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Adaptive response surface by kriging using pilot points for structural reliability analysis

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Abstract: *Structural reliability analysis aims to compute the probability of failure by considering system uncertainties. However, this approach may require very time-consuming computation and becomes impracticable for complex structures especially when complex computer analysis and simulation codes are involved such as finite element method. Approximation methods are widely used to build simplified approximations, or metamodels providing a surrogate model of the original codes. The most popular surrogate model is the response surface methodology, which typically employs second order polynomial approximation using least-squares regression techniques. Several authors have been used response surface methods in reliability analysis. However, another approximation method based on kriging approach has successfully applied in the field of deterministic optimization. Few studies have treated the use of kriging approximation in reliability analysis and reliability-based design optimization. In this paper, the kriging approximation is used an alternative to the traditional response surface method, to approximate the performance function of the reliability analysis. The main objective of this work is to develop an efficient global approximation while controlling the computational cost and accurate prediction. A pilot point method is proposed to the kriging approximation in order to increase the prior predictivity of the approximation, which the pilot points are good candidates for numerical simulation. In other words, the predictive quality of the initial kriging approximation is improved by adding adaptive information called "pilot points" in areas where the kriging variance is maximum. This methodology allows for an efficient modeling of highly non-linear responses, while the number of simulations is reduced compared to Latin Hypercubes approach. Numerical examples show the efficiency and the interest of the proposed method.*

Keywords: *response surface, reliability-mechanical coupling, Kriging, pilot points, approximation.*

I. INTRODUCTION

The civil structures are an important heritage. To better manage these assets, it is imperative to assess their reliability. The reliability of a structure is designed to measure the conventional safety (probability of failure), taking into account the various uncertainties, under a probabilistic angle. Thus, if we assume that R is the resistance of the structure and the applied stress S, the limit state function in the physical space is $H(\bar{X}) = R - S$. The reliability is then defined by:

$$F = P[H(\bar{X}) > 0] = \int_{H(\bar{X})>0} f_{\bar{X}}(\bar{x})\, dx_1 \dots dx_n \tag{1}$$

Where n is the number of random variables. $f_{\bar{X}}(\bar{x})$ is a joint probability density of the random vector $\bar{X}$. The probability of failure is given by:

$$P_f = 1 - F = \int_{H(\bar{X})\leq 0} f_{\bar{X}}(\bar{x})\, dx_1 \dots dx_n \tag{2}$$

The problem solving is essentially the evaluation of the performance function, which is usually very expensive because it is an issue of a finite element code. The probability of failure of a structure must be evaluated. The selected alternative is the use of mechanical reliability based method, proven method today in many areas of civil nuclear, offshore structures, but also in the civil engineering works of exceptional arts and existing buildings. The method of Monte Carlo simulation is the most widely used technique for the analysis of uncertainty. The major drawback of this technique is the large number of simulations required to achieve an acceptable level of confidence desired output, although the procedure is simple to perform.

Using a meta-model overcomes the problem of the cost of computation time related to direct coupling. There are several types of metamodel, including the Response Surface Methodology (RSM) and Kriging are cited. The application of the kriging technique for assessing the reliability of structures is very recent [8, 22]. Several studies [3, 14, 10, 21, 23] have shown growing interest in this type of metamodel for assessing the probability of failure. Kriging is probabilistic. The technique therefore has the advantage over other metamodel (quadratic response surfaces, polynomial chaos, support vector machine ...) to provide a measure of uncertainty a priori prediction without additional mechanical calculation.

The need to develop a modeling approach while controlling the cost of simulation and prediction of the model is the ambitious goal of this article. To achieve this goal, we propose a modeling methodology based on geostatistical methods with the addition of pilot points. This method reduces the number of simulations required for adequate assessment of uncertainties. The advantage of the method and its performance is demonstrated on existing examples in the literature. On the other hand, the impact parameter of the method that is the coefficient and the correlation function were examined on several examples in order to find the right tool for good accuracy.

II. DIFFERENT APPROACHES TO MODELING RESPONSE

The shape of the approximate function and points of experiments were chosen in various ways by different researchers for the implementation of the method of response surfaces.

Das and Zheng [13] proposed a cumulative response surface. A first surface is formed in a linear response for determining the point of design. Proposed gradient technique is then used to generate the calibration points. Then in a second setting, the surface is enriched with linear response quadratic terms. Points defining the linear surface calibrations are reused for the quadratic surface. Additional points are generated around the design point (DP) of the linear surface. Mixed terms can be added to the response surface if necessary.

Gayton [12] proposed a method of quadratic response surface called CQ2RS (Complete Quadratic Response Surface with resampling) method. This method is based on a statistical approach is to consider the estimation of the coordinates of the DP as a random variable, each achievement is the result of a new iteration of the experimental design. A confidence interval is assigned to the mean value of the estimation. The width of this range is taken as the iterations stop experimental design criterion. The leading experience is a factorial design constructed from pre-reduced field of research point. With this approach, the computational cost can be reduced DP.

Kaymaz and McMahon [9] used a linear response surface for the first iteration and a quadratic response surface without mixed terms for subsequent iterations. The control points are generated from the central point. They are selected from the region where the design point is the more likely, depending on the evaluation of the sign of the random variables. The control points defining the linear surface are reused for the quadratic surface. The coefficients of the response surface are determined by the weighted regression. A particular system of weighting the values of the function of limit state is used which allows penalizing control points away from the real to limit state surface and give more importance to the closest of the surface points.

Duprat and Sellier [6] proposed a method of adaptive quadratic response surface, taking into account, from one iteration to the other of the experimental design, the position of the DP compared to other points. As the PC is outside the item of experiments, it has refocused on DP, according to a mesh conditioned by the sensitivity of the response surface to the impact of various variables axes. These meshes of points are positioned to the failure zone. After the DP point is inside the experimental design, all points towards failure are preserved, while new points are symmetrical to the former points compared to DP. From an initial design of experiments carefully chosen, this method allows a rapid convergence towards the DP issue.

Mohammadkhani [18] proposed an algorithm for construction of the response surface. The fundamental objective in finding the surface of the answer lies in the estimation of the probability of failure of the structure. This calls for a particular interest in the region where the probabilities are higher in the area of failure, an area that is around the design point; this point is not known a priori. An iterative search technique is used to initially estimate the design point.

In another research work presented by Nguyen [24], the surface is constructed in response given space standardized in two variables, firstly taking a linear shape, and then mixed with a quadratic terms. The method minimizes the computational cost of ensuring the accuracy of the results by two weighting schemes for the control points.

Roussouly [25] proposed an adaptive method for constructing a surface predictive response. The method is based on the region of interest to enrich the sample and validate the results. It has three stages: (a) locate the region of interest, (b) improve the RS and (c) validate the RS. At the initial iteration, the sample is distributed according to a Latin hypercube. This area is used to enrich the sample until the coefficient of Q^2 of the RS reaches a desired value noted ε_{model}. The final validation is based on the bootstrap.

III. METHODOLOGICAL APPROACH

The approach we propose aims to increase the accuracy of the results by minimizing the computational cost. The main features of the proposed method lie in the choice of the expression of the response surface and control the quality of the model prediction. It allows the control of the quality of the response surface by analyzing the predictive index Q^2 quality. To achieve this goal, we propose a coupled modeling methodology based on regression methods and geostatistical modeling. The approach is to model the response by the sum of a regression and a kriging which keeps an average trend where there are no comments. At first, the reliability index is evaluated by a quadratic response surface using the centered composite experimental design for the first iterations. In a second step, the reliability index is estimated by kriging response surface for subsequent iterations in the space of standardized variables. Another key step of the proposed method is to control and increase the predictability of the approximation from fictitious information. Technical pilot point allows to improve the approximation of the experimental in the areas domain portions where the kriging variance is maximum. The values of the responses at the pilot points have been optimized by maximize the coefficient of predictability priori to approximation Q^2.

With this method, new values of responses are added to the actual values without any simulation has been performed. Adding pilot points can effectively reduce the number of simulations required for a good approximation.

IV. GENERAL APPROACH

4.1 INITIAL STAGE

The first step is related to the construction of a surface response surface (RS) to evaluate the error that is made by the approximation model. It is fundamental because it tells us to know about the quality of the predictions provided from the new set of data and details of the design point (DP). The issue then is concerned with the choice of the best regression method that minimizes the error. In this first stage the response surface is built from a design of experiments (DE), the coordinates of the data points define finite element calculation. One type of the most comprehensive plan in terms of the representativeness of the experience data is the centered composite design, which is the union of a star-shaped plan and a factorial design. It comprises (2^n +2 n +1) points where n are the number of random variables. This experimental design is used for the rest of our study.

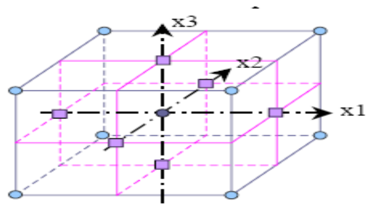

Fig.1 centered Composite Map

The Response Surface Methodology (RSM) consists to represent a response H as the sum of a low level polynomial metamodel (about one or two) and an error term ε having a normal distribution with zero mean $(E(\varepsilon) = 0)$ and a variance equal to σ^2.

Response H of a polynomial of second order model with two input variables x_1 and x_2 will be represented as follows:

$$A = A_0 + A_1x_1 + A_2x_2 + A_{11}x_1^2 + A_{22}x_2^2 + A_{12}x_1x_2 + \varepsilon \tag{3}$$

In general, H is written as a function of n variables:

$$H = A_0 + \sum_{j=1}^{n} A_jx_j + \varepsilon \tag{4}$$

Now, if we assume that we know the values of $m > n$ responses of H function with m different values of the input variables. for each observation i there are H_i values of the response and x_{ij} the $j^{ème}$ variable regression of body. Considering whereas the average error is zero $E(\varepsilon) = 0$ and its variance is: $X_i = (x_{ij})_{j=1,n} Var\ (\varepsilon) = \sigma^2$ the solution H_i has a mean of:

$E(H_i) = A_0 + \sum_{j=1}^{n} A_jX_{Ij} + \varepsilon$ and a variance equal to $Var\ (H_i) = \sigma^2$ where:

$$H_i = A_0 + \sum_{j=1}^{n} A_jX_j + \epsilon_i, \forall\ i = 1, \dots n \tag{5}$$

In another form the equation in matrix form is: $Y = X\beta + \varepsilon$ with:

$$Y = \begin{bmatrix} H_1 \\ H_2 \\ \cdot \\ \cdot \\ \cdot \\ H_m \end{bmatrix}, X = \begin{bmatrix} 1 & X_{11} & X_{12} & X_{1n} \\ \vdots & & \ddots & \vdots \\ 1 & X_{m1} & X_{m2} & X_{mn} \end{bmatrix}, A = \begin{bmatrix} A_0 \\ A_1 \\ \cdot \\ \cdot \\ \cdot \\ A_n \end{bmatrix} \text{ and } \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_m \end{bmatrix} \quad (6)$$

Minimizing the function of the least-squares error with respect to $A_{j=1\ldots n}$ the vector of regression coefficients is obtained:

$$\hat{A} = (X^T X)^{-1} X^T Y \quad (7)$$

The response surface method is based on an adjustment of an approximate function $h(x)$, which replaces a priori unknown function performance $H(x)$. To determine the coordinates of the design point (DP), it is necessary to use one of isoprobabilistic transformations (Rosenblatt or Nataf) U = T (X), which allows the passage of random input variables in standard normal space. The limit state function is approximated by an equivalent function in the standardized space:

$$H(U) = a_0 + \sum_{i=1}^{n} a_i U_i + \sum_{i=1}^{n} a_{ii} U_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_{ij} U_i U_j \quad (8)$$

With $\bar{a} = \{a_0, a_i, a_{ii}, a_{ij}\}$ coefficients are obtained by the method of least squares regression and *Ui, Uj* are random variables in standardized space.

$$\bar{a} = (M_U^T . M_U)^{-1} . M_U^T . \bar{y} \quad (9)$$

$$M_U = \begin{bmatrix} 1 & u_{1(1)} \ldots u_{n(1)} & u_{1(1)}^2 & \cdots u_{n(1)}^2 & u_{i(1)} u_{j(1)} \\ & \vdots & & \ddots & \vdots \\ 1 & u_{1(N)} \ldots u_{2(N)} & u_{1(N)}^2 & \cdots u_{n(N)}^2 & u_{i(N)} u_{j(N)} \end{bmatrix} \quad (10)$$

$\bar{y}$ is the vector of values of the limit state at different points of the experimental design. The classical algorithm Hasofer-Lind-Rackwitz Fiessler (HLRF) [17] is used for determining the coordinates of the design point and the reliability index is simply in the standard normal space, the number of standard deviations between the average linearized limit states (Method FORM) as shown in Fig.2

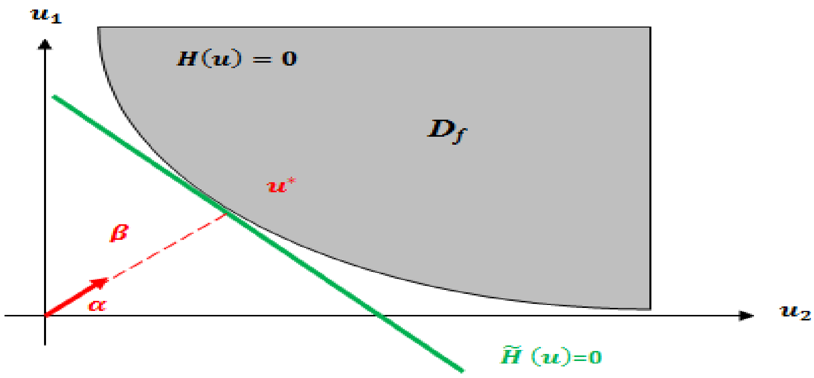

Fig.2 FORM approximation

4.2 Second stage

An interpolation using a polynomial of degree one or both is not sufficient in most cases. For this purpose, it is necessary to add a patch polynomial fitting term. This is random; hence our model is a combination of a quadratic regression model and a realization of a stochastic process Z.

This model is called the adaptive composite model (ACM).

$$Y(x) = {}^T X(x) A + Z(x) \quad (11)$$

The first term is the adjustment of the deterministic part of the model. The second term is a realization of a Gaussian stochastic process with zero mean. The estimated residual process is obtained using kriging.

The algorithm is written in Matlab, and the kriging model is built using the toolbox DACE [19] used and cited references in articles [8, 2]

Fig.3 shows the importance of kriging in the case of complex phenomena obtained by numerical calculation.

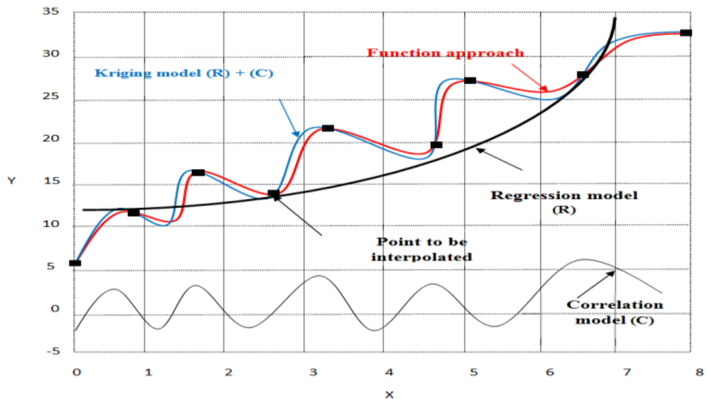

Fig.3 The importance of kriging in the complex phenomena

Centered composite plan is not suitable for kriging because the points are placed at the edges of the experimental field which does not allow detecting possible irregularities in the response. Plans that are currently used in kriging are Latin hypercube which best represent their points the experimental field, hence the name of "space filling design". Each edge of the experimental domain is divided into n equal segments so as to obtain a mesh size of the domain. Jourdan and Collombier [1, 5] showed that optimal hypercube is robust to changes in the correlation parameter θ which, remember, is not known when the construction plan. Latin hypercubes have many advantages; these points are uniformly distributed on each axis of the domain. On this, we use the optimal plan in the wake of the second phase to improve the response surface polynomial asked to be corrected by kriging.

V. Estimated Residuel Kriging Part

A polynomial model is used to bridge the gap between the model prediction and simulation; it is for this reason that we do next kriging on the residuals. This composite model is used to correct the response surface, which enables robust approximation especially in areas not sampled.

Kriging is to determine the weight assigned λ_i to estimate the value $\hat{Z}(x_0)$ function in any x_0 interpolating the data $\hat{Z}(x_i)$ already observed.

The kriging predictor of response which is the residue Z in a point $x_0$where no simulation has been performed is a linear function of *n* observations $\hat{Z}(x_i)$ to point's x_i plane.

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i(x_0) Z(x_i) \tag{12}$$

By minimizing the variance of the error at the point x_0, optimal weight λ_i depend on the covariance between observations. After demonstration, we get the following equation:

$$\hat{Z}(x_0) = {}^T\lambda Z = {}^T c(x_0) C^{-1} Z = {}^t Z C^{-1}\, c(x_0) \tag{13}$$

or:

$$C = \left[Cov(Z(x_i), Z(x_j)), i, j = 1, \dots \dots \dots n\right] \tag{14}$$

$${}^T c(x_0) = \left[Cov(Z(x_0), Z(x_i)), i = 1, \dots \dots \dots \dots n\right] \tag{15}$$

C is the covariance matrix between the points of the experimental design. The equation is then written:

$$\begin{aligned} \hat{Z}(x_0) &= {}^T c(x_0) C^{-1} Z \\ &= {}^T c(x_0) C^{-1}[Y - X\hat{A}] \end{aligned} \tag{16}$$

$X\hat{A}$ the polynomial regression is associated with the sample.

In our approach to adaptive modeling, we propose to estimate the response of a composite predictor of the form:

$$\hat{Y}(x_0) = X(x_0)\hat{A} + {}^T c(x_0) C^{-1}[Y - X\hat{A}] \tag{17}$$

The highly adaptive composite model will depend the covariance structure between residues. The estimate of the latter is very important in the construction of a predictor and therefore in our approach to adaptive modeling.

$$C\left(Z(x), Z(x+h)\right) = \exp\left[-\theta \sum_{i=1}^{D} (X_i - (x_i + h))^2\right] \tag{18}$$

With the unknown parameter θ of the correlation model. Note that the correlation between observations depends on the correlation parameter θ and the distance between the observations. The correlation increases as θ decreases and it decreases as the distance increases between observations.

So far we did not specify how to calculate from the experiences of the various model parameters kriging map, namely the β coefficients of the "regression" which describes the average trend and variance process σ_y^2

$$\hat{A} = (X^T C^{-1} X)^{-1} X^{Tt} C^{-1} Y \quad (19)$$

$$\sigma_y^2 = \frac{1}{n}(Y - X^T A)^T C^{-1} (Y - X^T A) \quad (20)$$

To overcome the many difficulties of estimating the covariance structure of the residuals, we propose: The residual process Z must be stationary assume that all points are depares the same distance h in a given direction, have the same covariance.

$$Cov\big(Z(x), Z(x+h)\big) = Cov\,(h) \;\; \forall\, x \in D \quad (21)$$

Covariance is positive definite. This assumption implies that the variance of the kriging estimator is positive or zero:

$$Var\,\big(Z(x)\big) = Cov(Z(x), Z(x) = Cov(0) = \sigma^2 \quad (22)$$

To build a better predictive interpolator, we propose to use an innovative approach of adding fictitious reversal (technical pilot points) according to a terminology introduced by RamaRao [4]. These points are considered as pilot data that no simulation has been carried out.

VI. Principle Of The Technique Of Pilot Points

We opt for the result of our method using the technique of pilot points and propose a modification of the latter to contribute to reducing uncertainties in the approximation model and increase the predictability of the approximation current.

The method is to add the n points of the Latin hypercube map for which response values $Y = (Y_1, \dots, Y_n)$ obtained by simulation, number n_p pilot points whose values $Y^{pp} = (Y_1^{pp}, \dots \dots \dots, Y_{n_p}^{pp})$ responses are unknown.

The location of the pilot points is done empirically. The pilot point is placed every half correlation lengths. To ensure that the pilot points are located on the side of the failure surface in relation to the design point (PC), we opt for the formula (Nguyen [24]) and we propose a slight modification. The amendment concerns the correlation distance between the base and the point or there is a significant gradient estimates.

$$DC_i = (DC_0)^{0.5}\left(\frac{-1/2}{\|grad\tilde{H}(\bar{U})\|} \cdot \frac{\partial \tilde{H}(\bar{U})}{\partial U_i}\right) \quad (23)$$

Where DC_0 is the basic correlation distance selected according to the estimated value of significant gradients. $\tilde{H}(\bar{U})$ is the equivalent function of the response surface in the standardized area outside.

We now show how to define the fictitious values $Y^{pp} = (Y_1^{pp}, \dots \dots \dots, Y_{n_p}^{pp})$ to n_p point whose pilot positions $x^{pp} = (x_1^{pp}, \dots \dots \dots, x_{n_p}^{pp})$ have been set.

Kriging is an exact interpolation method, conventional waste $e_{(i)} = Y_i - \hat{Y}(x_i)$ provide no information on the predictability of the approximation. (Cook 1982 [15]) to use a different type of residue called prediction residue. In the prediction residual is given by: $e_{(i)} = Y(x_i) - Y_{-i}(x_i) = Y_i - \hat{Y}(i)$ or $\hat{Y}(i)$ represents the estimated point x_i a model fitted without x_i issue.

The calculation of prediction residuals at each point of the plan will allow Latin hypercube calculating the predictability of the approximation. Called the predictive power of the model sum of squared prediction errors or other name the PRESS coefficient.

$$PRESS = \sum_{i=1}^{n} e_{(i)}^2 = \sum_{i=1}^{n} [Y_i - \hat{Y}_{(i)}]^2 \quad (24)$$

According to the equation of adaptive composite model $\hat{Y}_{(i)} = X_{(i)} \hat{A}_{(i)} + {}^T \lambda_{(i)} (Y_{(i)} - X_{(i)} \hat{A}_{(i)})$

Adjusting *n* regression model and *n* kriging model should be performed. The prediction residue at point *i* is given by:

$$\begin{aligned} e_{(i)} &= Y_i - \boldsymbol{Y}_{(i)} \\ &= \boldsymbol{Y}_i - X_{(i)} \hat{A}_{(i)} - {}^T (Y_{(i)} - X_{(i)} \hat{A}_{(i)}) \\ &= e_{(i)}^{reg} - \sum_{j \neq 1} \lambda_{(i)}^j (\boldsymbol{Y}_J - X_{(i)}^j \hat{A}_{(i)}) \\ &= e_{(i)}^{reg} - e_{(i)}^{krig} \end{aligned} \quad (25)$$

We proceed to the calculation of the first term $e_{(i)}^{reg}$:

$$e_{(i)}^{reg} = \{Y_i - {}^T X^i \hat{A}/1 - h_{ii}\} \text{ with } h_{ij} = {}^T X^i ({}^T XX)^{-1} X^j \quad (26)$$

Or $X_{(i)}$ and $Y_{(i)}$ are the vectors X and Y obtained without the $i^{\acute{e}me}$ observation.

The numerator of $e_{(i)}^{reg}$ is the classical residue e_i an ordinary least square approximation.

This implies that:

$$e_{(i)}^{reg} = e_i / 1 - h_{ii} \quad (27)$$

We now determine the coefficient PRESS for adaptive composite model.

$$e_{(i)}^{reg} = \sum_{j\neq 1} \lambda_{(i)}^{j} \left(Y_j - X_{(i)}^{j} \hat{\beta}_{(i)}\right) = \sum_{j\neq 1} \lambda_{(i)}^{j} [Y_j - X^i \hat{\beta} + \{{}^T X^i ({}^T XX)^{-1} X^i e_i / 1 - h_{ii}\}$$

$$= \sum_{j\neq 1} \lambda_{(i)}^{j} \left(e_j + \frac{h_{ij} e_i}{1-h_{ii}}\right) \quad (28)$$

$$\text{Where: } \sum_{i=1}^{n} \left(e_{(i)}\right)^2 = \sum_{i=1}^{n} \left(e_{(i)}^{reg} - e_{(i)}^{krig}\right)^2$$

$$= \sum_{i=1}^{n} \{ e_{(i)}^{reg} - \sum_{j\neq 1} \lambda_{(i)}^{j} \left(e_j + \frac{h_{ij} e_i}{1-h_{ii}}\right) \}^2$$

$$= \sum_{i=1}^{n} \{ \frac{e_i}{1-h_{ii}} - \sum_{j\neq 1} \lambda_{(i)}^{j} \left(\frac{(1-h_{ii}) e_j}{1-h_{ii}} - \frac{h_{ij} e_i}{1-h_{ii}}\right) \}^2 \quad (29)$$

Finally:

$$PRESS = \sum_{i=1}^{n} \frac{1}{(1-h_{ii})^2} \left\{ e_i - \sum_{j\neq 1} \lambda_{(i)}^{j} \left[(1-h_{ii}) e_j - h_{ij} e_i\right] \right\} \quad (30)$$

We find that this ratio depends only on conventional residues from the regression coefficient h_{ij}. We use this expression to calculate the value of the pilot points to improve the predictability of the approximation based on fictitious values.

A key point in the construction of a response surface is whether our model is representative of the system as it seeks to simulate the one hand and if he can predict the various points that have not been simulated to secondly. We consider predictive index that measures the quality Q^2 predictive model that is related to the coefficient PRESS by the equation:

$$Q^2 = 1 - \frac{PRESS}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} \quad (31)$$

From the above equation, the predictive quality index Q^2 can be written as:

$$Q^2 = 1 - \frac{\sum_{i=1}^{n} \frac{1}{(1-h_{ii})^2} \left\{ e_i - \sum_{j\neq 1} \lambda_{(i)}^{j} \left[(1-h_{ii}) e_j - h_{ij} e_i\right] \right\} 2}{\sum_{i=1}^{n} (Y_i - Y)^2} \quad (32)$$

VII. Purpose And Principle Of The Approach

The main objective of our approach is to improve the quality of the RS in the neighboring area of the state limit of DP. It is necessary to enrich the learning of the RS sample in the region of interest. However, whenever the pilot points are added, the RS is updated and the region of interest, determined from the latter is changed. Our approach is based on two tests of convergence (convergence in the region of convergence and the response surface).

7.1Tests of convergence of the region:

Checking the convergence criterion is performed on two values of the reliability index β and two points in terms of successive DP. The convergence is achieved when the difference between the reliability index of two consecutive iterations is less than the convergence criterion. This convergence criterion is described in the equation:

$$\begin{cases} |\beta_m - \beta_{(m-1)} \leq \varepsilon_\beta| \\ \| PC_m . PC_{m-1} \quad \| \leq \varepsilon_p \end{cases} \quad (33)$$

Where ε_p and ε_β are the convergence tolerances ranging between 10^{-4} and 10^{-7} according to the probabilistic model and studied numerical model and m is the number of iteration. The region of interest is stable if the convergence criterion is satisfied. Otherwise, a new response surface (SR) is constructed for which the number of pilot points in the sample in the region of interest is increased ($N_{i+1} = Ni + N_{add}$ where N_{add} is the number of pilot points added).

7.2Tests convergence of the response surface

This tests the quality of our composite model adaptive (ACM) and especially the quality of the RS and the value of Q^2 obtained by ordinary kriging and regression residual + adding drivers issue. The model will be even more predictive than this ratio will approach 1 and therefore provides information on the predictive power of the model. In this case, the algorithm and the iterative process is stopped if the final RS is identified with an index of predictive quality Q^2 close to 1.

The principle of indirect coupling as well as the flowchart of the algorithm for constructing response surfaces and the calculation of the reliability index is shown in Figure 4. This algorithm has been programmed in Matlab code (7.9.0) R2009b.

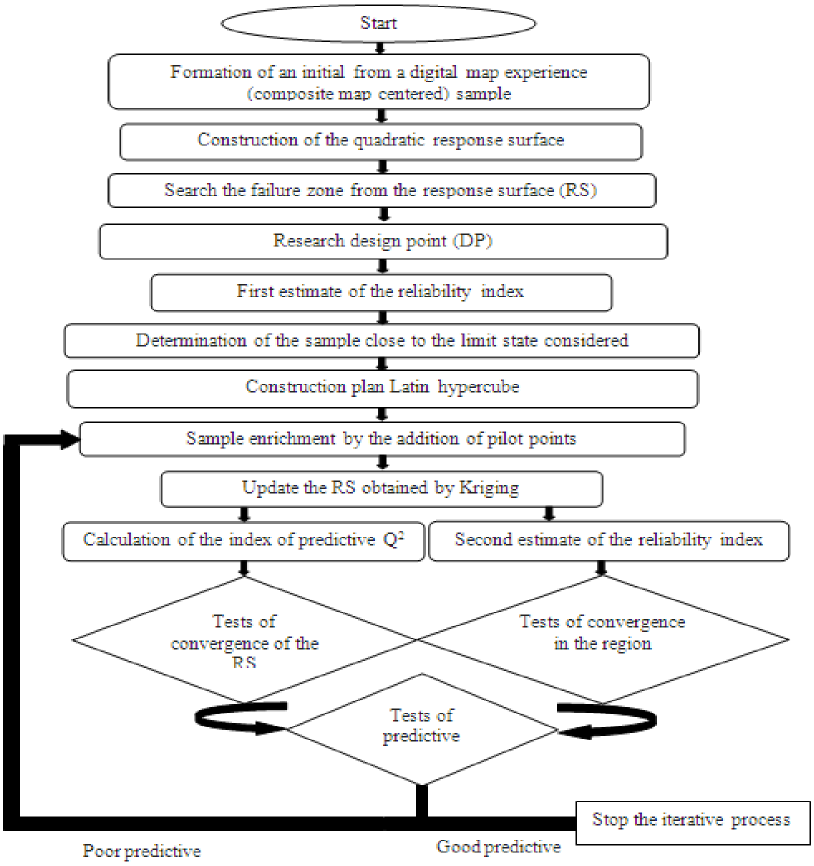

Fig.4: Algorithm for constructing the Adaptive Response Surface by Kriging using Pilot Points (ARSKPP)

VIII. Validation And Comparison Results Of The Proposed Method

The comparison of the proposed method is performed with the results of some references and the Monte Carlo method. The criteria for comparing different methods are available when the value of the reliability index Hasofer Lind β_{HL}, the number of calculations of the limit state function Ne (indicator computation time associated with method), the coefficients of the response surface and the value of the statistic Q^2.

8.1 Example1

To demonstrate the effectiveness of the proposed method (ARSKPP: Adaptive Response Surface by Kriging using Pilot Points), we use the example of a truss bridge treated by Blatman and Sudret [7]. It is a lattice whose structure is shown in Figure 5.

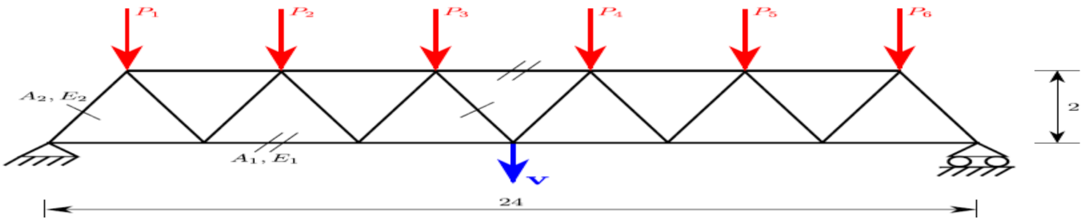

Fig.5: Truss Bridge (unit: m)

The model consists of 23 elements and 13 nodes each with two degrees of freedom before applying boundary conditions. The maximum vertical displacement of the structure indicated by (V) in Figure 5 is obtained by solving a finite element model of plane and straight bars. Random input variables are the sections, Young modules and strengths, which corresponds to 10 variables. They are considered independent of each other. Table 1 presents the probabilistic model.

Variable	Distribution laws	Average	Standard deviation
E_1-E_2 (Pa)	Log-normale	2.1×10^{11}	2.1×10^{10}
A_1(m2)	Log-normale	2×10^{-3}	2×10^{-4}
A_2(m2)	Log-normale	1×10^{-3}	1×10^{-4}
P_1-P_6 (N)	Gumbel	5×10^{4}	7.5×10^{3}

Table 1: Probabilistic Model truss bridge

It assesses the reliability of the structure with function $H(x) = 0.12 - [V(x)]$ where x are realizations of the variables x isoprobabilistic obtained by transformation of the input variables and (V) refers to the vertical displacement of the grid center.

The following curves show the good convergence of the proposed strategy. Figure 6 shows the reliability indices by iteration correspond to indices reliabilities assessed on the bounds of the interval to 95% on the prediction of the states limit. The results obtained by the proposed strategy presented above are compared with the results obtained by two other strategies based on double loop FORM and ordinary kriging. The results are given in Table 2.

Méthod	β_{HL}	Number of calls to h	Q^2	
ARSKPP	3.05	335	13 points	0.9219
	3.07	332	15 points	0.9432
	3.04	329	17 points	0.9675
	3.06	326	20 points	0.9812
	3.04	320	23 points	0.9998
FORM	3.04	1628	ND	
Kriging	2.90	620	0.716	

Table 2: Results of Example 1

According to the results, we can see that the ARSKPP method provides a value of reliability index close to that obtained by FORM.

The number of calls to the function limit state is of the order of 320 is very much lower than in other strategies.

The coefficient Q^2equals to 0.9998 means excellent predictive model. Figure 7 shows the impact of the number of pilot point on the predictability of the approximation. We started by calculating the coefficient Q^2 for the plane containing only the simulated values.
This coefficient is of the order of 0716 by cons after adding 23 pilot points it is very close to 1.
We also see from Figure 8 the influence of adding drivers focus on the number of calls to the function of limit state. To this end, the number of drivers is important, the number of calls to the limit state function h decreases and halved without the addition of these points (from 320 to 620 points with drivers without a point drivers) .

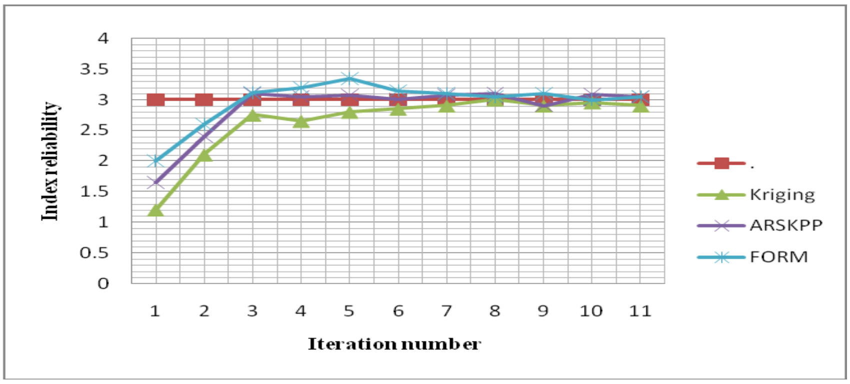

Fig.6: Number of iteration function β_{HL}

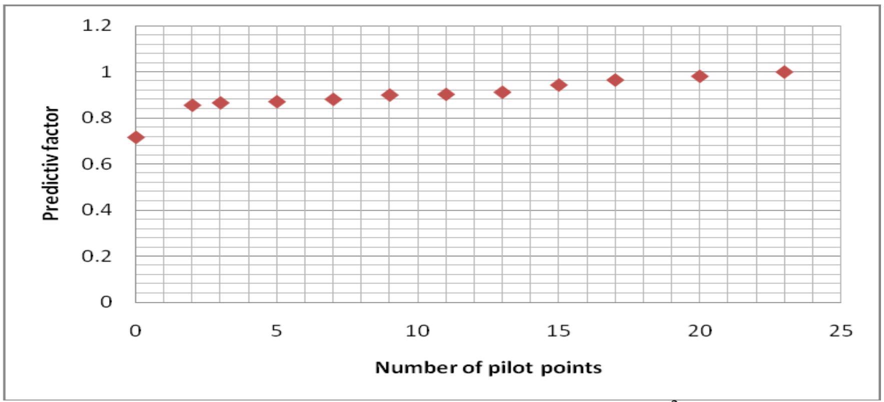

Fig.7: Number of drivers points based on Q^2

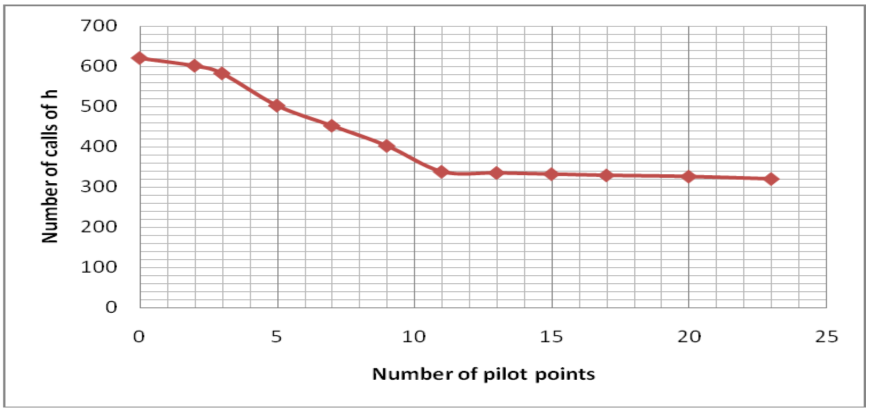

Fig.8: Number of pilots based on the number of items to call h

8.2 Example

This two-dimensional analytical example was analyzed by (Kaymaz 2005 Duprat and Sellier 2006 Nguyen et al 2009, Kang et al 2010[8 6 24 20]). The state boundary function is defined by:

$$H = exp[0.4(U_1 + 2)] - exp[0.3U_2 + 5] - 200 \tag{34}$$

All random variables have a standard normal distribution. The results are given in Table 3

Reference / Méthod	β_{HL}	U_1^*	U_2^*	$H(\bar{U})^*$	Ne
Adaptive M C S	2.71	-2.531	0.969	$-2.515x10^{-2}$	indis
Kaymaz	2.742	-2.648	0.710	4.37	indis
Duprat and Sellier	2.71	-2.538	0.951	$-7.035x10^{-2}$	21
Nguyen	2.708	-2.572	0.847	$6.536x10^{-1}$	13
MCAPP (Q^2=0.9987)	2.705	-2.569	0.839	0.8476	12 with 14 pilot points

Table 3: Results of Example 2

It is noted that the values of the reliability index are very close to each other for all methods. The ARSKPP method is much more accurate, without the number of calls to the calculation of H does not increase significantly. The coefficient Q^2 equal to 0.9987 shows that the quality of RS is excellent. As shown in Figure 9, the ARSKPP method is a method of interpolation accurate with the addition of driver points.

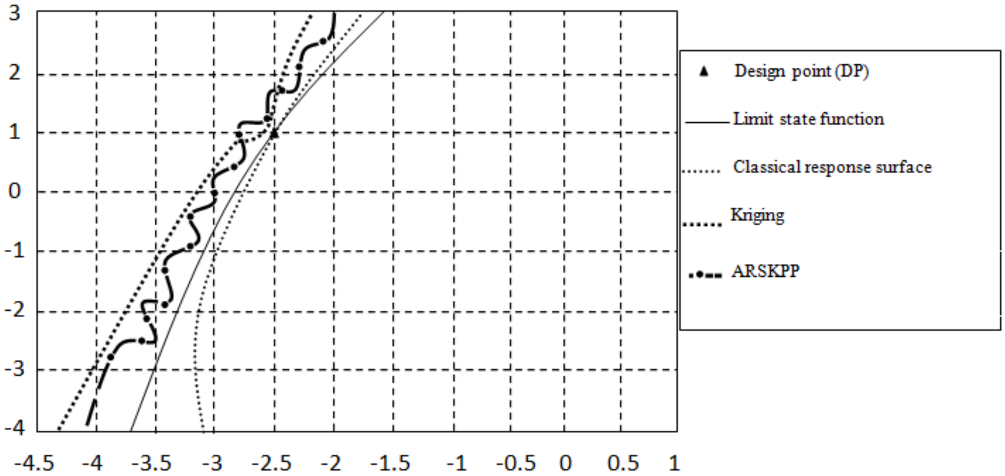

Fig.9: Response surface by kriging with and without pilot points

8.3 Example3

This example is proposed by (Wang and Gandhi 1996 [11]). The limit state function is highly nonlinear. It is given by the equation:

$$h(x) = x_1^3 + x_1^2 x_2 + x_2^3 - 18$$

Both variables x_1, x_2 are normally distributed, with mean values of 10 and 9.9 respectively and a standard deviation of 5. The results are given in the following table:

reference / method	β_{HL}	U_1^*	U_2^*
Adaptive MCS	2.53	-1.660	-1.591
Wang 1996	2.479	-1.454	-2.007
Kaymaz	2.527	-1.660	-1.905
MCAAPP (Q^2=0.9997) Ne=13 with 15 pilots point	2.519	-1.6587	-1.829

Table 4: Results of Example 3

The reliability index β_{HL} and coordinates (U_1^*, U_2^*) obtained by the methods are quite similar. However ARSKPP method is the most effective method in terms of number of calls to the calculation of the limit state function and improving the quality of approximation. Nevertheless, the effectiveness in terms of accuracy of the results is strongly dependent on the correlation parameter θ and the correlation function. In what follows, we present the influence of these parameters on the quality of the regression results.

IX. Impact Parameters Kriging

9.1Correlation parameter θ

The study of the types of regression and correlation was interesting for model robustness. Changes in correlation coefficients following a change in the type of regression or correlation is quite expected. In the literature, the construction of an optimization scheme based on maximum likelihood is used to find the value of θ giving the same weight to the experimental points. The choice of the value of θ at a very important impact on the accuracy of results. Several values of θ were tested using the same technique by giving the same weight this time the experimental points and pilot points while controlling parallel precision of results by an iterative approach. The results are shown schematically in both Figures 10 and 11 for examples 2 and 3.

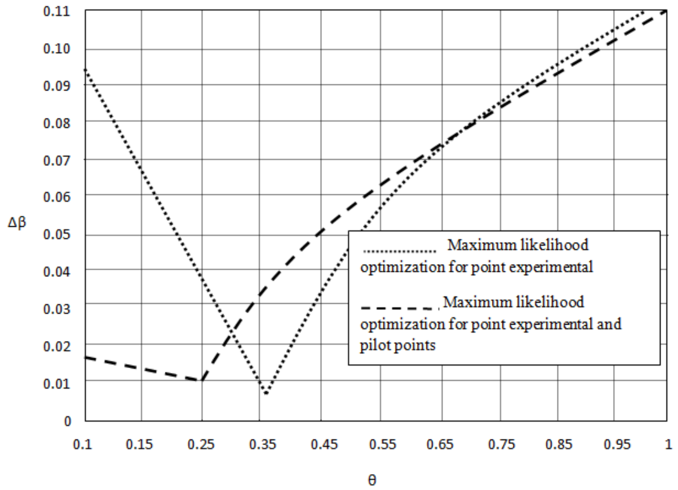

Fig.10: Different values of θ for Example 2

Figure 10 shows that the value of θ = 0.25 significantly reduces the difference between the reliability indices, which gives a value of β very close to the calculated value and this is achieved by giving the same weight to the point experimental and pilot.

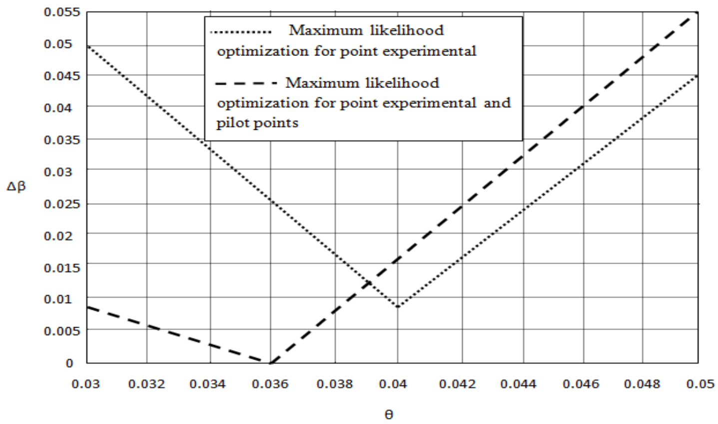

Fig.11: Different values of θ for Example 3

Similarly to Figure 11 Example 3, θ is in the range of 0.034 and a difference of β less than 0.01 for MCAAPP model. θ = 0.04 with a difference of β 0.05 without taking into account the pilot points where the impact of the contribution of these fictitious points on the accuracy of results.

9.1 Impact of the correlation function

The second most important parameter of our model is the correlation function. In this study, three types of correlation functions are tested in Examples 2 and 3 are shown in Figures 12 and 13.

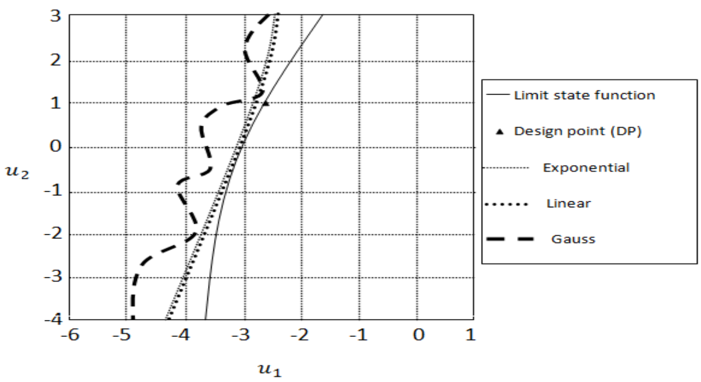

Fig.12: ARSKPP for different correlation function of Example 2

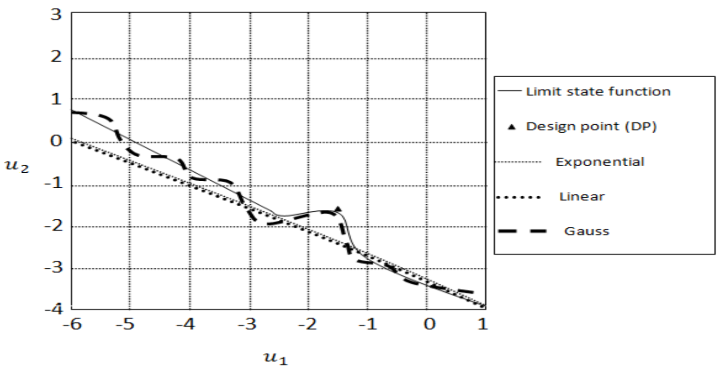

Fig.13: ARSKPP for different correlation function of Example 3

From these two figures, we find that the correlation function of Gauss is more suitable for problems with a nonlinear limit state functions and linear and exponential correlation function for the limit state functions of the same type as example 2 (A hypothetical nonlinear limit state)

X. Conclusion

This article presents the results of a research with the objective, using the kriging technique with the addition of driver points for the evaluation of the reliability of structures. The proposed method aims to improve the quality of the approximation and therefore the predictive quality of the model. Examples discussed in the literature were used to demonstrate the effectiveness of the method. Using this technique (kriging + pilot point), the method we proposed can:

- Quality control of the response surface by analyzing the predictive index Q^2 quality. -Improve approximation parts of experimental field.
- Reduce the number of simulations required for a good approximation and therefore saves computing time.
- The method is extensible: If the point of the simulated experimental design is not enough to get a proper index Q^2, additional observations are required. The proposed algorithm is able to add drivers points and put them in places to improve the approximation and prediction of the model.
- The addition of pilot points to current experimental design can significantly increase the value of the prediction coefficient Q^2 without using the finite element code.

On the other hand, the use of this method is conditioned by a good choice of these parameters (correlation parameter θ and correlation function).

The technique of maximum likelihood, giving the same weight to the experimental points and pilot points to determine the parameter θ while controlling parallel accurate results by iterative approach in the area of interest where the probability of failure is mainly calculated and close to the real value. The choice of the correlation function must be based on the type of the (linear or nonlinear) limit state function.

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