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Amplitude Effects on Natural Convection in a Porous Enclosure having a Vertical Sidewall with Time-varying Temperature

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Abstract: *Numerical investigation of unsteady natural convection flow through a fluid-saturated porous medium in a cubic enclosure which is induced by time-periodic variations in the surface temperature of a vertical wall was considered. The governing equations were written under the assumption of Darcy-law and then solved numerically using finite difference method. The problem is analyzed for different values of amplitude a in the range $0.2 \le a \le 0.8$, the Rayleigh number, Ra=200, Period, $\tau = 0.01$, time, $0 \le t \le 0.024$. It was found that heat transfer increases with increasing the amplitude. The location of the maximum fluid temperature moves with time according to the periodically changing heated wall temperature. Two main cells rotating in opposite direction to each other were observed in the cavity for all values of the parameters considered. The amplitude of Nusselt number increases with the increase in the oscillating amplitude. All the results of the problem were presented in graphical form and discussed.*

***Keywords**: Natural convection, Porous media, Time-periodic boundary conditions, Numerical study*

Nomenclature

A'	Amplitude
a	Dimensionless amplitude
g	Acceleration Due to Gravity
K	Permeability
k_f	Thermal Conductivity of the Fluid
k_m	Effective Thermal Conductivity of the Porous Medium
Lx	Length of Box in x-Dir.
Ly	Length of Box in y-Dir.
Lz	Length of Box in z-Dir.
Nu	Nusselt Number
p	Dimensionless Pressure
p'	Pressure
Ra	Rayleigh Number
T	Dimensionless Temperature
T_o	Reference Temperature
T'	Temperature
Th'	Hot Wall Temperature
$\overline{T}h'$	Mean Hot Wall Temperature
t	Dimensionless Time
t'	Time
u	Dimensionless Component of Velocity in x-Dir.
u'	Component of Velocity in x-Dir.
v	Dimensionless Component of Velocity in y-Dir.
v'	Component of Velocity in y-Dir.
w	Dimensionless Component of Velocity in z-Dir.
w'	Component of Velocity in z-Dir.
x	Dimensionless Length in x-Dir.
x'	Length in x-Dir.
y	Dimensionless Length in y-Dir.
y'	Length in y-Dir.
z	Dimensionless Length in z-Dir.
z'	Length in z-Dir.

α_m	Thermal Diffusivity of the Porous Medium
β	Volume Coefficient of Expansion
υ	Velocity Vector
ϕ	Vector Potential
ϕ_x, ϕ_y, ϕ_z	Vector Potential Component in x,y and z Dir. Respectively
μ	Dynamic Viscosity
ν_f	Kinematic Viscosity of the Fluid
ρ	Density
ρ_f	Fluid Density
τ	Dimensionless Period
τ'	Period

I. Introduction

Convective heat transfer through porous media has been a subject of great interest for the last three decades. An upsurge in research activities in this field has been accelerated because of a broad range of applications in various disciplines, such as geophysical, thermal and insulation engineering, the modeling of packed sphere beds, the cooling of electronic systems, groundwater hydrology, chemical catalytic reactors, grain storage devices, fiber and granular insulation, petroleum reservoirs, coal combustors, and nuclear waste repositories. Since the pioneering work of Cheng and Minkowycz [1] on boundary-layer free convection from a vertical flat plate embedded in a fluid-saturated porous medium this configuration model has been progressively refined to incorporate various boundary conditions, inertial effects, conjugate heat transfer effects, layering, etc. The work of Cheng and Minkowycz [1] and Johnson and Cheng [2] were especially noteworthy as they introduced the mathematical technique of boundary-layer theory into the subject and identified similarity solutions of the governing equations. The existence and identification of similarity solutions have been central to a number of further developments, particularly in the examination of free convection resulting from the use of Darcy's law. Several comprehensive reviews and books of the literature pertinent to this area are due to Cheng [2], Bejan [3], Tien and Vafai [4], Nakayama [5] and Nield and Bejan [6].

Hossain and Pop [7] considers the unsteady free convection boundary layer flow which is induced by time-periodic variations in the surface temperature of a vertical surface embedded in a porous medium. The basic steady flow is that of a power-law distribution where the surface temperature varies as the *n*th power of the distance from the leading edge. Small-amplitude time periodic disturbances are added to this basic distribution. Both the low- and high-frequency limits are considered separately, and these are compared with a full numerical solution obtained by using the Keller-box method. Attention is restricted to the cases $n \leq 1$; when $n = 1$, the flow is locally self-similar for any prescribed frequency of modulation.

Numerical study of natural convection in a porous cavity is carried out by Nawaf H. Saeid [8]. Natural convection is induced when the bottom wall is heated and the top wall is cooled while the vertical walls are adiabatic. The heated wall is assumed to have spatial sinusoidal temperature variation about a constant mean value which is higher than the cold top wall temperature. The non-dimensional governing equations are derived based on the Darcy model. The effects of the amplitude of the bottom wall temperature variation and the heat source length on the natural convection in the cavity are investigated for Rayleigh number range 20–500. It is found that the average Nusselt number increases when the length of the heat source or the amplitude of the temperature variation increases. It is observed that the heat transfer per unit area of the heat source decreases by increasing the length of the heated segment.

Yasin Varol and Hakan [9] numerically investigates the steady natural convection flow through a fluid-saturated porous medium in a rectangular enclosure with a sinusoidal varying temperature profile on the bottom wall were conducted. All the walls of the enclosure are insulated except the bottom wall which is partially heated and cooled. The governing equations were written under the assumption of Darcy-law and then solved numerically using finite difference method. The problem is analyzed for different values of the Rayleigh number Ra in the range $10 \leq Ra \leq 1000$, aspect ratio parameter AR in the range $0.25 \leq AR \leq 1.0$ and amplitude λ of the sinusoidal temperature function in the range $0.25 \leq \lambda \leq 1.0$. It was found that heat transfer increases with increasing of amplitude λ and decreases with increasing aspect ratio AR. Multiple cells were observed in the cavity for all values of the parameters considered.

Gang Wang and Qiuwang Wang [10] studied the unsteady natural convection for the sinusoidal oscillating wall temperature on one side wall and constant average temperature on the opposing side wall. The present article is on the unsteady natural convective heat transfer in an inclined porous cavity with similar temperature boundary conditions as those of Kalabin et al. The inclined angle ϕ of the cavity is varied from 0 to

80. The flow field is modeled with the Brinkman-extended Darcy model. The combined effects of inclination angle of the enclosure and oscillation frequency of wall temperature are studied for $Ra = 103$, $Da = 10^{-3}$, $\varepsilon = 0.6$, and $Pr=1$. Some results are also obtained with the Darcy–Brinkman–Forchheimer model and Darcy's law and are compared with the present Brinkman-extended Darcy model. The maximal heat transfer rate is attained at the oscillating frequency $f = 46.7\pi$ and the inclined angle $\phi = 42.2^{\circ}$.

The problem of unsteady natural convection in a square region filled with a fluid-saturated porous medium having non-uniform internal heating and heated laterally is considered by Saleh and Hashim [11]. The heated wall surface temperature varies sinusoidally with the time about fixed mean temperature. The opposite cold wall is maintained at a constant temperature. The top and bottom horizontal walls are kept adiabatic. The flow field is modelled with the Darcy model and is solved numerically using a finite difference method. The transient solutions obtained are all periodic in time. The effect of Rayleigh number, internal heating parameters, heating amplitude and oscillating frequency on the flow and temperature field as well as the total heat generated within the convective region are presented. It was found that strong internal heating can generate significant maximum fluid temperatures above the heated wall. The location of the maximum fluid temperature moves with time according to the periodically changing heated wall temperature. The augmentation of the space-averaged temperature in the cavity strongly depends on the heating amplitude and rather insensitive to the oscillating frequency.

Mansour and Abd-Elaziz **[12]** studied The problem of double-diffusive convection in inclined triangular porous enclosures with sinusoidal variation of boundary conditions in the presence of heat source or sink was discussed numerically. The dimensionless governing equations of the problem were solved numerically by using finite difference method. The effects of governing parameters, namely, the dimensionless time parameter, various values of the inclination angle, Darcy number, the heat generation/absorption parameter, the buoyancy parameter and the amplitude wave length ratio on the streamlines, temperature and concentration contours as well as the velocity component in the x-direction at the triangle mid-section, the average Nusselt and Sherwood numbers at the bottom wall of the triangle for various values of aspect ratio were considered. The present results are validated by favorable comparisons with previously published results. All the results of the problem were presented in graphical and tabular forms and discussed.

The mean objective of this paper is to study the effect of various boundary conditions on natural convection inside porous cavities heated laterally with a sinusoidal time variation. Numerical solution based on finite difference method was employed to solve the governing equations. Some graphical results were presented to illustrate the different influences of the problem parameters on heat and fluid motion.

II. Mathematical Formulations

Consider unsteady, three-dimensional natural convection flow in a cubic region filled with a fluid-saturated porous medium (Fig.1). The co-ordinate system employed is also depicted in this figure. The top surface is held at constant cold reference temperature T_o and the bottom surface is held at constant hot temperature Th'. The temperature of the vertical surfaces varies sinusoidally in time about a mean value of hot temperature $\overline{Th}'$ with amplitude A' and period τ'. The hot wall is greater than the cold reference wall at all times, as graphically depicted in Fig.2.The fluid and porous medium properties are assumed to be constant except for the variation of density with temperature in the buoyancy term in Darcy's equation for the fluid flow (Boussinesq approximation). The convective fluid is assumed to remains a single-phase. Under these assumptions, the non-dimensional equations governing convection in a volume of porous medium are the conservation of mass, momentum(darcy's law), and energy:

$$\nabla \cdot \upsilon = 0 \tag{1}$$

$$\upsilon = -\nabla p - G + R \tag{2}$$

$$\frac{\partial T}{\partial t} + \upsilon \cdot \nabla T = \nabla^2 T \tag{3}$$

Where:

$$\left.\begin{aligned} G &= [0, 0, C1] \\ R &= [0, 0, Ra\,T] \\ C1 &= \frac{K\, Lx\, g}{\nu_f\, \alpha_m} \end{aligned}\right\} \tag{4}$$

Ra is a modified Rayleigh number for a porous medium saturated with a fluid

$$Ra = \frac{g\,\beta\,\mathrm{K}\,\Delta \mathrm{T}_O\,\mathrm{Lx}}{\nu_\mathrm{f}\,\alpha_\mathrm{m}} \qquad (5)$$

(1)-(3) are non-dimensionalized by introducing the following variables:

$$x = \frac{x'}{Lx} \quad y = \frac{y'}{Lx} \quad z = \frac{z'}{Lx} \quad u = \frac{u'}{\alpha_m / Lx} \quad v = \frac{v'}{\alpha_m / Lx}$$

$$w = \frac{w'}{\alpha_m / Lx} \quad T = \frac{T' - T_o}{\Delta T_o} \quad t = \frac{t'}{(\rho\,\mathrm{cp})_m\,\mathrm{Lx}^2 / \mathrm{k}_\mathrm{m}} \quad p = \frac{p'}{\alpha_m\,\mu_f / K} \quad a = \frac{A'}{\Delta T_o}$$

$$\tau = \frac{\tau'}{Lx^2 \Big/ \left(\frac{k_m}{(\rho\,cp)_m} \right)}$$

where the primes denote dimensional quantities.

Due to the solenoidal form of the velocity in (1), there is an alternative formulation described by Horne [13] in which a vector potential (ϕ) is introduced such that

$$\upsilon = \nabla \times \phi \qquad (6)$$

Then, Taking the curl of (2),

$$\nabla \times \upsilon = \nabla \times (-\nabla p) - \nabla \times G + \nabla \times R \qquad (7)$$

And introducing (6) yields the following set of equations:

$$\nabla^2 \phi_x = -Ra \frac{\partial \mathrm{T}}{\partial \mathrm{y}} \qquad (8)$$

$$\nabla^2 \phi_y = Ra \frac{\partial \mathrm{T}}{\partial x} \qquad (9)$$

$$\nabla^2 \phi_z = 0 \qquad (10)$$

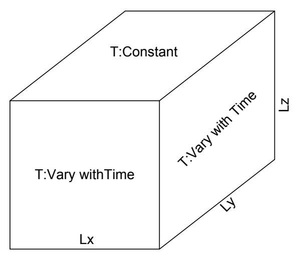

fig. (1): the cubic region boundary conditions

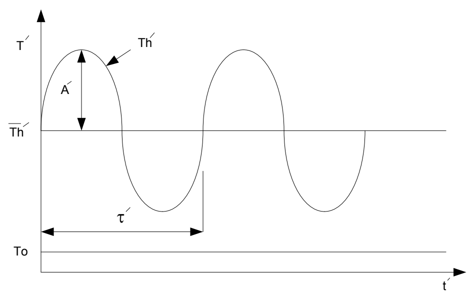

fig.(2) : periodically variation of temperature with time

2.1 Boundary Conditions

For rigid boundaries, the boundary conditions in terms of the vector potential are derived by Hirasaki [14] :

$$\phi_x = \phi_y = \frac{\partial \phi_z}{\partial z} = 0 \qquad \text{at } z = 0, \frac{Lz}{Lx} \qquad (11)$$

$$\phi_x = \frac{\partial \phi_y}{\partial y} = \phi_z = 0 \qquad \text{at } y = 0, \frac{Ly}{Lx} \qquad (12)$$

$$\frac{\partial \phi_x}{\partial x} = \phi_y = \phi_z = 0 \qquad \text{at } x = 0, 1 \qquad (13)$$

The solution to (10) with the boundary conditions for ϕ_z is $\phi_z = 0$ everywhere.
The non-dimensional thermal boundary conditions are

$$T = 0 \qquad \text{at } z = \frac{Lz}{Lx} \qquad (14)$$

$$T = 1 \qquad \text{at } z = 0 \qquad (15)$$

$$T = 1 + a\sin(2\pi t/\tau) \qquad \text{at } x = 0, 1 \qquad (16)$$

$$T = 1 + a\sin(2\pi t/\tau) \qquad \text{at } y = 0, \frac{Ly}{Lx} \qquad (17)$$

Fig.(1) illustrates these boundary conditions.

2.2 Initial Conditions
The flow is initiated by changing the top boundary temperature to T_o at time t = 0 and maintaining the lower boundary temperature at Th'. Therefore

$$\left.\begin{array}{ll} T = 1 & \text{at } z = 0 \\ T = 0 & \text{at } z = \frac{Lz}{Lx} \\ u = v = w = \phi_x = \phi_y = \phi_z = 0 & \text{everywhere} \end{array}\right\} \qquad (18)$$

III. Numerical Method and Validation

A standard finite difference numerical method is employed to solve (3),(8) and (9) subjected to conditions in (11)-(18). The parabolic part of the formulation (3) is solved using the alternating-direction implicit (ADI) method. The resulting algebraic equations are solved by the tri-diagonal matrix algorithm. The elliptic part corresponding to (8) and (9) is solved using the Successive Over-Relaxation (SOR) method. In order to validate the computation code, the previously published problems on natural convection in cavity filled with a fluid- saturated porous medium were solved. Table (1) shows the average Nusselt number on the bottom wall,

$$Nu = -B1 \int_0^{C} \int_0^{1} \left.\frac{\partial T}{\partial z}\right|_{z=0} dx\, dy \qquad (19)$$

Where: $B1 = \frac{Lz}{Ly}, C = \frac{Ly}{Lx}$

is in good agreement with the solutions reported by the literature. Equation (19) is integrated numerically for each instant of time until steady state is reached within prescribed error.

Table (1): Comparison with Previous Work

Ra	Nu			
	Constant wall temperature b.c.			
	Stamps[15]	Holst[16]	Schubert[17]	Present work
60	-	1.67	-	1.66
80	-	-	-	2.30
100	2.66	-	2.651	2.67

IV. Results and Discussion

The analysis in the undergoing numerical investigation are performed in the following domain of the associated dimensionless groups: the heating amplitude, $0.2 \le a \le 0.8$,the Rayleigh number, Ra=200,Period, τ=0.01,time, $0 \le t \le 0.024$. The results of this analysis are shown in Figs. 3,4,5,6,7,8,9,10,11,12,13 and 14.

4.1 Representative Temperature and Flow Fields

Figs. 3-12 present the time-dependent isotherms and flow patterns over the duration of the amplitude (0.2,0.4,0.6,0.8) at Ra=200 and τ=0.01. Figs. 3, 5, 7 shows the temperature evolution during one period in vertical section in the cavity (x=0.214) while Figs. 4,6,8 presents temperature contours evolution in horizontal section (z=0.35).Initially, at t=0, the walls are cold and there is no fluid motion inside the enclosure. As heating started, the fluid temperatures adjoining the hot wall rise. The fluid moves due to buoyancy force from bottom region of the cavity to the top region of the cavity as we can dedicated in the velocity vector plot in Figs. 9-12.This movement creates a clockwise circulation cell inside the left enclosure and an anticlockwise circulation cell inside the right enclosure (Figs. 9 and 11). At the very beginning, t=0.006, the isotherms are almost curvilinear as shown in Fig. 3 and the conduction or diffusion mode is dominant. For t=0.012 and t=0.024, it is observed that the thermal boundary layer develops at the hot and cold boundaries as shown in Figs. 5 and 7 implies that the convection mode is dominant and the isotherms become more curvilinear. At amplitude =0.2, the maximum temperature occurs at the heated wall. Increasing the amplitude result the maximum temperature occurs at the fluid inside the enclosure. Careful inspection of Figs 3-8 discloses that the temporal maximum fluid temperature location moves away from the heated wall appropriate with time increasing.

4.2 Effect of Amplitude on Nusselt number

The periodic oscillations of Nusselt number Nu at Ra=200 and τ=0.01 are shown in Figs. 11 and 12. It can be shown that the Nu varies with the identical values of the positive and negative amplitudes to indicate the net zero heat flux from the oscillating temperature wall to the constant temperature wall. As the amplitude increases, the Nusselt number becomes larger than zero when it is integrated in time. This suggests that the heat flux is positive from the vertical walls to the horizontal walls. Fig. 11 clearly shows that the amplitude of Nusselt number increases with the increase in the oscillating amplitude. It is seen that Nu is positive when the heat is transferred into the enclosure and negative when it is transferred from the enclosure to the environment.

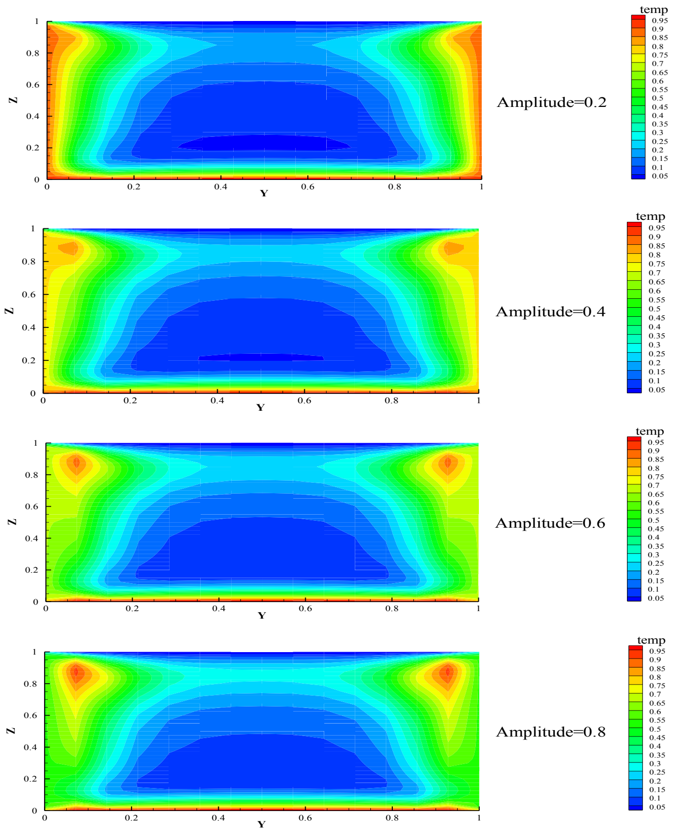

fig. (3): three dimensional transient temperature distribution of different amplitude at section (x=0.214) for Ra=200, period=0.01, t=0.006

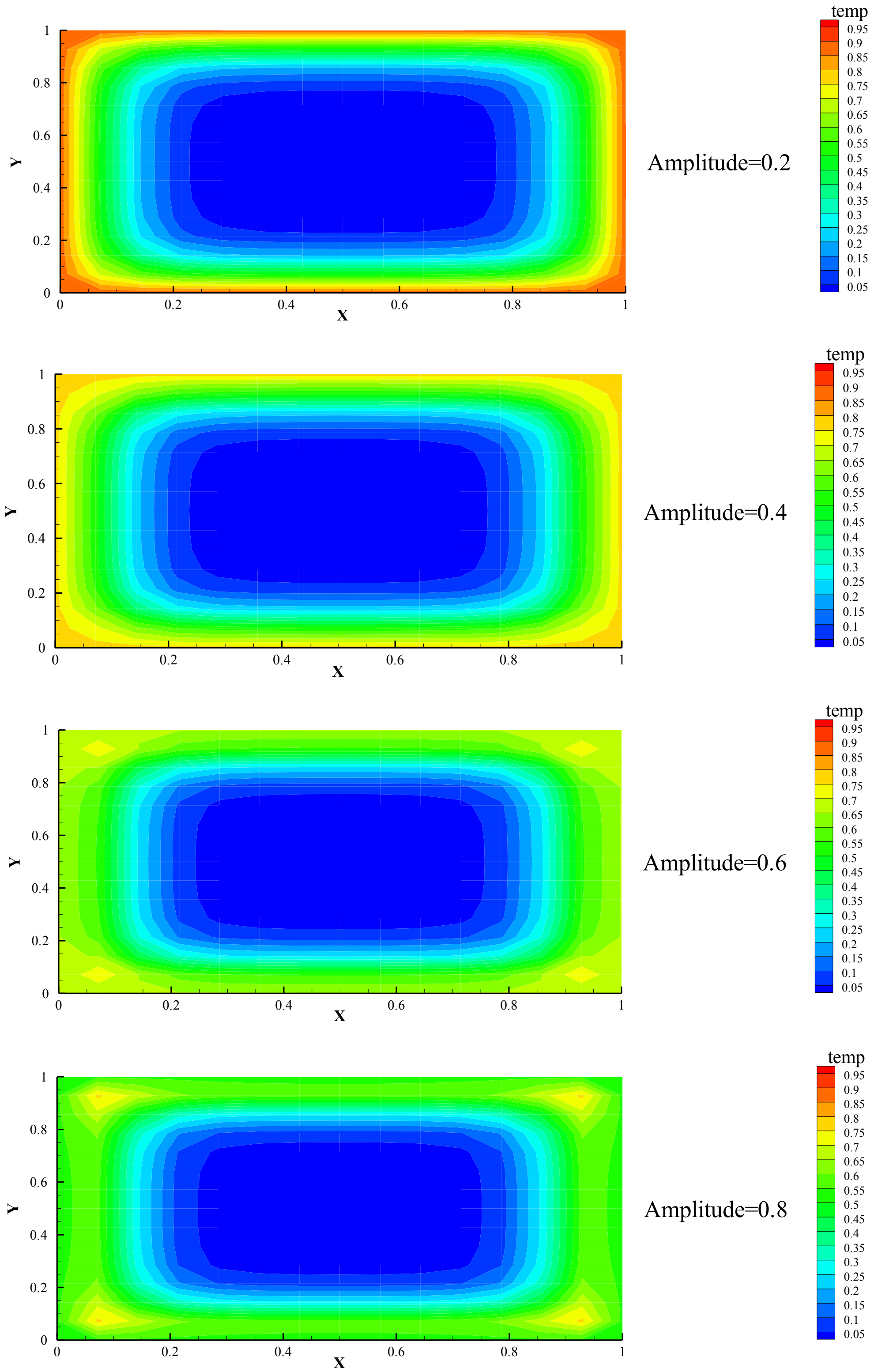

fig. (4): three dimensional transient temperature distribution of different amplitude at section (z=0.35) for Ra=200, period=0.01, t=0.006

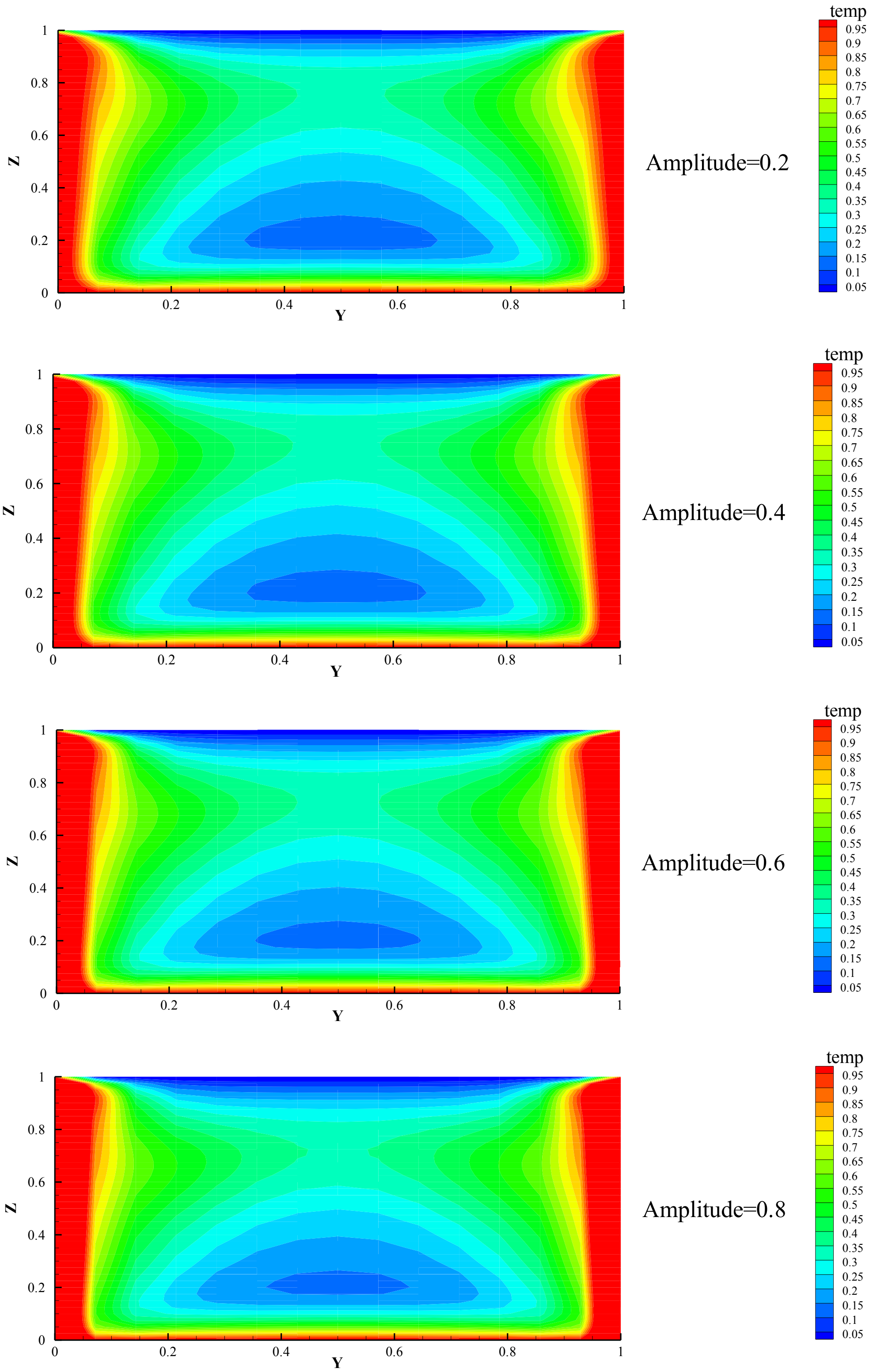

fig. (5): three dimensional transient temperature distribution of different amplitude at section (x=0.214) for Ra=200, period=0.01, t=0.012

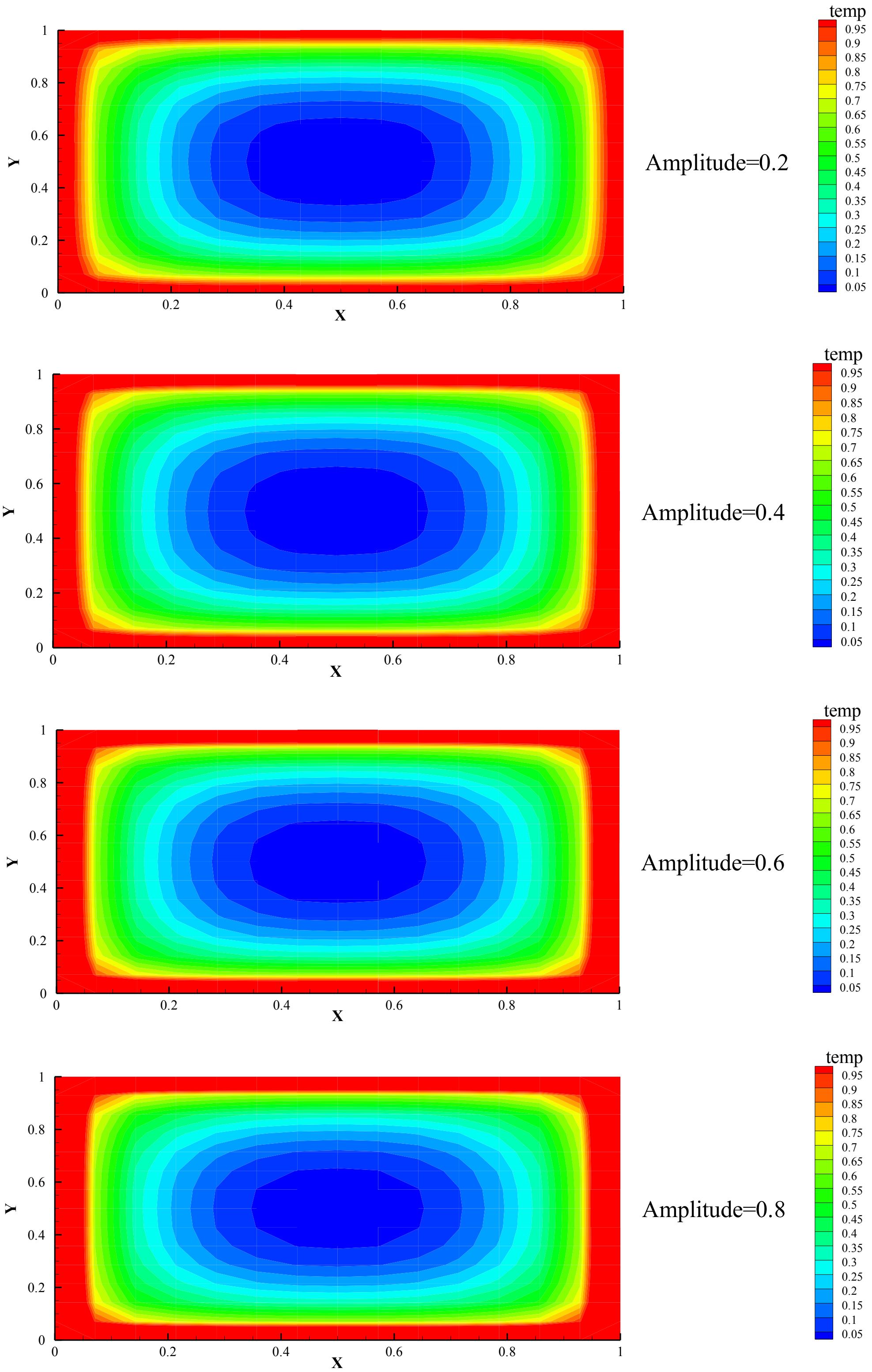

fig. (6): three dimensional transient temperature distribution of different amplitude at section (z=0.35) for Ra=200, period=0.01, t=0.012

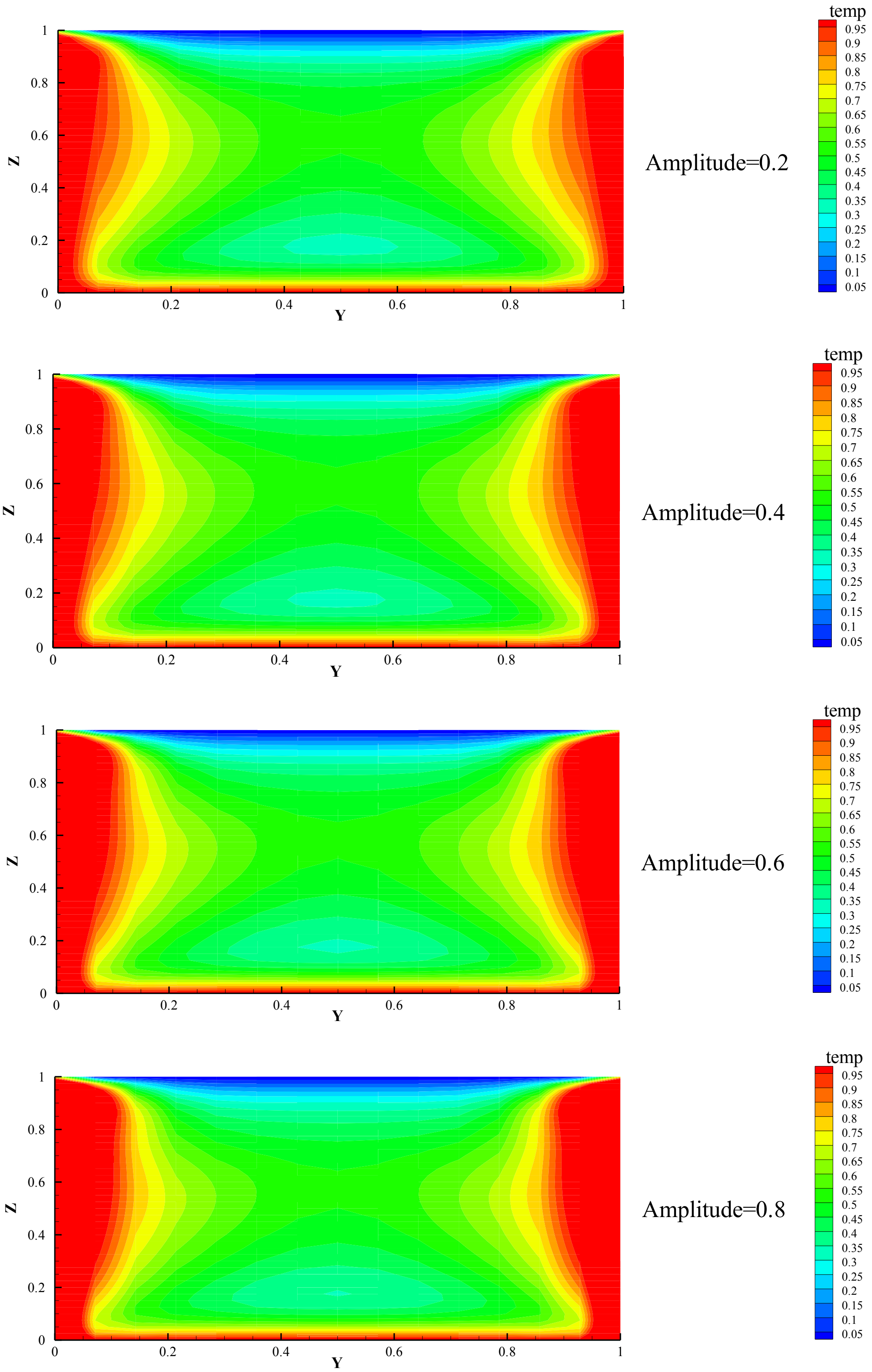

fig. (7): three dimensional transient temperature distribution of different amplitude at section (x=0.214) for Ra=200, period=0.01, t=0.024

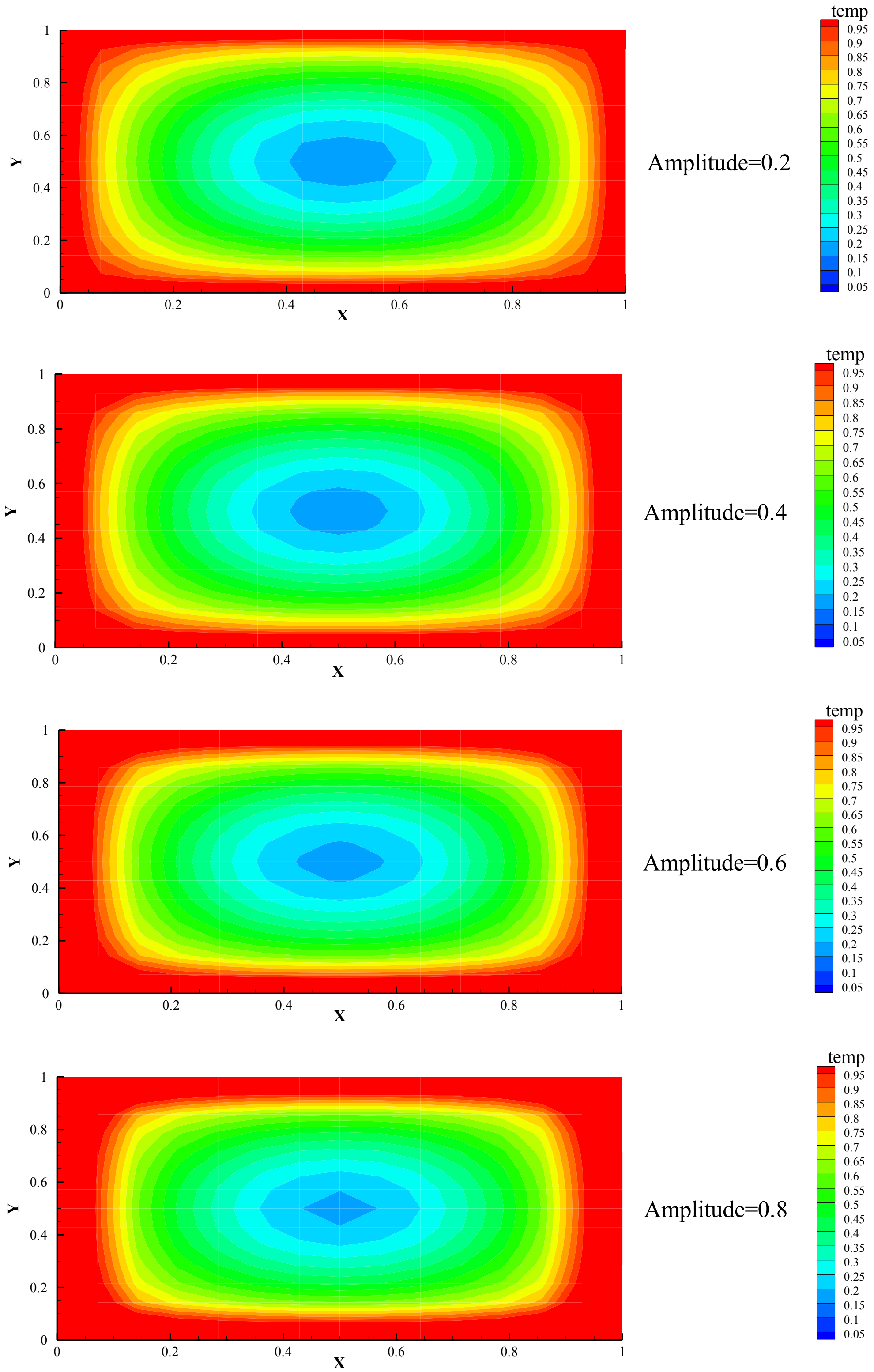

fig. (8): three dimensional transient temperature distribution of different amplitude at section (z=0.35) for Ra=200, period=0.01, t=0.024

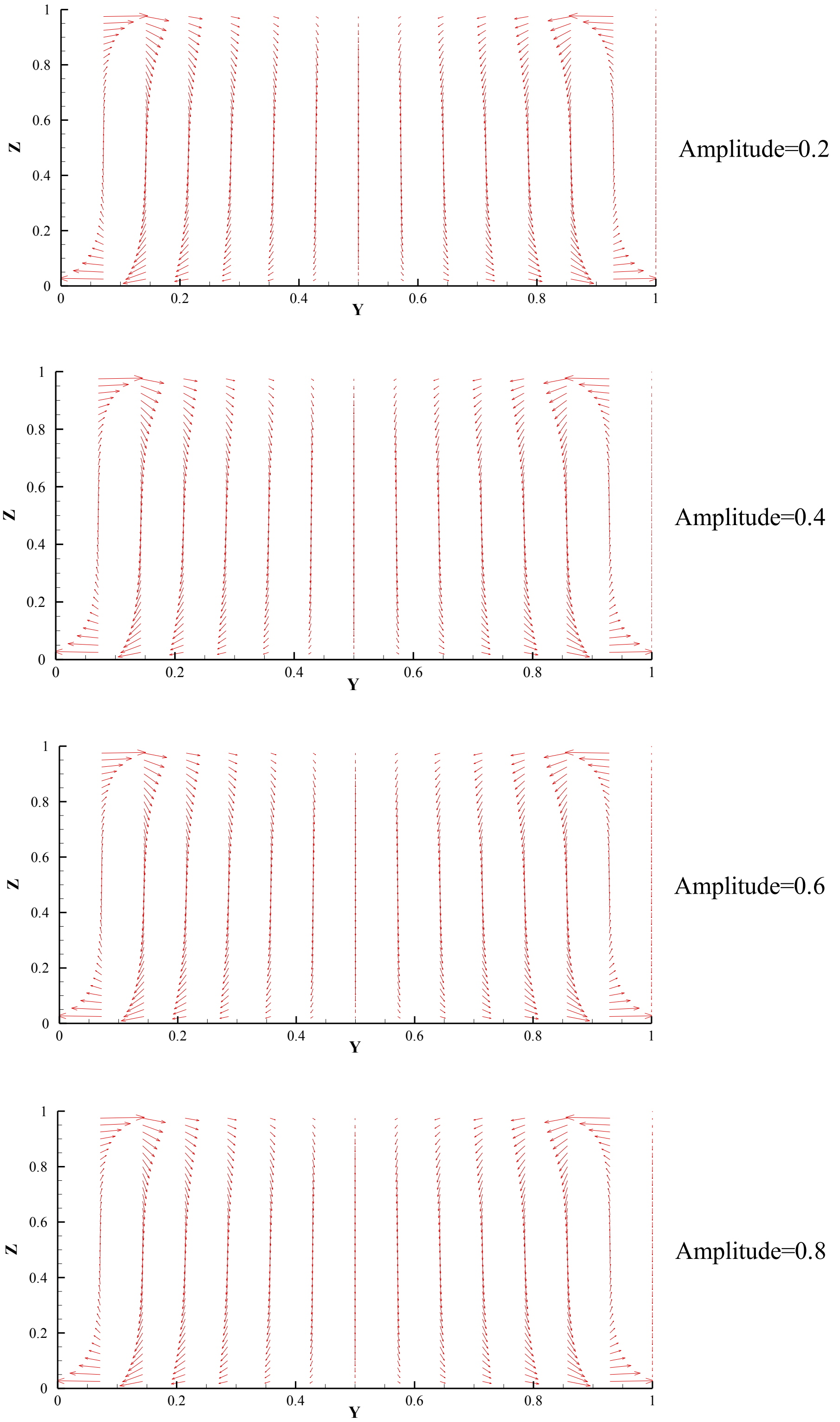

fig. (9): velocity distribution of different amplitude at section (x=0.214) for Ra=200, period=0.01, t=0.006

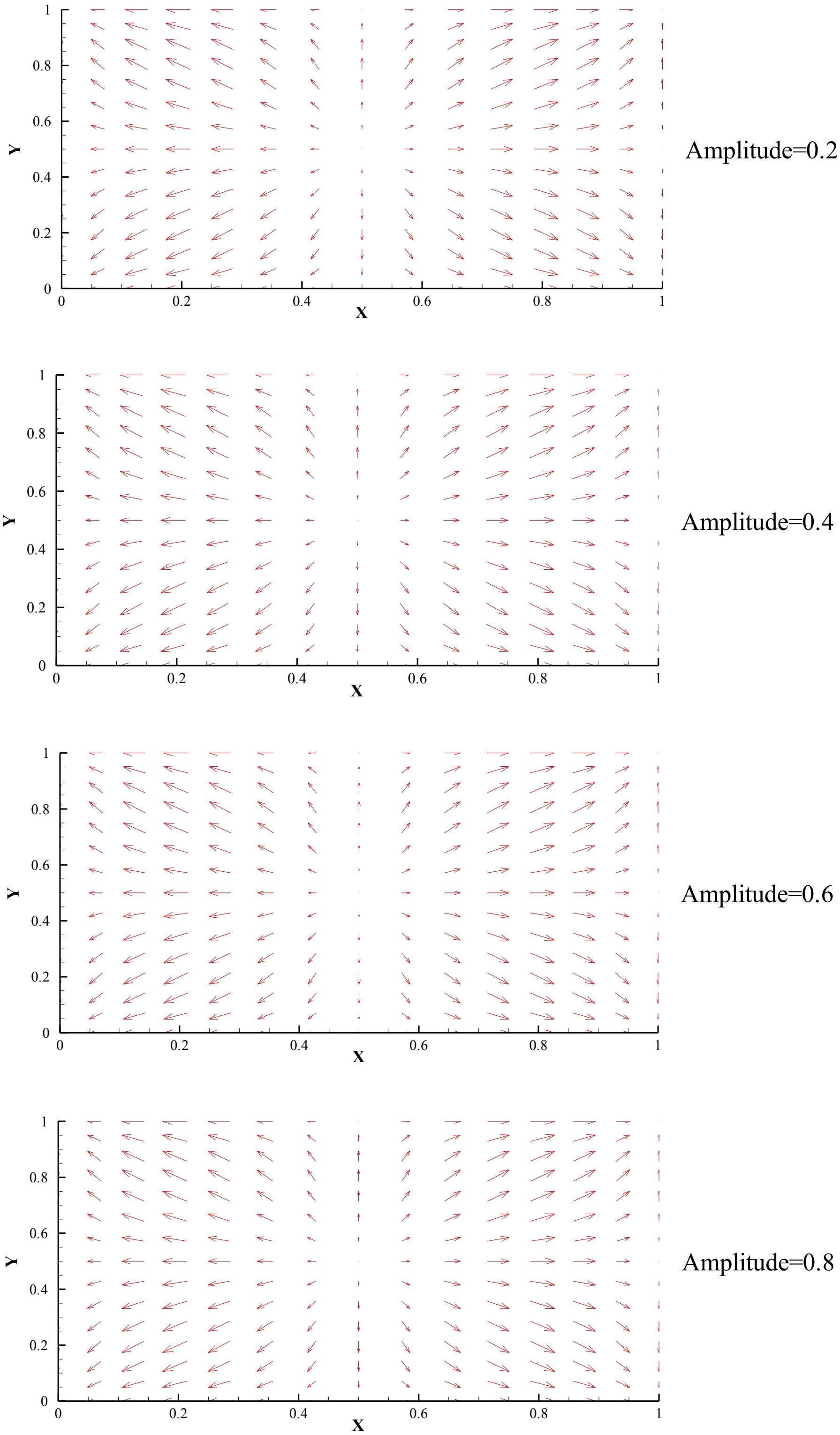

fig. (10): velocity distribution of different amplitude at section (z=0.35) for Ra=200, period=0.01, t=0.006

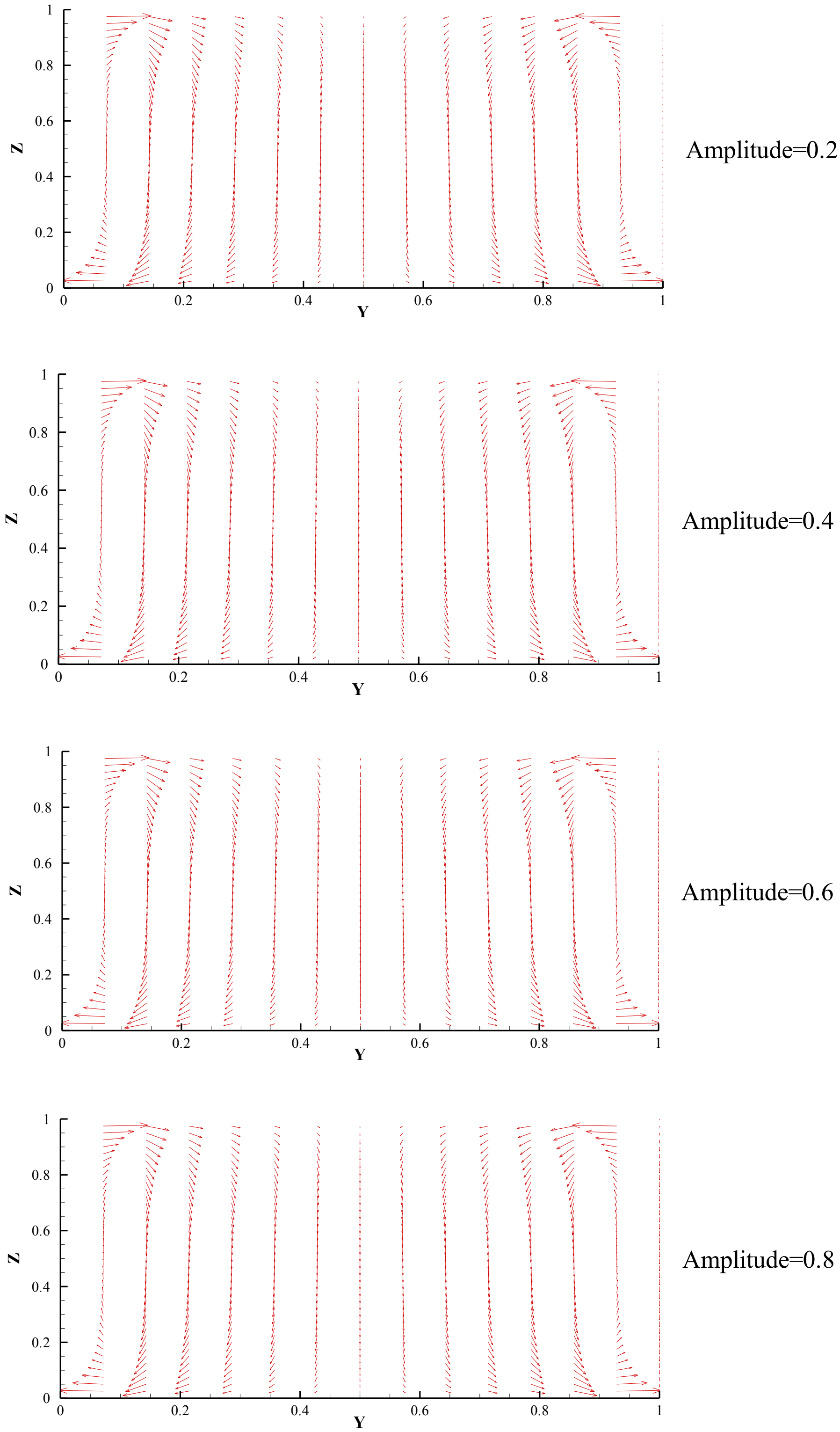

fig. (11): velocity distribution of different amplitude at section (x=0.214) for Ra=200, period=0.01, t=0.024

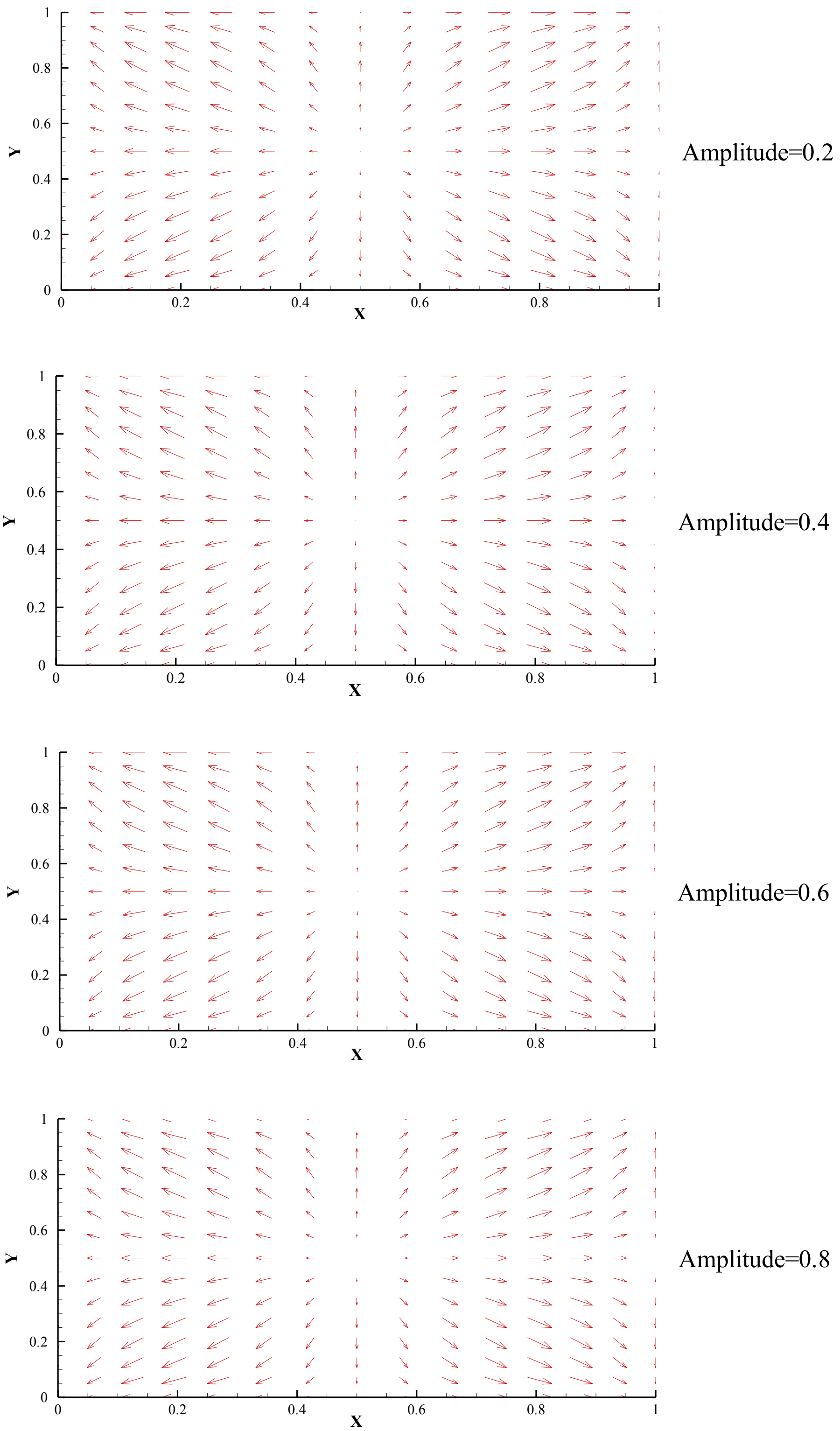

fig. (12): velocity distribution of different amplitude at section (z=0.35) for Ra=200, period=0.01, t=0.024

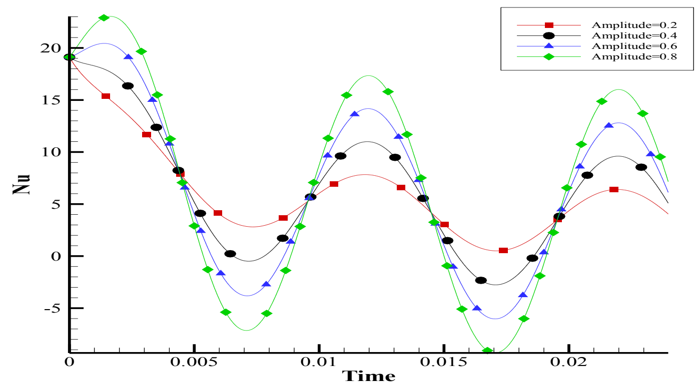

fig.(13): variation of Nusselt number with time at the vertical wall for various values of amplitude

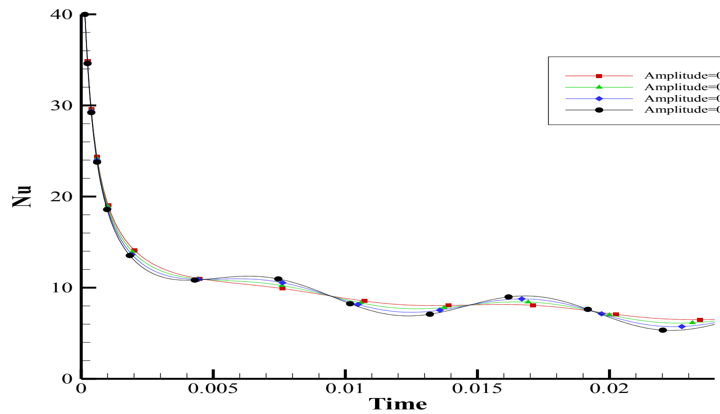

fig.(14): variation of Nusselt number with time at the bottom wall for various values of amplitude

V. Conclusions

The present numerical study modeled the effect of a time periodic boundary conditions on natural convection in a square enclosure filled with a porous medium . The dimensionless forms of the governing equations are solved numerically using finite difference method. The computation code is validating with the published data for a fixed mean hot wall temperature. Detailed numerical results for flow and temperature field, time-dependent behavior of the temperature in the cavity have been presented in the graphical form. The main conclusions of the present analysis are as follows:

1. The time required to reach the basic steady state is longer for low amplitude than for high amplitude.
2. The location of the maximum fluid temperature moves with time according to the periodically changing the heated wall temperature.
3. Amplitude of Time sinusoidal oscillating boundary conditions have drastically been changing the temperature in the cavity.
4. The flow field is characterized by two main cells rotating in opposite direction to each other.
5. Changing the amplitude does not affect the two cell pattern of the flow field.

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