Optimization of Free-form Surfaces

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Abstract: Optimum machining parameters are of great concern in manufacturing environments, where economy of machining operation plays a key role in competitiveness in the market. The surface finish and material removal rate have been identified as quality and quantity attributes and efforts have been made to bridge the gap between them. The present study highlights the optimization of CNC milling process parameters (i.e Feed Rate, Spindle Speed and Side Step) in order to maximize the material removal rate and to minimize the surface roughness such that these multi-criterions could be fulfilled simultaneously to the expected level. The effect of these selected parameters on the above quality and productivity attributes were investigated using Response Surface Methodology (RSM) with Central Composite design of experiment. The effect of these parameters were also studied for two different profiles (inclined surface and free form 3D profile) and models for MRR and Surface Roughness were predicted for these profiles.

Keywords: Material removal rate (MRR), Surface Roughness, Multi Objective Optimization, Response Surface, Pareto Chart.

I. INTRODUCTION

1.1 Nature of Problem

Nowadays due to competitive nature of market, selection of proper cutting parameters has become a great concern for manufacturing industries as they have to consistently respond to the customer demand, ensuring high productivity without compromising with the quality. Productivity can be interpreted in terms of material removal rate in the machining operation and quality represents satisfactory yield in terms of product characteristics as desired by the customers of which one of the characteristics can be surface roughness. Increase in productivity results in reduction in machining time which may result in quality loss. On the contrary, an improvement in quality results in increasing machining time thereby, reducing productivity. So there is need of proper selection of machining parameters as to maximize productivity and at the same time ensuring the proper quality standard.

1.2 Previous Work

Recent investigation performed by Alauddin *et al.* [1] has revealed that when the cutting speed is increased, productivity can be maximised and, meanwhile, surface quality can be improved. According to Hasegawa *et al.* [2], surface finish can be characterised by various parameters such as average roughness (Ra), smoothening depth (Rp), root mean square (Rq) and maximum peak-to-valley height (Rt). The present study uses average roughness(Ra) for the characterisation of surface finish, since it is widely used in industry. By using factors such as cutting speed, feed rate and depth of cut, Hashmi et al. [3, 4] developed the surface roughness models and determined the cutting conditions for 190 BHN steel and Inconel 718. EI-Baradie [5] and Bandyopadhyay [6] have shown that by increasing the cutting speed, the productivity can be maximised and, at the same time, the surface quality can be improved. The present study uses average roughness for the characterisation of surface roughness. Mital and Mehta [9] have conducted a survey of the previously developed surface roughness prediction models and factors influencing the surface roughness. They have found that most of the surface roughness prediction models have been developed for steels.

1.3 Process Description

In the proposed research work, experiments were carried for prediction of model for material removal rate (MRR) and surface roughness in which the product was prepared by CNC milling operation using Response surface methodology (RSM) for design of experiments.

As we work on three factors for prediction of model for MRR and Surface roughness, instead of using a three factorial level design of experiment, we opt for central composite design method, useful in response surface methodology, for building a second order (quadratic) model for the response variable. This design of experiments take in two levels (high and low) for each factor and the experiment runs by taking median value of the two levels and the set of axial points one below and above the median value.

Now as the optimised parameter achieved for high MRR and low Surface Roughness using RSM technique may depend on the profile of the surface machined, this was further investigated using the same RSM technique

carrying out two different sets of experiment for two different profiles i.e. inclined and free form surface and separate models predicted for them were further validated leaving no hypothesis in prediction of these models.

1.4 Scope of Work

We have chosen our problem so as to provide maximum benefit to industries. Moreover, we have not yet find any research paper on such an objective. Many researchers have focused on optimization of parameters for steels and its alloys and not for Aluminium. Though the experiment concerned with surface roughness prediction model via Artificial neural network method has already been carried out but none of the work is concerned with optimization of MRR and surface roughness together to benefit industries.

II. Methodology

2.1 Introduction

One of the most important steps in any DOE is to select the design such that we have limited number of experiments and at the same time cost is also less. Factor selection and level selection is the next step which aims at choosing the levels at which we need to perform the tests. Confirmation is the final step which tells us whether the model obtained from the tests is acceptable or not.Going through various papers on parameter optimization, we have found that most important factors that affect surface roughness and material removal rate (MRR) are-

- 1. Feed Rate
- 2. Cutting Speed
- 3. Side Step/ Axial Depth/ Radial Depth

2.2 Trade Selection and Design

Response Surface Methodology: It is the design of experiment technique that explores the relationship between several explanatory variables and one or more response variables. The central theme of this method is to use set of designed experiments and obtain an optimal point. Set of designed experiment implicates use of fractional factorial experiment performing selected experiments rather than experiments for all combination of factors.

Full Factorial Design: Design in which experiments at all combinations of factors are performed. Since number of experiments to be performed in this will be very large, this is a costly technique.

Fractional Factorial Design: Design in which certain combinations of factors are experimented and analysis is done based on these experiments only. Since the numbers of experiments involved are small, this is less costly in comparison to Full Factorial design. Hence, we selected this for our research.

Fitting a polynomial model to the results of experiments is the first step of response surface methodology. An easy way to estimate a first-degree polynomial model is to use a factorial experiment or a fractional factorial designs. This is sufficient to determine which explanatory variables have an impact on the response variable(s) of interest. Once it is suspected that only significant explanatory variables are left, then a more complicated design, such as a central composite design can be implemented to estimate a second-degree polynomial model, which is still only an approximation at best.

Central Composite design contains an imbedded factorial or fractional factorial design with centre points that is augmented with a group of `star points' that allow estimation of curvature. If the distance from the centre of the design space to a factorial point is ± 1 unit for each factor, the distance from the centre of the design space to a star point is $\pm alpha$ with |alpha| > 1. The precise value of alpha depends on certain properties desired for the design and on the number of factors involved. The star points establish new extremes for the low and high settings for all factors [10].

2.3 CCD applications for Inclined Surface

2.3.1 Design of experiment plans

Step 1:Model- To fasten the machining process model was made of smaller sizes. Shape was given to upper surface only and the model was made with dimensions slightly greater than the original dimensions. This was done to ensure that the cutting speed and feed rate does not remain in transitional phase while machining the surface as the tool approaches end. SOLIDWORKS was used to make the CAD model and file was saved in IGS format.

Step 2:Procurement of material: Aluminium rod of cross section 19mm X 19mm. The rod was procured and pieces were then cut at 2cm marks by an electric cutter.

Step 3: CAM and CNC milling: Cimatron is the CAM software used to form the GM codes for the model which was made by us in Solidworks. CNC machine used is COSMOS.

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Fig 2. 1: CNC milled 15 aluminium pieces

Fig 2. 2: CNC milled aluminium piece

Step 4: Readings: For material removal rate, initial weight was taken in Chemistry laboratory in Block VI. Final weight, after milling, was also taken from same weighing machine. Total time spent by machine in giving out the final design was also noted at the site. Surface roughness was measured using Talysurf machine in Metrology lab in workshop. Three readings were taken on each specimen at different places. Ra values were taken as indicator of surface roughness.

MRR formula is as given $MRR = \frac{W_i - W_f}{\rho_s \times t}$ Density of Aluminium is found to be 2.7 gm/cm³.

Step 5:Response Surface Analysis on Minitab: Random Experiments were performed with the help of combinations generated by Minitab 15 using Central Composite Design in DOE section. The values were then input to the software and analysis of the model was done.

Step 6:Mathematical model formulation: Minitab gave a model of response surface. Terms with low P value were selected and regressed separately in Minitab to find out the quadratic model between response surface and explanatory variables. Response surface optimizer was used to find out the optimal values for our pieces to mill. Step 7:Multi-objective optimization (MOPP): Our mathematical model is to Maximize MRR and Minimize Surface roughness (SR). Matlab was used to solve this optimization problem by using Genetic Algorithm: This gave Pareto front indicating the efficient frontier where our optimal values can lie depending on the weights given to each objective function.

Step 8: Confirmation Runs: Optimum values obtained were then put to test by milling 8 new pieces again but this time all at the same parameters i.e. the parameters obtained from the solution of Matlab MOPP using GA. Step 4 was repeated to get the reading of new pieces

Step 9: Hypothesis Testing: Claim of the response optimizer was checked using hypothesis testing for unknown variance. Since we do not have variance of the population and we have sample of 8 pieces with us we can know the variation as well mean of our sample. This will require use of Student's t test to check the claim of solution of MOPP. Outer array and inner array are summarized in Table 2.3.

Table2. 1. Outer and finiter Array of DOE				
Outer Array	Coolant	Supply kept constant		
		Coolant used: Karpol		
	Power Supply	Assumed constant		
	Operating Temperature	40° Centigrade		
	Tool	MAFORD 8mm diameter ball		
		mill with 4 flutes		
	Vice	Constant pressure on pieces		
Inner Array	Feed Rate	Low: 800 mm/min		
		High: 1000mm/min		
	Cutting Speed	Low: 4000RPM		
		High: 5000RPM		
	Side Step	Low: 0.15		
		High: 0.25		

Table2. 1:	Outer	and	Inner	Array	of DOE
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2.3.2 Data Analysis

Denoting:- Feed Rate by A, Cutting Speed by B, Side Step by C. Their respective square terms by A^2 , B^2 and C^2 and their respective interactions are given by AB, AC and BC. Data related to observations of initial and final weights is as given in Table 3.5. Observed time includes time for roughing and semi finishing along with final finishing.

Table2. 2: Observations from various experiments for inclined surface setup					
Run Order	Initial Weight	Final Weight	Time to machine	MRR(mm ³ /second)	
	(gm)	(gm)	(s)		
1	20.784	17.8133	241	35.1242	
2	20.600	17.7124	348	23.6338	
3	20.189	17.7281	302	23.2134	
4	19.616	17.3942	178	37.3141	
5	20.170	17.8922	262	24.7843	
6	20.100	17.8140	250	26.0218	
7	20.547	18.3166	254	24.9839	
8	20.975	18.6240	317	21.1223	
9	20.721	18.2167	322	22.1524	
10	20.095	17.8335	292	22.0832	
11	20.764	18.3795	252	26.9584	
12	19.527	16.8922	213	35.2550	
13	21.150	18.8176	334	19.9237	
14	20.589	17.5903	255	33.5852	
15	20.397	17.6872	272	28.3508	

Final model achieved is

StdOrder	RunOrder	Feed Rate	Cutting	Side	MRR	Surface
		(mm/min)	Speed	Step	(mm ³ /second)	Roughness
			(RPM)	(mm)		(µm)
6	1	1000	4000	0.25	35.1242	0.5251
3	2	800	5000	0.15	23.6338	0.4622
4	3	960	5000	0.15	23.2134	0.4986
8	4	1000	5000	0.25	37.3141	0.5704
12	5	900	5340	0.20	24.7843	0.5803
13	6	900	4500	0.12	26.0218	0.4236
5	7	800	4000	0.25	24.9839	0.4644
11	8	900	3660	0.20	21.1223	0.5081
9	9	732	4500	0.20	22.1524	0.4355
2	10	1000	4000	0.15	22.0832	0.4931
7	11	800	5000	0.25	26.9584	0.5000
14	12	900	4500	0.28	35.2550	0.5107
1	13	800	4000	0.15	19.9237	0.4422
10	14	1070	4500	0.20	33.5852	0.5163
15	15	900	4500	0.20	28.3508	0.4967

$$\label{eq:MRR} \begin{split} MRR &= 121 - 0.014*A + 0.0768*B - 443*C - 0.000015*A^2 - 0.000008*B^2 + 336*C^2 - 0.000004*A*B - 0.0042*A*C + 0.449*B*C \end{split}$$

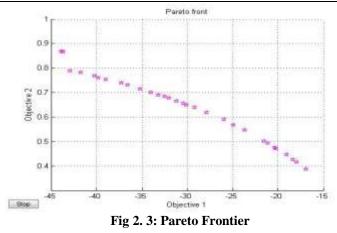
 $SR = 1.04 + 0.0013596*A - 0.0006289*B + 0.230 *C -0.00000072*A^2 +0.00000007 *B^2 - 4.17*C^2 +0.0002573*A*C+0.0009825*B*C$

2.3.3 Response optimizer

Any optimization problem first requires description of the variables and then mathematical formulation of the problem. Mathematical Formulation is:

MINIMIZE (-MRR, SR)

Subjected to: $MRR > 10mm^3$ /second; SR < 1 micrometer (Result has been in next section 3.2.3) Using Genetic Algorithm: GA gave Pareto frontier as shown in figure 2.8(Using MATLAB)



2.4 CCD application for freeform surface

Freeform surface is used in industries in order to create aesthetics that also perform functions i.e. car outer bodies or consumer product outer forms or technical surfaces for components such as gas turbine blades and other aerodynamic components.

There are two basic methods for creating freeform surface in CAD softwares. The first begins with construction curves from which the 3D surface is then swept (section along guide rail) or meshed (lofted) through. This is the way we have created our free form surface as our model consists of two skew lines at some distance which are then joined through surface. Other method is direct creation of the surface with manipulation of the surface poles/control points.

2.4.1 Design of experiment plans

All the steps are same as described in section 3.4.1. There are slight changes in values chosen for inner arrays, spindle speed now varies in range 3000 to 5000 rpm.





Fig 2.4: CNC milled freeform surface

2.4.2 Data analysis.

StdOrder	RunOrder	Feed Rate	Cutting	Side	MRR	Surface
		(mm/min)	Speed	Step	(mm ³ /second)	Roughness
			(RPM)	(mm)		(µm)
14	1	1000	4000	0.20	25.0051	1.79600
8	2	1500	5000	0.20	27.1917	1.92000
9	3	500	4000	0.15	14.4359	2.03850
3	4	500	5000	0.10	8.5605	0.82450
1	5	500	3000	0.10	8.6255	1.35425
7	6	500	5000	0.20	20.9251	2.72620
12	7	1000	5000	0.15	22.3566	2.64820
10	8	1500	4000	0.15	28.3934	1.34667
6	9	1500	3000	0.20	34.1424	3.34160
13	10	1000	4000	0.10	16.1900	1.00750
11	11	1000	3000	0.15	23.0613	1.25700

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					optimizatio	n oj ji ce jerm
4	12	1500	5000	0.10	24.1027	2.58575
5	13	500	3000	0.20	16.4125	2.97625
15	14	1000	4000	0.15	19.8583	2.72620
2	15	1500	3000	0.10	22.3228	2.33900

Table2.4: Responses of surface milling setup

Final model achieved is

MRR= -26.6+0.0313*A+318*C-0.000002*A*B-0.0262*A*C-543*C² SR= -0.00444*B+110*C-0.00297*B*C-0.0169*A*C+0.000001*A²+0.000001*B²-247*C² These models have excluded all those values which have P values greater than 0.1.

2.4.3 Response optimizer: Mathematical Formulation is:

MINIMIZE (-MRR, SR)

Subjected to: $MRR > 10 \text{ mm}^3/\text{second}$; SR < 6 micrometer (Result has been shown in next section 3.3.3)

2.5 Confirmation analysis

2.5.1 For Freeform surface

Validation runs have observed values as given in table below. Similarly doing the validation runs for Inclined surface.

Wi(gm)	Wf(gm)	Wi-	Delta	Time(s)	MRR	SR
		Wf(gm)	$V(mm^3)$		(mm ³ /sec)	(µm)
21.344	18.7683	2.3757	6772.235	229	29.57308	1.806
21.456	18.4224	2.8336	8077.537	228	35.42779	2.096
21.109	18.4481	2.4609	7015.108	229	30.63366	2.345
21.17	19.0322	1.9378	5523.945	231	23.91318	1.7974
21.577	19.3266	2.0504	5844.926	230	25.41272	2.696
21.781	19.3357	2.2453	6400.513	228	28.07243	2.234
21.664	19.6295	1.8345	5229.475	229	22.83614	2.462

Table2, 51:	Observed	values from	validation runs
1 abic 2. 51.	Observeu	values if offi	vanuation runs

2.6 Evaluation of accuracy of CCD approach

2.6.2 For freeform surface

Null Hypothesis Ho: $\mu(MRR) = 35.87586566$ and $\mu(SR) = 4.090181947$

We have taken a sample of 8 and performed the test at the optimal parameter given by Matlab. Since the variance is unknown, we will perform t-test with 8 degrees of freedom and level of significance as 0.05 i.e. 95% confidence interval. Corresponding table of observations is shown in table given below.

Table2.6: Decision making table					
Statistics	MRR		SR		
Sample mean	27.73639427		2.1685		
Sample variance	4.1058		0.105745189		
Mean of model	35.8758		2.090182		
to	-1.982411564		1.959521317		
t(8,0.025)[18]	2.306		2.306		
Decision	Accept	Null	Accept	Null	
	hypothesis		Hypothesis		

Similarly for Inclined surface null hypothesis is accepted for both MRR and SR.

III. Analysis and results

As in the proposed research we carried out different sets of experiment for two different profiles i.e. inclined surface and free form surface and predicted model for each one of them and studied the effect of each selected parameter on MRR and Surface roughness and finally got the optimized parameters of milling for each profile.

Table 3. 1Approach for analysis				
Objective	Steps			
Analysis of Model 1. MRR 2. Surface Roughness	 Percentage contribution to the model of each variable Contour and surface plot of each response variable 			
Optimized parameters	 Response Optimizer Matlab : Multi Objective Optimization and Pareto Chart Formation 			

3.1 Approaches for Statistical Analysis:

We have shown our approach in the first part and for all other sections reader is assumed to suppose that the same approach is followed.

3.2 Inclined Surface

3.2.1 Surface Roughness

All three selected parameters (Feed Rate, Spindle Speed, Side Step) have significant effect on the surface roughness, even square of all these factors contribute to the surface roughness.(as seen in pie chart).

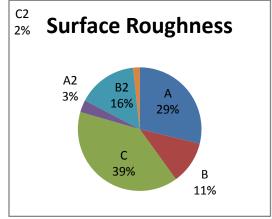


Fig 3. 1: Percentage contribution to model of SR

As expected surface roughness increases with increase in Feed rate, Spindle speed and Side step, with side step contributing the most. As seen from the p value for the surface roughness the only significant interaction is between feed rate and side step. R-square between the measured Surface roughness values and the parameters is approximately 98.3% whereas R-square Adjusted is 95.6%.

Estimated Regression Coefficients for Surface Roughness

Term	Coef	P value
Constant	1.03630	0.191
Feed Rate	0.00136	0.143
Cutting Speed	-0.00063	0.010
Side Step	0.22962	0.842
Feed Rate*Feed Rate	-0.00000	0.121
Cutting Speed*Cutting Speed	0.00000	0.007
Side Step*Side Step	-4.53891	0.044
Feed Rate*Cutting Speed	-0.00000	0.992
Feed Rate*Side Step	0.00098	0.229
Cutting Speed*Side Step	0.00026	0.118

R-Sq = 98.38% R-Sq(pred) = 86.74% R-Sq(adj) = 95.46%

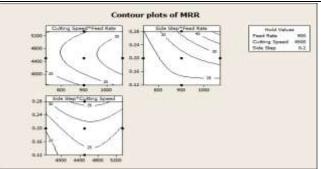


Fig 3. 2: Contour plots of MRR for inclined surface

3.2.2 MRR

In the case of inclined surface, <u>Side Step and Feed Rate</u> has significant effect on the MRR while the cutting speed has no such significant effect on it (as can be seen in the pie chart). The only significant interaction term in the model is product of <u>Feed Rate and Side Step</u>.

As expected and also confirmed by the model, MRR increases with increase in Feed Rate and Side Step while no such correlation exists with spindle speed. Rate at which tool moves over the surface is dependent on feed rate and side step only. Since the rate at which tool moves over the surface defines the MRR, it should not depend on cutting speed.

The R-square for this model is approximately 97.72% while R-sq. Adjusted as 93.61%.

As can be seen by the P value which represents the truthfulness of the result, factors having significant effect on the MRR(lower the value of P more truthful the data is) are side step, feed rate, feed rate*side step, Cutting speed*Cutting speed.

R-Sq = 97.72% R-Sq(pred) = 74.48% R-Sq(adj) = 93.61%

3.2.3 Optimized model

For the inclined surface the optimized milling parameter for maximum MRR and minimum Surface Roughness are-:

Tuble 5. 2. Optimum Vulues for menned surface		
Feed Rate	1070mm/min	
Spindle Speed	4372.7RPM	
Side Step	0.280	
MRR	47.0550	
S.R	0.5333	

Table 3. 2: Optimum Values for Inclined surface

The optimum feed rate and side step comes at the highest value within the specified range. Since optimum values occur at boundaries, our model could have been improved by selecting wider range of feed rates and side step.

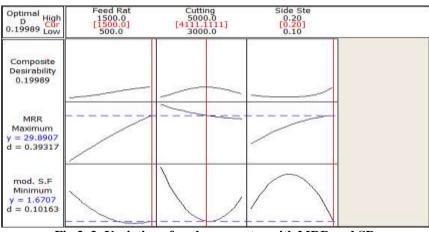


Fig 3. 3: Variation of each parameter with MRR and SR

3.3 Free Form Surface:

3.3.1 MRR

For the free form surface, MRR does not show dependence on cutting speed while showing significant dependence on Feed rate and side step and increases with increase in feed rate and side step and with negligible dependence on interaction among parameters.

The R-square for the model is approximately 96.20% and R-sq adjusted was 89.37%

As is visible from the contour plots, MRR is not depending on cutting speed or rather its dependence is negligible, while it depends mostly on federate and cutting speed.

3.3.2 Surface Roughness

In the case of free form surface, Surface roughness show dependence on Feed rate, cutting speed, side step and also on the interaction among factors like feed rate and side step.

The R-sq for the model is 96.32% while R-sq adjusted is 89.69%

3.3.3 Optimized model

Table 3. 3: Optimum values for freeform su	urface milling
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Feed Rate	1139
Spindle Speed	3984
Side Step	0.20
MRR	35.8751
SR	4.6707

The optimum side step comes at the highest value within the specified range, while feed rate and cutting speed takes the value in the mid range. Had the range of side step been wider, we might have got some interesting trend.

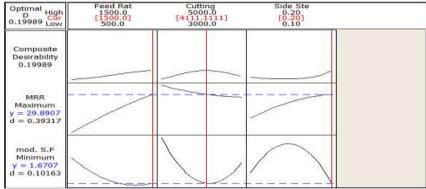


Fig 3. 1: Variation of each parameter with MRR and SR

3.2 Comparison between two Profiles

No such major difference is observed when two different profiles were inspected independently though the coefficients to the factors have changed. MRR value obtained for inclined surface is much more than the value obtained for freeform, mainly due to the amount of material removed is more for the inclined surface for the same time. Surface roughness trend is exactly reverse of MRR more for free form 3D surface than inclined surface.

Table 3. 4: Summary of results			
Response variable	Inclined Surface (Significant Factors affecting response variable)	Free Form Surface (Significant Factors affecting response variable)	
MRR	Feed rate, Side step Interaction term – feed rate and side step.	Feed rate, Side step Interaction term- No term	
Surface Roughness	Feed rate, Cutting Speed, Side step Interaction term- No term	Feed rate, Cutting Speed, Side step. Interaction term-Feed rate and side step.	

IV. Conclusion

After developing the model we can easily study and compare the results for MRR and surface roughness. Results show the significant dependence of MRR on feed rate and side step, increasing with increase in these parameters (FR and SS) and no such dependence on cutting speed was observed. Surface roughness show dependence on all the three factors, non linear relationship as indicated by the model (for example dependence on side step somewhat quadratic in nature) and also the interaction factors playing a major role in its correct prediction. The models have been validated by again machining the surface on the found optimized parameters than running the null test hypothesis using Student t-test. Null hypothesis has been accepted in four models.

Based on the model developed by us the time required for machining given the quality of the surface roughness required can be predicted before machining and then the optimized parameter can be generated for machining, minimizing time (i.e. maximizing (MRR)) without compromising the surface quality. This would be greatly helpful to the manufacturing industries to sustain in this competitive environment.

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