ *y*5  ${}^{0}[T]_{5-main} =$  $\omega 5(3,1) \quad \omega 5(3,2) \quad \omega 5(3,3)$ *z*5 0 0 0 1 ${}^{0}[T]_{5-\text{main}} \ge {}^{5}[T]_{\{\text{Tool}\}}(4.5)$ Also,  ${}^{0}[T]_{\{\text{Tool}\}}$ = Where, 0 Г1  $L5^{-}$ 0  $cos(\alpha 5)$ 0  $-sin(\alpha 5)$  ${}^{5}[T]_{\{Tool\}} =$ (4.6)0  $sin(\alpha 5)$ 0  $cos(\alpha 5)$ LO 0 0 1 The value of terms in <sup>0</sup>[**T**]<sub>5-main</sub>matrix are given as:  $\omega 5(1,1) = \cos(q3 + q4 + q5) * \cos(q1)$  $\omega 5(1,2) = -\sin(q3 + q4 + q5) * \cos(q1)$  $\omega 5(1,3) = sin(q1)$  $\omega 5(2,1) = \cos(q3 + q4 + q5) * \sin(q1)$  $\omega 5(2,2) = -\sin(q3 + q4 + q5) * \sin(q1)$  $\omega 5(2,3) = -\cos(q1)$  $\omega 5(3,1) = sin(q3 + q4 + q5)$  $\omega 5(3,2) = \cos(q3 + q4 + q5)$  $\omega 5(3,3) = 0$  $x5 = \cos(q1) * [\cos(q3 + q4) * (L4 + 2 * x) - d2 + x * \cos(q3)]$  $y_5 = sin(q_1) * [cos(q_3 + q_4) * (L_4 + 2 * x) - d_2 + x * cos(q_3)]$ z5 = L0 + L1 + sin(q3 + q4) \* (L4 + 2 \* x) + x \* sin(q3)

# For Supporting Chain

In this part of the robot one or more frames are assigned to each link starting from ground frame (Frame 0) to Frame 9 of the main chain that has been described earlier. This choice of end frame in this part of robot is essential, as it helps us in solving inverse kinematics for supporting chain. Each of the frame used in this part of robot is described as follows:

Frame 0: Ground Stationary Frame

Frame 1: Attached to base of "Base Plate". L0 distance from Frame 0 in +z axis.

**Frame 6**: At "Base Plate" and "Cylinder link" revolute joint. Attached to "Base Plate" at "Base Plate" and "Cylinder link" junction axis.

**Frame 7**: At same point as of Frame 6 but can rotate about z axis with respect to frame 6. Attached to "Cylinder Link".

Frame 8: At "Standard link 2" and "Piston link" revolute joint. Attached to "standard link 2" along x axis.

**Frame 9**: At "main link" and "Standard link 2" revolute joint. Attached to "main link" along x axis. Frame 9 is also 2\*X distance from frame 4 along x axis.

Denavit-Hartenberg Parameters for the Supporting chain of the proposed robot is given in the table 2.

# **D-H Parameters for Supporting Chain**

Tuble 2. France parameters of Supporting Cham				
	α (along axis)	a (along axis)	d (along axis)	<i>θ</i> (along axis)
Frame 0 → Frame 1	0	L0 (z axis)	0	q1 (z axis)
Frame 1 → Frame 6	+90 (x axis)	L1 (z axis)	d6 (x axis)	0
Frame 6 → Frame 7	0	0	0	q7 (z axis)
Frame 7 → Frame 8	0	xn (x axis)	0	q8 (z axis)
Frame 8 → Frame 9	0	X (x axis)	0	q9 (z axis)

 Table 2: Frame parameters of Supporting Chain

Therefore,

 ${}^{0}[T]_{4}$  ${}^{0}[T]_{1} x {}^{1}[T]_{6} x {}^{6}[T]_{7} x {}^{7}[T]_{8} x {}^{8}[T]_{9} (5.1)$ =  $[\omega s(1,1) \quad \omega s(1,2) \quad \omega s(1,3) \quad xs]$  $\omega s(2,1) \quad \omega s(2,2) \quad \omega s(2,3)$ ys (5.2)  ${}^{0}[T]_{4}$ =  $\omega s(3,1) \quad \omega s(3,2)$  $\omega s(3,3)$ ZS0 0 1 0 The values of the above terms in matrix are:  $\omega s(1,1) = \cos(q7 + q8 + q9) * \cos(q1)$ 

 $\omega s(1,2) = -\sin(q7 + q8 + q9) * \cos(q1)$   $\omega s(1,3) = \sin(q1)$   $\omega s(2,1) = \cos(q7 + q8 + q9) * \sin(q1)$   $\omega s(2,2) = -\sin(q7 + q8 + q9) * \sin(q1)$   $\omega s(2,3) = -\cos(q1)$   $\omega s(3,1) = \sin(q7 + q8 + q9)$   $\omega s(3,2) = \cos(q7 + q8 + q9)$   $\omega s(3,3) = 0$   $xs = \cos(q1) * (d6 + x * \cos(q7 + q8) + xn * \cos(q7))$   $ys = \sin(q1) * (d6 + x * \cos(q7 + q8) + xn * \cos(q7))$  $zs = L0 + L1 + x * \sin(q7 + q8) + xn * \sin(q7)$ 

#### V. Inverse Kinematics

The inverse kinematics is used to get required joint parameters for reaching an end effector configuration in cartesian space. So, we first need to figure out which variables in our forward kinematic matrices are constant and which one of them are free to manipulate. The list for variables and constant for the proposed robot design is given below.

I. For Main chain and manipulator

Table 3: Known and Unknown par	rameters of Main Chain
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Known Constants	Unknown Variables
LO	q1
L1	q3
x	q4
L4	q5
L5	α5
d2	

# **II.** For Supporting Chain

Known Constants	Unknown Variables
LO	q1
L1	xn (Prismatic joint)
х	q7 (in extended configuration)
d6	q8
q7 (in parallelogram configuration)	q9

Table 4: Known and Unknown parameters of Supporting Chain

During solving inverse kinematics for an orientation, the end effector configuration to be reached via end effector in cartesian space will also be known in the form of a 4x4 matrix. Let us assume a 4x4 orientation matrix that has to be reached via end effector.

$$[T]given = \begin{bmatrix} qxx & qxy & qxz & px \\ qyx & qyy & qyz & qy \\ qzx & qzy & qzz & rz \\ 0 & 0 & 0 & 1 \end{bmatrix} (5.3)$$

We assume every term in the above matrix is known and is given as an input to solve inverse kinematic equations. We will use these terms to derive solutions for each unknown variable that are being listed in above tables.

#### Solution for Main chain with manipulator

In this part the values of q1, q3, q4, q5, a5 are needed by the robot to reach required end-effector position and orientation.

By comparing  ${}^{0}[T]_{\{Tool\}}$  and [T]given.

 ${}^{0}[T]_{\{Tool\}} = [T]given$ 

By equating these two matrices we get twelve equations. From these twelve equations we get,

q1 = atan2(qy, px)(5.4)  $\alpha 5 = atan2(-qzz, qzy)$ (5.5)

Now since the value of  $\alpha 5$  is known and the value of L5 is already known. Therefore, the value of  ${}^{5}[T]_{\{Tool\}}$  is  $\begin{bmatrix} 1 & 0 & 0 \\ L5 \end{bmatrix}$ 

given by  $\begin{bmatrix} 0 & \cos(\alpha 5) & -\sin(\alpha 5) & 0 \\ 0 & \sin(\alpha 5) & \cos(\alpha 5) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ . With the help of  ${}^{5}[\mathbf{T}]_{\{\text{Tool}\}}$  and equation  ${}^{0}[\mathbf{T}]_{\{\text{Tool}\}} = {}^{0}[\mathbf{T}]_{5\text{-main}} \mathbf{x}$ 

 ${}^{5}[T]_{\text{Tool}}$  we have,

 $[{}^{0}[T]_{5-main}]given = [T]given \ x \ [{}^{5}[T]_{[Tool]}]^{-1}$  (5.6) Here  $[{}^{0}[T]_{5-main}]given$  gives the required orientation of main link frame 5. Now, since the value of  $[{}^{0}[T]_{5-main}]given$  is known for a required end-effector position and orientation. Let its value be,

$$\begin{bmatrix} {}^{0}[T]_{5\text{-main}} \end{bmatrix} given = \begin{bmatrix} q5xx & q5xy & q5xz & p5x \\ q5yx & q5yy & q5yz & q5y \\ q5zx & q5zy & q5zz & r5z \\ 0 & 0 & 1 \end{bmatrix} (5.7)$$
  
By comparing  $\begin{bmatrix} {}^{0}[T]_{5\text{-main}} \end{bmatrix} given \text{ and } {}^{0}[T]_{5\text{-main}} \text{ we get}$ 

 $\mathbf{q3} = \mathbf{sin}^{-1}(\frac{d - a \ast \mathbf{sin}(\epsilon)}{\epsilon}) \tag{5.8}$ 

$$a_{1} = c \quad a_{2}^{2} \qquad (5.0)$$

$$q4 = \epsilon - q3$$
(5.9)  
$$q5 = \sin^{-1}(q5zx) - q3 - q4$$
(5.10)

Here in equation 5.8, 5.9 and 5.10

$$a = L4 + 2 * X$$

$$b = X$$

$$c = \left(\frac{q5y}{\sin(q1)}\right) orc = \left(\frac{p5x}{\cos(q1)}\right)$$
(5.11)
(5.12)
(5.13)

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$$d = r5z$$
(5.14)  

$$\epsilon = \cos^{-1}\left(\frac{a^2 + c^2 + d^2 - b^2}{2 * a * \sqrt{(d^2 + c^2)}}\right) + atan2(d,c)$$
(5.15)

#### Solution for Supporting chain

This robot is being designed to work in two main configurations as already being described in Design section. They are distorted parallelogram configuration and supported extended configuration. In distorted parallelogram configuration angle q7 is zero. The main advantage of this configuration is secondary support from ground surface via "cylinder link" extrusion. Due to this extra support the robot can lift heavier weight, while at the same time being very stable. But the major drawback of this configuration is that it cannot be used above certain height (maximum reachable height = L0 + L1 + X + L9 + L5). One such typical distorted parallelogram configuration is shown in figure 6.



Figure 6: Parameters in Parallelogram (distorted) configuration

In this configuration since q7 = 0, therefore the required parameters to reach the end effector configuration is xn, q8, q9. Some of the parameters to reach end-effector configuration is already known from inverse kinematic solution of main chain and manipulator. Those parameters are q1, q3, q4, q5,  $\alpha$ 5.

Another main configuration in which this robot can work is supported extended configuration. In this configuration the "main link" of the robot is supported by "standard link 2" of the supporting chain. The mid beam of "standard link 2" becomes tangential to "main link" beams. In this configuration angle q9 becomes constant and can be very easily determined by geometry of the robot. This configuration is designed to help robot reach higher end-effector configurations and also increase the maximum liftable load which is one of the main aim of this project. An arbitrary supported extended configuration is shown as



Figure 7: Parameters in Parallelogram extended configuration

In this configuration the required unknown parameters are q7, q8 and length xn (piston-cylinder length). While the known parameters form solution of main link and manipulator inverse kinematics are q1, q3, q4, q5,  $\alpha$ 5. The inverse kinematic of the supporting chain is derived by geometric method. While deriving inverse kinematic of supporting chain parameters q1, q3, q4, q5 and  $\alpha$ 5 are taken as input with other known constants of the robot as described above in table 1 and 3. In this derivation parameters q7, q8, q9 and xn are needed to be derived using the listed known constants and derived parameters in main chain inverse kinematic section as described above. The inverse kinematic is needed to be derived for both the extended and parallelogram cases. We need the values of frame 9 and frame 6 to find the inverse kinematics of secondary chain using geometry. The value of frame 9 with respect to ground can be found as

$${}^{0}[T]_{9} = {}^{0}[T]_{1} x^{1}[T]_{2} x^{2}[T]_{3} x^{3}[T]_{4} x^{4}[T]_{9} \qquad (5.16)$$

$${}^{0}[T]_{9} = \begin{bmatrix} \omega 9(1,1) & \omega 9(1,2) & \omega 9(1,3) & x9 \\ \omega 9(2,1) & \omega 9(2,2) & \omega 9(2,3) & y9 \\ \omega 9(3,1) & \omega 9(3,2) & \omega 9(3,3) & z9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.17)$$

The value of the above matrix is completely known in numeric terms. The values of q1, q3, q4, q5,  $\alpha$ 5 are already derived. And other known constants which are used to get  ${}^{0}[\mathbf{T}]_{9}$  are listed in table 1. The another required matrix is  ${}^{0}[\mathbf{T}]_{6}$ . And it can be derived as

$${}^{0}[T]_{6} = {}^{0}[T]_{1} x^{1}[T]_{6} \\ {}^{0}[T]_{6} = \begin{bmatrix} \omega 6(1,1) & \omega 6(1,2) & \omega 6(1,3) & x6 \\ \omega 6(2,1) & \omega 6(2,2) & \omega 6(2,3) & y6 \\ \omega 6(3,1) & \omega 6(3,2) & \omega 6(3,3) & z6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(5.18)

Value of above matrix is also completely known as the value of q1 is the only variable required to derive  ${}^{0}[T]_{6}$ . The other constants used to derive  ${}^{0}[T]_{6}$  are listed in table 2.

Distance between  ${}^{0}[\mathbf{T}]_{6}$  and  ${}^{0}[\mathbf{T}]_{9}$  is given as

$$(g, h, i) = (x9 - x6, y9 - y6, z9 - z6)$$
  
horizontaldistance =  $k = \sqrt{g^2 + h^2}$   
verticaldistance = i

#### i. <u>Case of Distorted Parallelogram (q7 = 0)</u>

In this configuration q7 is zero. Therefore, q8, q9 and xn are needed to be derived. By geometry of the proposed robot design

$$xn = \frac{2 * n * \cos(\gamma) \pm \sqrt{(2 * n * \cos(\gamma))^2 - 4 * (n^2 - x^2)}}{2}$$
(5.19)

In equation 5.19

 $xn \in \left[\left(\sqrt{5}-1\right) * x - d6 - d2, 2 * x - d2 - d6\right]$ 

(5.22)

and

$$q8 = \cos^{-1}\left(\frac{n^2 - (xn)^2 - x^2}{2 * xn * x}\right)$$
(5.20)  
$$q9 = \varepsilon 2 + \varepsilon 3 + \varepsilon 1$$
(5.21)

In the equation 5.19, 5.20 and 5.21 m = k + d2 + d6

$$n = \sqrt{k^2 + i^2}$$
(5.23)  
(5.24)

$$u = \sqrt{m^2 + i^2}$$
(5.24)  

$$\gamma = \cos^{-1}\left(\frac{k}{\sqrt{k^2 + i^2}}\right) = \sin^{-1}\left(\frac{i}{\sqrt{k^2 + i^2}}\right)$$
(5.25)

$$\varepsilon 1 = \cos^{-1}\left(\frac{3 * x^2 + u^2}{4 * x * u}\right)$$
 (5.26)

$$\varepsilon 2 = \cos^{-1}\left(\frac{n^2 + u^2 - (d6 + d2)^2}{2 * u * n}\right)$$
(5.27)

$$\varepsilon 3 = \cos^{-1}\left(\frac{n^2 + x^2 - (xn)^2}{2 * x * n}\right)$$
(5.28)

#### ii. <u>Case of Extended configuration (q9 = known constant)</u>

In this case q9 will be used as a known constant. While q7, q8, xn are needed to be derived. By geometry of the proposed robot design

$$xn = \sqrt{x^2 + n^2 - 2 * x * n * \cos(\varepsilon 5)}$$
 (5.29)

Here  $xn \in [(\sqrt{5}-1) * x - d6 - d2, 2 * x - d2 - d6]$ 

$$q7 = \gamma + \varepsilon 4$$
(5.30)  
$$q8 = \varepsilon 4 + \varepsilon 5$$
(5.31)

Here in the above equations

$$\gamma = \cos^{-1}\left(\frac{k}{\sqrt{k^2 + i^2}}\right) = \sin^{-1}\left(\frac{i}{\sqrt{k^2 + i^2}}\right)$$
(5.32)  
$$\varphi = \cos^{-1}\left(\frac{m}{\sqrt{m^2 + i^2}}\right) - \varepsilon 1$$
(5.33)

$$\varepsilon 5 = \gamma - q9 - \varphi \tag{5.34}$$

$$\varepsilon 4 = \cos^{-1}\left(\frac{(xn)^2 + n^2 - x^2}{2 * xn * n}\right)$$
(5.35)

k, i, m, n, ɛl from previous "distorted parallelogram case (case I)"

### VI. Structural Analysis

In structural analysis two aspects of the robot are needed to be studied. First part is static structural analysis of robot in which structural strength of robot in various configurations are calculated. And in another partthe required acceleration or force required to manipulate the robot is being calculated. This can be very helpful in determining the required actuator strength of the robot.

For this analysis of the robot, we need to assign values to the constant parameters of the robot. The assumed values and specifications are listed below in table 5, table 6 and table 7.

Parameters	ters Assigned Values	
LO	10 mm	
L1	133 mm	
х	670 mm	
L4	66 mm	
L5	95 mm	
d2	53 mm	
d6	33 mm	
10	66 mm	

 Table 5: Assigned ParametersValues

Table 0. Link Properties				
Link	Weight (in Kg)	Density (Kg/m^3)	Material	
Ground Plate	9.221	7850	AISI 4130 Steel	
Base Plate	24.622	7850	AISI 4130 Steel	
Standard Link 1 & 2	12.323	7850	AISI 4130 Steel	
Main Link	22.124	7850	AISI 4130 Steel	
Roll Plate	2.122	2700	Aluminium Alloy 6061-T6 (SS)	
Gripper Plate	1.741	2700	Aluminium Alloy 6061-T6 (SS)	
Gripper Jaw	1.208	2700	Aluminium Alloy 6061-T6 (SS) with Silicon Rubber friction Plate	
Cylinder Link	8.370	7850	AISI 4130 Steel	
Piston Link	5.748	7850	AISI 4130 Steel	

Table 6: Link Properties

Table 7: Material Proper	ties for Structural Analysis
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$\frac{Material}{Properties} →$	AISI 4130 Steel	Aluminium Alloy 6061-T6 (SS)	Silicon Rubber
Youngs Modulus ( in GPa)	205	68.9	0.05
Poisson's Ratio	0.29	0.33	0.47
Tensile Yield strength (in MPa)	435	276	5.5
Tensile Ultimate Strength (in MPa)	670	310	

With the help of table 6 we can figure out that the total weight of the robot is 101.03 kg. These parameters can be changed for further optimization of the design. Thus, our proposed robot can be scaled up or scaled down according to requirements.

# Static Structural Analysis

Static structural analysis is the first analysis that is required to test the structural strength of the robot in both of the configurations. The robot was analysed in both the configuration with varying loads. In the analysis we have found a sweet spot for maximum lifting capacity of the robot. **And that turns out to be 150 kg**. Figure 8 and 9 shows application of 150 kg load in vertically downward direction on gripper jaws in both the cases on the gripper jaws.



Figure 8: Application of load in Parallelogram Configuration



Figure 9: Application of load in Extended Configuration

The result of the above analysis is shown in following sub-sections. a). Total Deformation

Maximum Deformation in parallelogram position is minimal approx. 4mm at gripper teeth.



Figure 10

In second position i.e. when main link is supported on standard link 2. This is shown in figure 11.



Figure 11

# b). Equivalent Stresses

In both position 1 maximum stress generate at linkage of piston and cylinder i.e. 89.883 MPa which is far less than yield strength of material used i.e. AISI 4130 Steel (435 MPa). Maximum stress at this point due to support it provide to lift weight and act like fixed point of cantilever beam.



Figure 12

In second position maximum equivalent stress generated is at joint of baseplate and standard link i.e. 228.73 MPa. Majority of stress is compressive stress due to self-weight and load applied.



Figure 13

# c). Maximum Principal Stresses

The result for maximum and minimum principal stresses are shown in image 14 and image 15 for both the configurations.



Figure 14



Figure 15

# d). Factor of safety

The factor of safety for a link is the ratio of ultimate strength of material to the actual working stress or maximum permissible strength when in use. The factor of safety is calculated for the purpose to know whether structure is able to bear 150kg load without failure or not. The results of factor of safety looks quite promising in

this scenario. In parallelogram configuration the minimum factor of safety is 4.83 while in more bizarre extended configuration the minimum factor of safety is 1.90. The results are being shown in image 16 and image 17.



Figure 16



Figure 17

# **Actuator Motion Analysis**

In this analysis we aim to study required velocity and acceleration of each joint to move from one endeffector configuration to another. During this analysis we calculated required velocity and acceleration for empty robot with no load in normal gravity of  $9.80665 \text{ m/s}^2$ . The initial configuration of the robot is parallelogram configuration and the final configuration is extended configuration. Both of the starting and final configuration with a mid-configuration is shown in image 18, 19 and 20. The entire movement takes 10 second to complete. In this analysis secondary support for the robot has been removed. This is achieved by removing extrusion from cylinder link. The weight of the cylinder link is adjusted to original weight by increasing its density in simulation environment. The support has been removed to calculate the actuator velocity, acceleration and force without any secondary support. Thus, the required actuator force to move the robot from one configuration to another will usually be less than the result of this analysis in zero load configuration.



Figure 18: Starting Configuration at time 0.0 seconds



Figure 19:Mid Configuration at 5.00 seconds



Figure 20: Final Configuration at 10.00 seconds

The velocity acceleration and position graph of the actuators over the entire timeline from zero to tenth seconds is shown in figure 21 to figure 29. The force and torque required for each actuator can be calculated by multiplying acceleration to manipulated weight (body weight with load on robot). Thus, following graphs can be used to select or design the actuators for the proposed robot in manufacturing phase. The robot has nine actuators excluding gripper jaws actuation system. Eight of these actuators are used to manipulate rotary joints. While only one very large hydraulic actuator (in figure 26) is used to manipulate prismatic hydraulic joint between piston and cylinder link in supporting chain.



Figure 21: Ground and base plate Joint at frame 1



Figure 22: Base Plate and Standard Link 1 joint at frame 3



Figure 23: Standard link 1 and Main link joint at frame 4



Figure 24: Main Link and Standard Link 2 joint at frame 9



Figure 25: Standard Link 2 and Piston link joint at frame 8



Figure 26: Hydraulic Joint between Piston and Cylinder Link

![](_page_16_Figure_3.jpeg)

Figure 27: Cylinder Link and Base Plate joint at frame 7

![](_page_16_Figure_5.jpeg)

Figure 28: Main Link and Roll Plate joint at frame 5

![](_page_16_Figure_7.jpeg)

Figure 29: Roll Plate and Gripper Plate joint at frame {Tool}

### VII. Conclusion

a) Through this robot design we have tried to counter many problems of normal serial link manipulators. Our proposed design is strong enough to lift 150 kg of weight while its own body is of 101.03 kg thus achieving a maximum lifting capacity to body weight ratio of more than one.

b) The proposed robot also has easy manipulation as shown in inverse kinematics section. The inverse kinematics of this robot can be achieved by analytical and geometrical methods. Which makes it very easy for a controller to control.

c) The required actuator strength for most of the actuators in this robot is also low. But the number of required actuators is more. Thus, one can say that a greater number of low strength actuator is required to manipulate this robot.

We want to present this robot as a future platform for development of parallel link manipulators. We call this proposed robot design as "Parallelo Arm Robot". Many future developments are required for this robot to become a reality. We as a group would be happy to see future work being done on this robot platform.

![](_page_17_Figure_6.jpeg)

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