Neutrosophic Feebly Connectedness and Compactness

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Abstract: The aim of this paper is to construct the concepts related to connectedness in neutrosophic topological spaces. Here we introduce the concepts of neutrosophic feebly connected, neutrosophic feebly disconnected, neutrosophic feebly C_i - disconnected (i = 1,2,3,4), neutrosophic feebly C_i – connected (i = 1,2,3,4) and neutrosophic feebly compact in neutrosophic topological spaces and obtain several preservation properties and some characterizations concerning connectedness in these spaces.

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I. Introduction

Ever since the introduction of fuzzy sets by L.A.Zadeh [17], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C.L.Chang [3]. Atanassov[2] introduced the notion of intuitionistic fuzzy sets, Coker [4] introduced the intuitionistic fuzzy topological spaces. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and Coker[15]. The neutrosophic set was introduced by Smarandache [15] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012, Salama, Alblowi [1, 13-15], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2014, Salama,Smareandache and Valeri [16] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

Iswary et al. [9] defined the concept of neutrosoftic semi open sets in neutrosophic topological spaces. Jeyapuvaneswari et al. [10-12] defined neutrosophic feebly open sets, neutrosophic feebly closed setsand neutrosophic feebly continuous functions in neutrosophic topological spaces. In this paper, the concepts of neutrosophic feebly connected, neutrosophic feebly disconnected, neutrosophic feebly C_i- connected (i = 1, 2, 3, 4) and neutrosophic feebly compact spaces are discussed in neutrosophic topological spaces and studied several preservation properties and some characterizations concerning connectedness in these spaces.

This paper is organized as follows. Section II gives the basic definitions of neutrosophic feebly open sets, neutrosophic feebly closed sets and their properties which are used in the later sections. The Section III deals with the concept of neutrosophic feebly connected, neutrosophic feebly disconnected spaces, neutrosophic feebly C_i-connected (i = 1, 2, 3, 4), and neutrosophic feebly C_i – disconnected (i = 1, 2, 3, 4) spaces.Section IV explains neutrosophic feebly compact in neutrosophic topological spaces and their properties.

II. Preliminaries

First we shall present the fundamental definitions. The following one is obviously inspired by Smarandache [5-7] and Salama[13-15]

Definition 2.1. [15] Let X be a non-empty fixed set. A neutrosophic set A is an object having the form A = {x, μ_a(x), σ_a(x), γ_a(x) : x ∈ X} where μ_a(x), σ_a(x) and γ_a(x) which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element x ∈ X to the set A.
Neutrosophic sets $0_{n}$ and $1_{n}$ in $X$ as follows:

0$\scriptstyle{_{n}}$ may be defined as:

1. $0_{n} = \{ x, 0, 0, 1 : x \in X \}$
2. $0_{2} = \{ x, 0, 1, 1 : x \in X \}$
3. $0_{3} = \{ x, 0, 1, 0 : x \in X \}$
4. $0_{4} = \{ x, 0, 0, 0 : x \in X \}$

1$\scriptstyle{_{n}}$ may be defined as:

1. $1_{n} = \{ x, 1, 1, 0 : x \in X \}$
2. $1_{1} = \{ x, 1, 0, 1 : x \in X \}$
3. $1_{2} = \{ x, 1, 1, 0 : x \in X \}$
4. $1_{4} = \{ x, 1, 1, 1 : x \in X \}$

Definition 2.2. [15] Let $A = \langle \mu_{A}, \sigma_{A}, \gamma_{A} \rangle$ be a neutrosophic set in $X$. Then the complement of the set $A$ for short may be defined as three kinds of complements:

1. $A^{C} = \{ x, 1-\mu_{A}(x), 0-\sigma_{A}(x), 1-\gamma_{A}(x) : x \in X \}$
2. $A^{C} = \{ x, \gamma_{A}(x), \sigma_{A}(x), \mu_{A}(x) : x \in X \}$
3. $A^{C} = \{ x, \sigma_{A}(x), \gamma_{A}(x), \mu_{A}(x) : x \in X \}$

One can define several relations and operations between neutrosophic sets as follows:

Definition 2.3. [15] Let $x$ be a non-empty set, and neutrosophic sets $A$ and $B$ be in the form $A = \{ \langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_{B}(x), \sigma_{B}(x), \gamma_{B}(x) \rangle : x \in X \}$. Then we may consider two possible definitions for subsets $A \subseteq B$.

$A \subseteq B$ may be defined as:

1. $A \subseteq B \Rightarrow \mu_{A}(x) \leq \mu_{B}(x)$, $\sigma_{A}(x) \leq \sigma_{B}(x)$ and $\gamma_{A}(x) \geq \gamma_{B}(x)$, for all $x \in X$.
2. $A \subseteq B \Rightarrow \mu_{A}(x) \leq \mu_{B}(x)$, $\sigma_{A}(x) \leq \sigma_{B}(x)$ and $\gamma_{A}(x) \geq \gamma_{B}(x)$, for all $x \in X$.

Definition 2.4. [15] Let $X$ be a non-empty set, and $A = \{ \langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x) \rangle, B = \{ \langle x, \mu_{B}(x), \sigma_{B}(x), \gamma_{B}(x) \rangle : x \in X \}$ are neutrosophic subsets. Then

1. $A \setminus B$ may be defined as:
2. $A \setminus B = \{ \langle x, \mu_{A}(x) \wedge \mu_{B}(x), \sigma_{A}(x) \wedge \sigma_{B}(x) \wedge \gamma_{A}(x) \wedge \gamma_{B}(x) \rangle : x \in X \}$
3. $A \setminus B = \{ \langle x, \mu_{A}(x) \wedge \mu_{B}(x), \sigma_{A}(x) \wedge \sigma_{B}(x) \wedge \gamma_{A}(x) \wedge \gamma_{B}(x) \rangle : x \in X \}$

We can easily generalize the operations of intersection and union in Definition 2.4 to arbitrary family of neutrosophic sets as follows:

Definition 2.5. [15] Let $\{A_{j} : j \in J \}$ be an arbitrary family of neutrosophic sets in $X$. Then

1. $\bigwedge_{j} A_{j}$ may be defined as:
2. $\bigwedge_{j} A_{j} = \{ \langle x, \mu_{A_{j}}(x), \sigma_{A_{j}}(x), \gamma_{A_{j}}(x) \rangle, V_{j} \} \}$
3. $\bigwedge_{j} A_{j} = \{ \langle x, \mu_{A_{j}}(x), \sigma_{A_{j}}(x), \gamma_{A_{j}}(x) \rangle, V_{j} \} \}$

Definition 2.6. [15] Let $A = \{ \langle x, \mu_{A}(x), \sigma_{A}(x), \gamma_{A}(x) \rangle, B = \{ \langle x, \mu_{B}(x), \sigma_{B}(x), \gamma_{B}(x) \rangle : x \in X \}$ be neutrosophic subsets of $X$ and $Y$ respectively. Then $A \times B$ is a neutrosophic set of $X \times Y$ defined by

1. $(P_{1})A \times B = \{ \langle x, y \rangle, \min (\mu_{A}(x), \mu_{B}(y)), \min (\sigma_{A}(x), \sigma_{B}(y)), \max (\gamma_{A}(x), \gamma_{B}(y)) \}$
2. $(P_{2})A \times B = \{ \langle x, y \rangle, \min (\mu_{A}(x), \mu_{B}(y)), \min (\sigma_{A}(x), \sigma_{B}(y)), \max (\gamma_{A}(x), \gamma_{B}(y)) \}$

Notice that

1. $(CP_{1})A \times B = \{ \langle x, y \rangle, \min (\mu_{A}(x), \mu_{B}(y)), \min (\sigma_{A}(x), \sigma_{B}(y)), \min (\gamma_{A}(x), \gamma_{B}(y)) \}$
2. $(CP_{2})A \times B = \{ \langle x, y \rangle, \min (\mu_{A}(x), \mu_{B}(y)), \min (\sigma_{A}(x), \sigma_{B}(y)), \min (\gamma_{A}(x), \gamma_{B}(y)) \}$

Definition 2.7. [15] A neutrosophic topology $[NT]$ is a non-empty set $X$ is a family $\tau$ of neutrosophic subsets in $X$ satisfying the following axioms:

1. $0_{n}, 1_{n} \in \tau$
2. $G_{1} \cap G_{2} \in \tau$ for any $G_{1}, G_{2} \in \tau$
3. $\forall G_{i} \in \tau$ for every $G_{i} : i \in I$ is $\tau$

In this case the pair $(X, \tau)$ is called a neutrosophic topological space. The elements of $\tau$ are called neutrosophic open sets.

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**Definition 2.8.** [15] The complement of \( A \) \((A^c \text{ for short})\) of a neutrosophic closed set is called a neutrosophic open set in \( X \).

**Definition 2.9.** [10] A neutrosophic subset \( A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle \) in a neutrosophic topological space \((X, \tau)\) is said to be a neutrosophic neighborhood of a neutrosophic point \( x \in X \), if there exists a neutrosophic open set \( B = \langle x, \mu_B, \sigma_B, \gamma_B \rangle \) with \( x, x, x, x \leq B \leq A \).

Now, we define neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces:

**Definition 2.10**[16] Let \( X \) and \( Y \) be two nonempty neutrosophic sets and \( f : X \to Y \) be a function.

(i) If \( B = \{ (y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y \} \) is a neutrosophic set in \( Y \), then the pre image of \( B \) under \( f \) is denoted and defined by \( f^{-1}(B) = \{ (x, f^{-1}(\mu_B(x)), f^{-1}(\sigma_B(x)), f^{-1}(\gamma_B(x)) : x \in X \} \).

(ii) If \( A = \{ x, \mu_A(x), \sigma_A(x), \gamma_A(x) \} : x \in X \} \) is a NS in \( X \), then the image of \( A \) under \( f \) is denoted and defined by \( f(A) = \{ (y, f(\mu_A(y)), f(\sigma_A(y)), f(\gamma_A(y)) : y \in Y \} \) where \( f(\lambda_A) = C \{ f(C(A)) \} \).

In (i), (ii), since \( \mu_B, \sigma_B, \gamma_B, \mu_A, \sigma_A, \gamma_A \) are neutrosophic sets, we explain that \( f^{-1}(\mu_B(x)) = \mu(f(x)) \), and if \( f^{-1}(\mu_B(x)) \neq \varnothing \)

\[
\text{and} \quad (\alpha_A(y) = \begin{cases} \sup \{ \alpha_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \varnothing \\ 0, & \text{otherwise} \end{cases}
\]

**Lemma 2.11.** [16] Let \( f : X \to Y \) be a function. If \( A \) is a neutrosophic subset of \( X \) and \( \mu \) is a neutrosophic subset of \( Y \). Then

(i) \[ f(f^{-1}(A)) \leq A \]

(ii) \[ f(f^{-1}(A)) = A \iff f \text{ is surjective.} \]

(iii) \[ f^{-1}(f(A)) \geq A \]

(iv) \[ f^{-1}(f(A)) = A \text{ whenever } f \text{ is injective.} \]

**Definition 2.12.** [10] A neutrosophic subset \( A \) of a neutrosophic topological space \((X, \tau)\) is neutrosophic feebly open if there is a neutrosophic open set \( U \) in \( X \) such that \( U \leq A \leq NSc(U) \).

**Lemma 2.13.** [10] (i) Every neutrosophic open set is a neutrosophic feebly open set.

**Definition 2.14.** [10] A neutrosophic subset \( A \) of a neutrosophic topological space \((X, \tau)\) is neutrosophic feebly closed if there is a neutrosophic closed set \( U \) in \( X \) such that \( NSint(U) \leq A \leq U \).

**Lemma 2.15.** [10] A neutrosophic subset \( A \) of a neutrosophic topological space \((X, \tau)\) is

(i) neutrosophic feebly closed if and only if \( Nc(Ncl(A)) \leq A \).

(ii) A neutrosophic subset \( A \) is neutrosophic feebly closed iff \( NSSc(Ncl(A)) \leq A \).

(iii) A neutrosophic subset \( A \) is a neutrosophic feebly closed set if and only if \( A^c \) is neutrosophic feebly open.

(iv) Every neutrosophic closed set is a neutrosophic feebly closed set.

**Definition 2.16.** [10] Let \((X, \tau)\) be neutrosophic topological space and \( A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle \) be a neutrosophic set in \( X \). Then neutrosophic feebly interior of \( A \) is defined by \( NFint(A) = \bigvee \{ G : G \text{ is a neutrosophic feebly open set in } X \text{ and } G \leq A \} \).

**Lemma 2.17.** [10] Let \((X, \tau)\) be neutrosophic topological space. Then for any neutrosophic feebly subsets \( A \) and \( B \) of a neutrosophic topological space \( X \), we have

(i) \( NFint(A) \leq A \)

(ii) \( A \) is a neutrosophic feebly open set in \( X \) iff \( NFint(A) = A \)

**Definition 2.18**[10] Let \((X, \tau)\) be neutrosophic topological space and \( A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle \) be a neutrosophic set in \( X \). Then the neutrosophic feebly closure is defined by \( NFcl(A) = \bigwedge \{ K : K \text{ is a neutrosophic feebly closed set in } X \text{ and } A \leq K \} \).

**Lemma 2.19.** [10] Let \((X, \tau)\) be a neutrosophic topological space. Then for any neutrosophic subset \( A \) of \( X \),

(i) \( NFcl(A)^c = NFcl(A^c) \)

(ii) \( NFcl(A)^c = NFcl(A^c) \).

**Lemma 2.20.** [10] Let \((X, \tau)\) be a neutrosophic topological space. Then for any neutrosophic subsets \( A \) and \( B \) of a neutrosophic topological space \( X \),

(i) \( A \leq NFcl(A) \)

(ii) \( A \) is a neutrosophic feebly closed set in \( X \) iff \( NFcl(A) = A \)

(iii) \( A \leq NFcl(A) \leq NFcl(B) \).

**Definition 2.21.** [11] Let \((X, \tau)\) and \((Y, \sigma)\) be neutrosophic topological spaces. Then a map \( f : (X, \tau) \to (Y, \sigma) \) is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in \((Y, \sigma)\) is neutrosophic open set in \((X, \tau)\).
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**Definition 2.22.**[12] Let \((X, \tau)\) and \((Y, \sigma)\) be two neutrosophic topological spaces. A function \(f: X \rightarrow Y\) is called neutrosophic feebly irresolute if the inverse image of every neutrosophic feebly open set in \(Y\) is neutrosophic feebly open in \(X\).

**III. Neutrosophic Feebly Connected Spaces**

In this section, we study the notion of neutrosophic feebly connected, neutrosophic feebly disconnected in neutrosophic topological spaces. Also, we introduced neutrosophic feebly \(C_i\)-connected \((i = 1,2,3,4)\) and neutrosophic feebly \(C_i\)-disconnected \((i = 1,2,3,4)\) and we give some properties and theorems of such concepts.

**Definition 3.1** A neutrosophic topological space \((X, \tau)\) is neutrosophic feebly disconnected if there exists neutrosophic feebly open sets \(A, B\) in \(X, \tau\) such that \(X\) \(AVB = 1_N\) and \(B\GammaA = 0_N\).

That is

(i) \(A < \mu_A \lor \sigma_B, \sigma_A \land \gamma_B > = 1_N\),
(ii) \(< x, \mu_A \lor \sigma_B, \sigma_A \land \gamma_B > = 1_N\),
(iii) \(< x, \mu_A \lor \sigma_B, \sigma_A \land \gamma_B > = 0_N\)

(iv) \(< x, \mu_A \lor \sigma_B, \sigma_A \land \gamma_B > = 0_N\)

The following example shows that \(X\) is neutrosophic feebly disconnected space.

**Example 3.2.** Let \(X = \{a, b, c\}, \tau = \{\{0_N, 1_N, A, B, C, D\}\}\) where

\[
A = \{x, (0, 0, 0), (1, 0, 1), (1, 1, 0)\}
\]

\[
B = \{x, (0, 1, 0), (1, 1, 0), (0, 0, 0)\}
\]

\[
C = \{x, (1, 0, 0), (1, 1, 0), (0, 0, 1)\}
\]

\[
D = \{x, (0, 0, 0), (0, 0, 1), (0, 1, 0)\}
\]

\(A\) and \(B\) are neutrosophic feebly open sets in \(X, \tau\), \(A \neq 0_N, B \neq 0_N,\) and \(AVB = 0_N\). Hence \(X\) is neutrosophic feebly disconnected.

If \(X\) is not neutrosophic feebly disconnected then it is said to be neutrosophic feebly connected.

**Example 3.3.** Let \(X = \{a, b\}, \tau = \{\{0_N, 1_N, G_1\}\}\) where \(G_1 = \{x, (0, 0, 1), (1, 0, 1), (1, 0, 0)\}\). Then \((X, \tau)\) is a neutrosophic topological space. Let \(G_2 = \{x, (0, 1, 0), (0, 0, 1), (1, 0, 0)\}\). Then \(G_2\) and \(G_3\) are neutrosophic feebly open sets in \(X, G_2 \neq 0_N, G_2 \neq 0_N,\) and \(G_2 \cap G_3 = G_2 \cap G_3 = 0_N\) Hence \(X\) is neutrosophic feebly connected.

**Example 3.4.** Let \(\{a, b\}\), \(\tau = \{\{0_N, 1_N, G_1\}\}\) where \(G_1 = \{x, (0, 2, 0), (0, 7, 0), (0, 7, 0)\}\). Then \((X, \tau)\) is a neutrosophic topological space. Let \(G_2 = \{x, (0, 1, 0), (0, 1, 1)\}\) and \(G_3 = \{x, (0, 0, 1), (0, 0, 0)\}\). It can be found that \(G_2\) and \(G_3\) are neutrosophic feebly open sets in \(X, G_2 \neq 0_N, G_3 \neq 0_N,\) and \(G_2 \cap G_3 = G_2 \cap G_3 = 0_N\). Hence \(X\) is neutrosophic feebly connected.

**Definition 3.5.** Let \(N\) be a neutrosophic subset in neutrosophic topological space \((X, \tau)\)

(a) If there exists neutrosophic feebly open sets \(U\) and \(V\) in \(X\) satisfying the following properties, then \(N\) is called neutrosophic feebly \(C_i\)-disconnected \((i = 1,2,3,4)\):

\[
\begin{align*}
& \cdot C_1: N \subseteq UV, U \lor V \subseteq N \oplus, N \land U \neq 0_N, N \land V \neq 0_N \\
& \cdot C_2: N \subseteq UV, N \lor U \land V \neq 0_N, N \land U \neq 0_N, N \land V \neq 0_N \\
& \cdot C_3: N \subseteq UV, U \land V \neq 0_N, U \neq N \oplus, V \neq N \oplus \\
& \cdot C_4: N \subseteq UV, N \lor U \land V \neq 0_N, U \neq N \oplus, V \neq N \oplus
\end{align*}
\]

(b) \(N\) is said to be neutrosophic feebly \(C_i\)-connected \((i = 1,2,3,4)\) if \(N\) is not neutrosophic feebly \(C_i\)-disconnected \((i = 1,2,3,4)\).

Obviously, we can obtain the following implications between several types of neutrosophic feebly \(C_i\)-connected \((i = 1,2,3,4)\),

1. neutrosophic feebly \(C_1\)-connected \(\Rightarrow\) neutrosophic feebly \(C_2\)-connected
2. neutrosophic feebly \(C_2\)-connected \(\Rightarrow\) neutrosophic feebly \(C_3\)-connected
3. neutrosophic feebly \(C_3\)-connected \(\Rightarrow\) neutrosophic feebly \(C_4\)-connected
4. neutrosophic feebly \(C_4\)-connected \(\Rightarrow\) neutrosophic feebly \(C_1\)-connected

**Example 3.6.** Let \(\{a, b\}, \tau = \{\{0_N, 1_N, G_1, G_2\}\}\) where

\[
G_1 = \{x, (0, 4, 0), (0, 6, 0), (0, 6, 0)\}
\]

\[
G_2 = \{x, (0, 5, 0), (0, 5, 0), (0, 5, 0)\}
\]

Consider the neutrosophic subset \(G_3 = \{x, (0, 3, 0), (0, 7, 0), (0, 7, 0)\}\). Then \(G_3\) is neutrosophic feebly \(C_2\)-connected, neutrosophic feebly \(C_3\)-connected, neutrosophic feebly \(C_4\)-connected but neutrosophic feebly \(C_1\)-disconnected.

**Example 3.7.** Let \(\{a, b\}, \tau = \{\{0_N, 1_N, G_1, G_2, G_3\}\}\) where

\[
G_3 = \{x, (0, 2, 0), (0, 6, 0), (0, 6, 0), (0, 6, 0)\}
\]

\[
G_2 = \{x, (0, 8, 0), (0, 2, 0), (0, 2, 0), (0, 2, 0)\}
\]

Consider the neutrosophic subset \(G_3 = \{x, (0, 1, 0), (0, 9, 0), (0, 9, 0)\}\). Then \(G_3\) is neutrosophic feebly \(C_3\)-connected but neutrosophic feebly \(C_3\)-disconnected.
Definition 3.8. A neutrosophic topological space \((X,\tau)\) is neutrosophic feebly \(C_2\)-disconnected if there exists neutrosophic subset \(A\) in \(X\) which is both neutrosophic feebly open and neutrosophic feebly closed in \(X\), such that \(A \neq 0_N\), \(A \neq 1_N\). If \(X\) is not neutrosophic feebly \(C_2\)-disconnected then it is said to be neutrosophic feebly \(C_2\)-connected.

Example 3.9. Let \(\{a, b\}, \tau = \{0_N, 1_N, G_1\}\) where \(G_1 = \{(x, (0.2, 0.6), (0.5, 0.5), (0.2, 0.6))\}\). Then \(G_1\) is neutrosophic feebly open and neutrosophic feebly closed set in \(X\), \(G_1 \neq 0_N\), \(G_1 \neq 1_N\). Thus \(X\) is neutrosophic feebly \(C_2\)-connected.

Theorem 3.10. Neutrosophic feebly \(C_2\)-disconnectedness implies neutrosophic feebly connectedness.

Proof. Suppose that there exists non-empty neutrosophic feebly open sets \(A\) and \(B\) such that \(AB = 0_N\). Then \(\mu_AV\mu_B = 1_N, \sigma_A\Lambda\sigma_B = 0_N, \gamma_A\Lambda\gamma_B = 0_N, \mu_A\Lambda\mu_B = 0_N, \sigma_A\Lambda\sigma_B = 1_N, \gamma_A\Lambda\gamma_B = 1_N\). In other words, \(B^c = A\). Hence \(X\) is neutrosophic feebly clopenwhich implies \(X\) is neutrosophic feebly connected.

Theorem 3.11. A neutrosophic topological space \((X, \tau)\) is neutrosophic feebly irresolute surjection and \(X\) be neutrosophic feebly connected. Then \(Y\) is neutrosophic feebly connected.

Proof. Suppose that \(Y\) is not neutrosophic feebly connected, then there exists non-empty neutrosophic feebly open sets \(U\) and \(V\) in \(Y\) such that \(UV = 1_N\) and \(U^c = 0_N\). Since \(f\) is neutrosophic feebly irresolute mapping, \(A = f^{-1}(U) \neq 0_N, B = f^{-1}(V) \neq 0_N\), which are neutrosophic feebly open sets in \(X\) and \(f^{-1}(U)Vf^{-1}(V) = f^{-1}(1_N) = 1_N\), which implies \(AVB = 1_N\). Also \(f^{-1}(U)Vf^{-1}(V) = f^{-1}(0_N) = 0_N\), which implies \(AAB = 0_N\). Thus \(X\) is neutrosophic feebly disconnected, which is a contradiction to our hypothesis. Hence \(Y\) is neutrosophic feebly connected.

Theorem 3.12. A neutrosophic topological space \((X, \tau)\) is neutrosophic feebly irresolute surjection and \(X\) be neutrosophic feebly disconnected. Then \(Y\) is neutrosophic feebly connected.

Proof. Suppose that \(Y\) and \(V\) are neutrosophic feebly open sets in \(X\) such that \(U \neq 0_N, V \neq 0_N\) and \(U = V^c\). Since \(U = V^c\), \(V\) is a neutrosophic feebly open sets and follows that \(V\) is neutrosophic feebly closed set and \(U \neq 0_N\) implies \(V \neq 1_N\). But this is a contradiction to the fact that \(X\) is neutrosophic feebly \(C_2\)-connected.

Conversely, let \(U\) be a both neutrosophic feebly open set and \(V\) be neutrosophic feebly closed set in \(X\) such that \(U \neq 0_N, U \neq 1_N\). Now take \(U^c = V\) is a neutrosophic feebly open set and \(U \neq 1_N\). This implies \(U^c = V \neq 0_N\) which is a contradiction. Hence \(X\) is neutrosophic feebly \(C_2\)-connected.

Theorem 3.13. A neutrosophic topological space \((X, \tau)\) is neutrosophic feebly connected if and only if there exists non-zero neutrosophic feebly open set \(U\) and \(V\) in \(X\) such that \(U = V^c\).

Proof. Necessity: Let \(U\) and \(V\) be two neutrosophic feebly open set in \((X, \tau)\) such that \(U \neq 0_N, V \neq 0_N\) and \(V = U^c\). Therefore \(V\) is a neutrosophic feebly closed set. Since \(U \neq 0_N, V \neq 1_N\), this implies \(V\) is a proper neutrosophic subset which is both neutrosophic feebly open set and neutrosophic feebly closed set in \(X\). Hence \(X\) is not a neutrosophic feebly connected space. But this is a contradiction to our hypothesis. Thus, there exist no non-zero neutrosophic feebly open sets \(U\) and \(V\) in \(X\), such that \(U = V^c\).

Sufficiency: Let \(U\) be both neutrosophic feebly open and neutrosophic feebly closed set in \(X\) such that \(U \neq 0_N, U \neq 1_N\). Now let \(V = U^c\). Then \(V\) is a neutrosophic feebly open set and \(V \neq 1_N\). This implies \(U^c = V \neq 0_N\), which is a contradiction to our hypothesis. Therefore \(X\) is neutrosophic feebly connected space.

Theorem 3.14. A neutrosophic topological space \((X, \tau)\) is neutrosophic feebly connected if and only if there exists non-zero neutrosophic subsets \(U\) and \(V\) in \((X, \tau)\) such that \(U = V^c\). Since \(V = (NFcl(U))^c\) and \(U = (NFcl(V))^c\).

Proof. Necessity: Let \(U\) and \(V\) be two neutrosophic subsets in \((X, \tau)\) such that \(U \neq 0_N, V \neq 0_N\) and \(U = V^c, V = (NFcl(U))^c\) and \(U = (NFcl(V))^c\). Since \((NFcl(U))^c\) and \((NFcl(V))^c\) are neutrosophic feebly open sets in \(X\), \(U\) and \(V\) are neutrosophic feebly open set in \(X\). This implies \(X\) is not a neutrosophic feebly connected space, which is a contradiction. Therefore, there exist no non-zero neutrosophic feebly open set \(U\) and \(V\) in \(X\), such that \(U = V^c, V = (NFcl(U))^c\) and \(U = (NFcl(V))^c\).
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Sufficiency: Let $U$ be both neutrosophic feebly open and neutrosophic feebly closed set in $X$ such that $U \neq 0_N, U \neq 1_N$. Now by taking $V=U^c$ we obtain a contradiction to our hypothesis. Hence $X$ is neutrosophic feebly connected space.

Definition 3.16. A neutrosophic topological space $(X, \tau)$ is neutrosophic feebly strongly connected, if there exists no nonempty neutrosophic feebly closed sets $A$ and $B$ in $X$ such that $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$. In other words, a neutrosophic topological space $X$ is neutrosophic feebly strongly connected, if there exists no nonempty neutrosophic feebly closed sets $A$ and $B$ in $X$ such that $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$.

Theorem 3.17. An neutrosophic topological space $(X, \tau)$ is neutrosophic feebly strongly connected, if there exists no nonempty neutrosophic feebly open sets $A$ and $B$ in $X$ such that $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$.

Proof. Let $A$ and $B$ be neutrosophic feebly open sets in $X$ such that $A \neq 1_B$ and $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$. If we take $C = A^c$ and $D = B^c$, then $C$ and $D$ become neutrosophic feebly closed sets in $X$, and $\mu_C + \mu_D \geq 1_N, \sigma_C + \sigma_D \geq 1_N, \gamma_C + \gamma_D \leq 1_N$. Hence $X$ is neutrosophic feebly strongly connected. Conversely, use a similar technique as above.

Example 3.18. Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, A, B\}$ where $A = \langle x, (0, 0, 0.6), (0.6, 0.6), (0.6, 0.9) \rangle$. Then $A$ and $B$ are neutrosophic feebly open sets in $X$, also $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$. Hence $X$ is neutrosophic feebly strongly connected.

Theorem 3.19. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a neutrosophic feebly irresolute surjection, $X$ be a neutrosophic feebly strongly connected. Then $Y$ is also neutrosophic feebly strongly connected.

Proof. Assume that $Y$ is not neutrosophic feebly strongly connected, then there exists nonempty neutrosophic feebly closed sets $U$ and $V$ in $Y$ such that $U \neq 0_Y, V \neq 0_Y$ and $UV = 0_Y$. Since $f$ is neutrosophic feebly irresolute mapping, $f^{-1}(U) \neq 0_Y, f^{-1}(V) \neq 0_Y$, which are neutrosophic feebly closed sets in $X$. Thus $X$ is a neutrosophic feebly strongly connected, which is a contradiction to our hypothesis. Hence $Y$ is neutrosophic feebly strongly connected.

Remark 3.20. Neutrosophic feebly strongly connected and neutrosophic feebly connected are independent as shown by the following example.

Example 3.21. Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, A, B\}$ where $A = \langle x, (0, 0.6), (0.6, 0.6), (0.2, 0.3) \rangle$, $B = \langle x, (0.3, 0.1), (0.2, 0.6), (0.2, 0.3) \rangle$. Then $A$ and $B$ are neutrosophic feebly open sets in $X$, also $\mu_A + \mu_B \geq 1_N, \sigma_A + \sigma_B \geq 1_N, \gamma_A + \gamma_B \leq 1_N$. Hence $X$ is neutrosophic feebly strongly connected. But $X$ is not neutrosophic feebly Connected, since $X$ is both neutrosophic feebly open and neutrosophic feebly closed in $X$.

Example 3.22. Let $\tau = \{0_N, 1_N, A, B\}$ where $A = \langle x, (0, 0.5), (0.3, 0.4), (0.6, 0.2) \rangle$, $B = \langle x, (0.5, 0.4), (0.2, 0.4), (0.2, 0.2) \rangle$. Hence $X$ is neutrosophic feebly $C_2$-connected, but $X$ is not neutrosophic feebly strongly connected.

IV. Neutrosophic Feebly Compact Spaces

In this section we introduce the concept neutrosophic feebly compact spaces using neutrosophic feebly open sets and study some of their basic properties.

Definition 4.1. A collection $B$ of neutrosophic feebly open sets in $X$ is called a neutrosophic feebly open cover of a subset $B$ of $X$ if $B \leq V\{U_a|U_a \in B\}$.

Definition 4.2. A topological space $X$ is said to be neutrosophic feebly compact if every neutrosophic feebly open cover of $X$ has a finite subcover.

Definition 4.3. A subset $A$ of a topological space $X$ is said to be neutrosophic feebly compact relative to $X$ if every neutrosophic feebly open cover of $A$ has a finite subcover.

Theorem 4.4. Every neutrosophic feebly compact space is neutrosophic compact.

Proof. Let $X$ be neutrosophic feebly compact. Suppose $X$ is not neutrosophic compact. Then there exists a neutrosophic open cover $B$ of $X$ that has no finite subcover. This is a contradiction to $X$ being neutrosophic feebly compact. Hence $X$ is neutrosophic compact.

Theorem 4.5. A neutrosophic feebly compact subset of a neutrosophic feebly compact space $X$ is neutrosophic feebly compact relative to $X$.

Proof. Let $A$ be a neutrosophic feebly closed subset of a neutrosophic feebly compact space $X$. Then $A^c$ is a neutrosophic feebly open in $X$. Let $\mathcal{B}=\{A_i: i \in I\}$ be a neutrosophic feebly open cover of $A$. Then $\mathcal{B}\cup\{A^c\}$ is a
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Neutrosophic feebly open cover of $X$. Since $X$ is neutrosophic feebly compact, it has a finite subcover say \( \{ P_1, P_2, \ldots, P_n \} \). Then \( \{ P_1, P_2, \ldots, P_n \} \) is a finite neutrosophic feebly open cover. Thus $A$ is neutrosophic feebly compact relative to $X$.

**Theorem 4.6.** Let $f: X \to Y$ be neutrosophic feebly continuous surjection and $X$ be neutrosophic feebly compact. Then $Y$ is neutrosophic compact.

**Proof.** Let $f: X \to Y$ be a neutrosophic feebly continuous surjection and $X$ be neutrosophic feebly compact. Let \( \{ V_i \} \) be a neutrosophic open cover for $X$. Since $f$ is neutrosophic feebly continuous, \( \{ f^{-1}(V_i) \} \) is a neutrosophic feebly open cover of $X$. Since $X$ is neutrosophic feebly compact, \( \{ f^{-1}(V_i) \} \) contains a finite subcover, namely \( \{ f^{-1}(V_{i_1}), f^{-1}(V_{i_2}), \ldots, f^{-1}(V_{i_n}) \} \). Since $f$ is surjection, \( \{ V_{i_1}, V_{i_2}, \ldots, V_{i_n} \} \) is a finite subcover for $Y$. Thus $Y$ is neutrosophic compact.

**Theorem 4.7.** Let $f: X \to Y$ be a neutrosophic feebly open function and $Y$ be neutrosophic feebly compact. Then $X$ is neutrosophic compact.

**Proof.** Let $f: X \to Y$ be a neutrosophic feebly open function and $Y$ be neutrosophic feebly compact. Let \( \{ V_i \} \) be a neutrosophic open cover for $X$. Since $f$ is neutrosophic feebly open, \( \{ f(V_i) \} \) is a neutrosophic feebly open cover of $Y$. Since $Y$ is neutrosophic feebly compact, \( \{ f(V_i) \} \) contains a finite sub neutrosophic feebly open cover, namely \( \{ f(V_{i_1}), f(V_{i_2}), \ldots, f(V_{i_n}) \} \). Then \( \{ V_{i_1}, V_{i_2}, \ldots, V_{i_n} \} \) is a finite subcover for $X$. Thus $X$ is neutrosophic compact.

**Theorem 4.8.** The image of a neutrosophic compact space under a neutrosophic continuous map is neutrosophic feebly compact.

**Proof.** Let \( f: (X, \tau) \to (Y, \sigma) \) be a neutrosophic feebly continuous map from a neutrosophic feebly compact space $X$ onto a feebly topological space $(Y, \sigma)$. Let $\{ A_i : i \in I \}$ be a neutrosophic open cover of $(Y, \sigma)$. Since $f$ is neutrosophic continuous, \( \{ f^{-1}(A_i) : i \in I \} \) is a neutrosophic feebly open cover of $(X, \tau)$. As $(X, \tau)$ is neutrosophic feebly compact, the neutrosophic open cover \( \{ f^{-1}(A_i) : i \in I \}$ of $(X, \tau)$ has a finite subcover \( \{ f^{-1}(A_i) : i=1,2,3,\ldots,n \} \). Therefore \( U = \bigcup_{i \in I} f^{-1}(A_i) \). Then \( f(U) = \bigcup_{i \in I} A_i \) that is $V = \bigcup_{i \in I} A_i$. Thus \( \{ A_1, A_2, \ldots, A_n \} \) is a finite subcover of \( \{ A_i : i \in I \}$ for $(Y, \sigma)$. Hence $(Y, \sigma)$ is neutrosophic feebly compact.

V. Conclusion

Neutrosophic feebly connectedness and neutrosophic feebly disconnectedness in the neutrosophic topological space have been introduced. Also, we have introduced the neutrosophic feebly compactness and we hope that the findings in this chapter will help researchers to enhance and promote the further study on neutrosophic topology to carry out general framework for their applications in practical life.

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