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Prediction of the Crushing Strength of Sda-Clay Fired Bricks Using Simplex Theory

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Abstract: *Waste materials have been found useful in the production of new building materials. The addition of sawdust ash (SDA) an industrial waste generated by the timber industry, to burnt bricks raw mixes leads to the production of bricks of lighter weight and higher refractive properties. It also solves the problem of waste disposal and management.*

However, the development of a model for the prediction of the crushing strength of the new product (ie SDA-clay burnt bricks) is a critical factor in the production and use of SDA-clay burnt bricks as cheap and affordable building material. This article presents a response function based on simplex theory, for the prediction of the crushing strength of SDA- clay burnt bricks when given a specific mix proportion of its components, and vice versa. The results obtained from the simplex response function developed, are in total agreement with the corresponding crushing strength test results. The use and application of the response function will eliminate the tedious, expensive and time consuming trial- mix design methods.

Keywords*: Clay, Sawdust Ash (SDA), Burnt bricks, Simplex theory, Response function and Crushing strengths.*

I. Intoduction

Sawdust is an industrial waste generated by the timber. It causes nuisance both to the health and environment when not properly disposed (Elinwa, 2006). The general trend for the industrial waste or by-product is to develop alternative ways for their exploitation in order to eliminate cost of disposal, and avoid soil and water contamination (Katsioti et al, 2005).

Various research works have been carried out on the use of the ash from sawdust in the production of clay bricks. Significant among them is the research by Elinwa (2006) on the effect of saw dust on clay bricks. He concluded that though the crushing strength of clay bricks decreases as the percentage by weight of sawdust ash increases, bricks of reasonable strength can be produced. In his own work, Malu et al (2007) established that a mixture of kaolin and sawdust in the ratio of 1:1 by volume was suitable for the production of good quality insulating bricks. Okumade (2008) was able to produce bricks of compressive strength of up to 18.48 MN/m^2 when sawdust and wood ash were mixed with clay.

Hence, there is need to determine the optimum amount of sawdust ash required to produce clay bricks of optimum strength. This paper presents a response function based on simplex theory for the prediction of the crushing strength clay bricks incorporating a specified amount of sawdust ash. The response function can also be used to predict various mix proportions that can give-SDA clay bricks of a particular crushing strength.

II. Materials

2.1 Sawdust

Sawdust, a waste-product from the timber industry, was collected from timber sawmills in Owerri. The predominant species of wood milled by the sawmills, are Iroko, Mahogamy, Abura, Afara and Obeche. The sawdust used in the production of the SDA clay fired bricks tested, was obtained by open burning in an open drum. It was then, ground to very fine powder using mortar and pestle, and sieved with a 212 um British test sieve. The characteristics of the SDA are given in Tables 1 and 2.

2.2 CLAY

The clay is obtained from Isu Ihime locality, which has a long standing history for brick making, and so it is called Isu clay. It was crushed and soaked in water for 2 days in order to remove unwanted organic materials. This was followed by drying and subsequent grinding to finer particles. The characteristics of the clay are given in Tables 1 and 2.

Table 1 :Physical properties of Clay and SDA

Parameter	Material	
	Clay	SDA
Specific Gravity	2.45	2.29
Bulk Density(Kg/m^3)	1610	831
Moisture Contents (%)	0.63	0.37
Loss on Ignition	-	4.67

Source: Elinwa (2006)

Table 2: Chemical properties of Clay and SDA

Constituents	Materials Percentage by weight	
	Clay	Sawdust
S_iO_2	48.5	67.2
Al_2O_3	16.4	4.1
Fe_2O_3	4.1	2.3
CaO	0.5	10.0
MgO	0.7	5.8
Na_2O	-	0.1
K_2O	1.4	0.1
SO_2	0.01	0.5
P_2O_5	0.01	0.5
MnO	0.02	0.01
NiO	0.1	0.05
Cr_2O_3	0.1	0.02
TiO_2	0.8	0.2
MO	0.1	0.04

Source: Elinwa (2006)

2.3 WATER

Water is the third material used in the production of the SDA-clay burnt bricks.

III. Methods

Two methods, namely, analytical and laboratory methods were adopted in the development of response function for the prediction of the crushing strength of SDA – clay bricks.

3.1ANALYTICAL METHOD

The analytical method is based on the simplex theory for experimental mixtures whose properties depend on the proportions of the constituents. Aggrawal (2002) described experimental mixtures as mixtures produced by mixing of various components required to make a different component. In this work, the raw mixture for SDA – clay bricks, consists of three components, viz, clay, SDA and water.

Let the total number of components in the mixture be 'q' and the proportion of the ith component in the mixture be such that:

$$X_i \geq 0 \ (i = 1, 2,3,...q) \quad \text{.........} (1)$$

If a unit quantity of the mixture is considered, then for a 3- component mixture,

$$X_1 + X_2 + X_3 = 1 \quad \text{.........}(2)$$

Eqn (2) gives a factor space of (q-1) dimension simplex. Writing Eqn (2) in a compact form yields:

$$\sum_{i=1}^{q} X_i = 1 \quad \text{.........}(3)$$

The number of coefficients, K, of the second degree polynomial can be obtained from:

$$K = \frac{(q + m -1)!}{(q-1)!\, m!} \quad \text{.........}(4)$$

where q = number of components

m = degree of the polynomial

3.1.2 SIMPLEX EQUATION

The general equation of the simplex function is given by Scheffe's (1958) as:

$$Y = b_0 + \sum b_i X_i + \sum b_{ii} X_i^2 + \sum b_{ij} X_i X_j + \sum X_{iij} X_i^2 X_j + \sum b_{ijj} X_i X_j^2 + \sum b_{ijk} X_i X_j X_k \quad \text{.......(5)}$$

Since SDA clay bricks have three components and the degree of the polynomial is two, then, the number of coefficients, K, of the polynomial is as follows:

$$K = \frac{(3+2-1)!}{(3-1)!} = 6 \quad \text{.......(6)}$$

For the SDA clay brick with three components, the response given by Eqn (5) becomes:

$$F(x) = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 \quad \text{.......(7)}$$

Eqn (7) is supposed to have six coefficients (i.e K = 6) as predicted from Eqn (6)

If Eqn (2) is multiplied by X_i, it becomes:

$$X_iX_1 + X_iX_2 + X_iX_3 + \ldots + X_i^2 + \ldots + X_iX_n = X_i \quad \text{.......(8)}$$

Also, multiplying Eqn (3) by b_0 gives the following product:

$$\sum X_i b_o = b_0 \quad \text{.......(9)}$$

From Eqn (8), Eqns (10) – (12) are obtained for a three component mixture

$$X_1^2 = X_1 - X_1X_2 - X_1X_3 \quad \text{.......(10)}$$

$$X_2^2 = X_2 - X_1X_2 - X_2X_3 \quad \text{.......(11)}$$

$$X_3^2 = X_3 - X_1X_3 - X_2X_3 \quad \text{.......(12)}$$

Expanding Eqn (9) yields Eqn (13),

$$b_0X_1 + b_0X_2 + b_0X_2 + b_0X_3 = b_0 \quad \text{.......(13)}$$

Substituting Eqn (10) – (13) into Eqn (5) gives:

$$F(x) = b_0X_1 + b_0X_2 + b_0X_3 + b_1X_1 + b_2X_2 + b_3X_3 + b_{11}(X_1 - X_1X_2 - X_1X_3) + b_{22}(X_2 - X_1X_2 - X_2X_3) + b_{33}(X_3 - X_1X_3 - X_2X_3) + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 \quad \text{.......(14)}$$

Rearranging Eqn (14) yields:

$$F(x) = X_1(b_0 + b_1 + b_{11}) + X_2(b_0 + b_2 + b_{22}) + X_3(b_0 + b_3 + b_{33}) + X_1X_2(b_{12} - b_{11} - b_{22}) + X_1X_3(b_{13} - b_{11} - b_{33}) + X_{23}(b_{23} - b_{22} - b_{23}) \quad \text{.......(15)}$$

Let the summation of the constants (given in brackets), be denoted as follows:

$$\propto_1 = b_0 + b_1 + b_{11} \quad \text{.......(16)}$$

$$\propto_2 = b_0 + b_2 + b_{22} \quad \text{.......(17)}$$

$$\propto_3 = b_0 + b_3 + b_{33} \quad \text{.......(18)}$$

$$\propto_{12} = b_{12} - b_{11} - b_{22} \quad \text{.......(19)}$$

$$\propto_{13} = b_{13} - b_{11} - b_{33} \quad \text{.......(20)}$$

$$\propto_{23} = b_{23} - b_{22} - b_{33} \quad \text{.......(21)}$$

Substituting Eqns (16) – (21) into Eqn (15), gives:

$$F(x) = \alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 + \alpha_{12}X_1X_2 + \alpha_{13}X_1X_3 + \alpha_{23}X_2X_3 \quad \text{.......(22)}$$

Eqn (22) can be written in a compact form as follows:

$$F(x) = \sum_{1 \le j \le q}^{3} \alpha_i X_i + \sum_{1 \le i \le j < q} \alpha_{ij} X_i X_j \quad \text{.......(23)}$$

The Eqn (23) is the response to the pure component, 'i' and the binary mixture "ij".

Thus, the response, y_i for the pure component and the response, y_i for the binary mixture components are respectively given as :

$$y_i = \sum \alpha_i X_i \quad \text{.......(24)}$$

and $y_{ij} = \sum \alpha_i X_i + \sum \alpha_{ij} X_i X_j$ ……………..…………………...……………(25)

3.1.3 COEFFICIENTS OF THE (3,2) SIMPLEX RESPONSE FUNCTION

For a three component mixture in a two dimensional factor space, the vertices of a simplex lattice represents pure components while the mid points of the borderlines connecting the vertices represents the binary components.

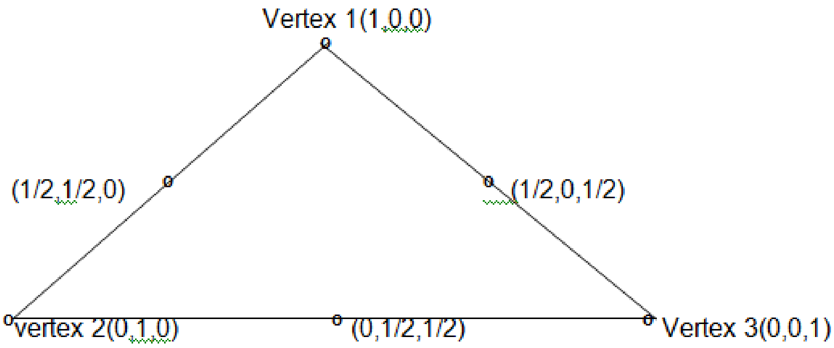

Fig 1: Simplex lattice for 2-dimensional factor space

At the first principal co-ordinate (i.e. at vertex 1), the quantity, $X_1 = 1$ while the other quantities X_2 and X_3 are all equal to zero. For the second vertex, the quantity, $X_2 = 1$ while all other quantities X_1 and X_3 are all equal to zero. Let $n = \sum \alpha_i X_i$

Substituting $X_1 = 1$ into Eqn (24) yields:

$n_1 = \alpha_1$ ……………………………………………………………………………………….(26a)

Also, substituting $X_2 = 1$ into Eqn (24) gives:

$n_2 = \alpha_2$ ……………………………………………………………..……..(26b)

Similarly

$n_3 = \alpha_3$ ………………………………………………………………...………(26c)

Generally, it can be represented thus:

$n_i = \alpha_i$ ……………………………………………………………………………(27)

and

$n_j = \alpha_j$ …………………………………………………………………….…(28)

For the midpoint of the borderline connecting the vertices 1 and 2 of the factor space, $X_1 = X_2 = ½$ while $X_3 = 0$.

Also, for the midpoint of the borderline connecting the vertices 1 and 3, $X_1 = X_3 = ½$ while $X_2 = 0$

And for the midpoint of the borderline connecting the vertices 2 and 3, $X_2 = X_3 = ½$ while $X_1 = 0$

Substituting into Eqn (25), the values of X_1, X_2 and X_3 at the midpoints of the borderline connecting the vertices 1 and 2, gives:

$$n_{12} = \frac{1}{2}\alpha_1 + \frac{1}{2}\alpha_2 + \frac{1}{4}\alpha_{12} \quad \text{……(29)}$$

Similarly

$$n_{13} = \frac{1}{2}\alpha_1 + \frac{1}{2}\alpha_3 + \frac{1}{4}\alpha_{13} \quad \text{……(30)}$$

and

$$n_{23} = \frac{1}{2}\alpha_2 + \frac{1}{2}\alpha_3 + \frac{1}{4}\alpha_{23} \quad \text{……(31)}$$

Where $n_{ij} = y_{ij}$

Generally, Eqns (29) to (31) can be given as:

$$n_{ij} = \frac{1}{2}\alpha_i + \frac{1}{2}\alpha_j + \frac{1}{4}\alpha_{ij} \quad \text{……(32)}$$

Rearranging Eqns (27), (28) and (32) gives respectively Eqn (33a), Eqn(33b) and Eqn(34)

$\alpha_i = n_i$ ……………………………………………………………………………….…(33a)

$\alpha_j = n_j$ ……………………………………………………………………...........(33b)

and

$\alpha_{ij} = 4n_{ij} - 2\alpha_i - 2\alpha_j$ ………………………………………………………….............(34)

substituting Eqns (33) into Eqn (34) yields:

$\alpha_{ij} = 4n_{ij} - 2n_i - 2n_j$ ……………………………………………………………………..(35)

Now, substituting Eqns (28) and (35) into Eqn (22) gives:

$F(x) = n_1X_1+n_2X_2+n_3X_3+X_1X_2(4n_{12}-2n_1-2n_2)$
$+ X_1X_3(4n_{13}-2n_1-2n_3)+X_2X_3(4n_{23}-2n_2-2n_3)$ …………………………….. …(36)

Expanding Eqn (36) and collecting like terms together yields:

$F(x) = n_1X_1 (1-2X_2-2X_3) + n_2X_2(1-2X_1-2X_3) + n_3X_3(1-2X_1-2X_2) + 4n_{12}X_1X_2$
$+ 4n_{13}X_1X_3+4n_{23}X_2X_3$…………………………………………...…………..…(37)

Now multiplying Eqn (2) by 2 gives Eqn (38) for a three component mixture

$2X_1 + 2X_2+2X_3 = 2$…………………………………………..…………………….(38)

Subtracting 1 from both sides of Eqn (38) yields:

$2X_1 + 2X_2+2X_3 -1 = 1$………………………………………..…….………………(39)

Simplifying further gives:

$2X_1 -1 = 1- 2X_2 – 2X_3$……………………………………………...…………….(40)

Similarly

$2X_2-1 = 1-2X_1 – 2X_3$………………………………………….……………….…(41)

$2X_3 -1 =1-2X_1 -2X_2$…………………………………………………...…………….(42)

Substituting Eqns (40) – (42) into Eqn (37) yields Eqn (43):

$$F(x) = n_1X_1(2X_1-1) + n_2X_2(2X_2-1) + n_3X_3(2X_3-1)$$
$$+ 4n_{12}X_1X_2 +4n_{13}X_1X_3 +4n_{23}X_2X_3 \quad ………………...……………(43)$$

Eqn (43) is the simplex response function for determination of the crushing strength of SDA – clay bricks consisting of three components

3.1.4:ACTUAL- PSEUDO MIX RATIOS OF COMPONENTS

Mixture components are subject to the requirement that the sum of all the components, must be equal to unity (Scheffe, 1958). This implies that the sum of the mix ratios of the three actual components of SDA clay burnt brick (i.e. clay, sawdust and water) must be equal to one. Three different trial mix ratios are prescribed for these actual components located at the vertices A_1, A_2 and A_3 of the triangular simplex lattice. The trial mix ratios are as follows:

A_1 (0.9, 0.1, 0.365), A_2 (0.8, 0.2, 0.35) and A_3 (0.75, 0.25 and 0.345).

Since the sum of the mix ratios of these actual components for each vertex is greater than one, the actual components, Z_i, must be transformed to pseudo components, X_i, using the following equation given by Scheffe's (1958)

Z = AX ………………………………………………………..(44)

From Eqn (44), Eqn (45) is obtained

X = BZ ………………………………………………………………..(45)

Where Z- = matrix of actual components
A = matrix of coefficients
X = matrix of pseudo components
B = inverse of matrix A

The actual, Z-, and Pseudo, X- component mix ratios are determined and presented in Table 3.

Table 3: Laboratory crushing strength test Result

Points of observation	Mix Nos	S_1	S_2	S_3	Response, Y	Z_1	Z_2	Z_3
n_1	Mix-1	9.5	3.2.07	3.7	Y_1	6.934	3.65	2.700
n_2	Mix-2	8.0	3.0	3.55	Y_2	5.904	1.476	2.620
n_3	Mix-3	7.0	2.0	3.50	Y_3	5.185	2.222	2.593
n_{12}	Mix-4	8.75	1.25	3.625	Y_{12}	6.422	9.17	2.661
n_{13}	Mix-5	8.25	1.75	3.60	Y_{13}	6.066	1.287	2.647

n_{23}	Mix-6	7.5	2.5	3.525	Y_{23}	5.545	1.848	2.606
c_1	Mix-C_1	8.09	1.91	3.548	Y_{c1}	5.971	1.410	2.619
c_2	Mix-C_2	0.84	1.6	3.6	Y_{c2}	6.177	1.177	2.647
c_3	Mix-C_3	8.82	1.18	3.64	Y_{c3}	6.466	8.65	2.669
c_4	Mix-C_4	8.925	1.07	3.648	Y_{c4}	6.542	7.84	2.674
c_5	Mix-C_5	7.385	2.6128	3.527	Y_{c5}	5.460	1.932	2.608
c_6	mix-C_6	7.875	2.125	3.563	Y_{c6}	5.806	1.567	2.627

3.2 LABORATORY TESTS

The test specimen used in this work, were cubic SDA – clay fired bricks each of dimension 76.2mm. They were prepared from the twelve mix ratios (given in Table 3), cured in water, dried and fired to a temperature of 600^0C as recommended by Elinwa(2006), and thereafter crushed in a crushing machine. Out of the twelve mix ratios used in the test, the first set of six mix ratios, were used to produce SDA – clay bricks for determining the responses (crushing strength) required to obtain the final simplex response functions for predicting the crushing strength of SDA clay bricks. The other set of six mix ratios were used in verifying the adequacy of the simplex response function. For each mix ratio, three SDA – clay fired bricks were tested, and average crushing strength of the bricks determined and presented in Table 4.

IV. Results And Analysis

The crushing strengths obtained from the laboratory tests are given in Table 4.

Table 4: Laboratory crushing strength test result

Points of observation	Mix Nos	Crushing Strengths (KN/mm^2)			Average crushing strength (KN/mm^2)
		Replicate 1	Replicate 2	Replicate 3	
n_1	Mix-1	28.10	28.37	28.48	28.32
n_2	Mix-2	18.32	17.84	17.93	18.03
n_3	Mix-3	14.37	14.46	14.32	14.35
n_4	Mix-4	24.43	24.77	24.81	24.67
n_5	Mix-5	20.61	20.53	20.88	20.67
n_6	Mix-6	17.59	17.68	17.58	17.62
n_7	Mix-C_1	19.20	19.05	18.77	19.01
n_8	Mix-C_2	22.26	22.25	22.23	22.25
n_9	Mix-C_3	25.39	25.03	24.84	25.08
n_{10}	Mix-C_4	25.25	25.39	25.48	25.37
N_{11}	Mix-C_5	16.13	16.15	15.99	16.09
n_{12}	mix-C_6	18.78	18.78	18.76	18.77

The final simplex response function i.e. Eqn (46) for predicting crushing strength of SDA – clay fired bricks, is obtained by substituting the first six crushing strength test results (presented in Table 4) into Eqn (43)

$$F(x) = 28.32X_1(2X_1-1) + 18.03X_2 (2X_2-1) +14.35X_3 (2X_3-1)$$
$$+ 98.68X_1X_2 + 90.28X_1X_3 + 70.48X_2X_3 \ldots\ldots (46)$$

The final response function was tested and found adequate at 95% confidence level using T-statistic and F-statistic tests. The results of these tests showed that the difference between the predicted and laboratory crushing strengths, are quite insignificant. The comparison of the predicted results in Tables 5, showed that the maximum percentage difference is 4.1%, which is negligible.

Table 5: Comparison of Laboratory and predicted crushing strength of SDA – clay fired bricks

Points of observation	Mix Nos	Laboratory results KN/mm^2(A)	Predicted results KN/mm^2 (B)	Absolute difference, (A-B)	Percentage difference, (A-B)/A
C_1	Mix-C1	19.01	18.23	0.78	4.1%
C_2	Mix-C2	22.25	22.42	0.17	0.8%
C_3	Mix-C3	25.08	24.47	0.61	2.4%
C_4	Mix-C4	25.37	25.19	0.18	0.71%
C_5	Mix-C5	16.09	16.42	0.33	2.1%
C_6	Mix-C6	18.88	19.22	0.45	2.4%

V. Conclusions

The following conclusions were drawn from the work

- The development of a response function for predicting the crushing strength of any new construction material, is a critical factor in the production and use of cheap and affordable construction materials.
- Simplex theory can be used successively to develop response function for predicting the crushing strength of building materials obtainable from a given mix proportion.The response functions can be used to predict accurately the crushing strength of SDA- clay burnt bricks when the mix proportions are known.
- Conversely, the response function can be used to predict the mix ratios of components that will yield a specified crushing strength.

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