Portraying Pedagogical Content Knowledge (PCK) of Novice Mathematics Teachers Using Vignette

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Abstract: This study aimed to describe the pedagogical content knowledge of novice mathematics teachers on the material system of linear equations in two variables. The approach used is qualitative descriptive. The main instrument is the researchers themselves and the supporting instruments is a vignette and interview guides. Procedure research includes: granting vignette to the subject and continued the interview. Data were analyzed using Karahasan frameworks. The results showed that the pedagogical content knowledge of the subject 1 are knowledge of teaching, knowledge of the learners, and content knowledge they are all on level 1. As for the subject 2, knowledge of teaching and knowledge of the learners are at level 2 and content knowledge is at level I.

Keywords: pedagogical content knowledge, system of linear equations in two variables (SLETV), vignette

I. Introduction

The teacher's knowledge is a sufficient condition for successful learning particular material. The teacher's knowledge includes content knowledge and pedagogical knowledge. Both must be owned by a teacher well. However, the facts show that not a few teachers who have a good content knowledge but lacking in pedagogical knowledge, or otherwise have a good pedagogical knowledge but lacking in content knowledge. Ideally, a teacher must possess both the knowledge (content and pedagogical) with both at once can combine the two in instructional practices. The merger of content knowledge and pedagogical initiated by Shulman in terms of Pedagogical Content Knowledge (PCK).

According to Shulman (1986) The category of pedagogical content knowledge includes the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstration-in a word, ways of representing and formulating the subject that makes it comprehensible to others. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to learning of those most frequently taught topics and lessons.

While (Loughran et al., 2006) is defined the pedagogical content knowledge (PCK) of a teacher as their knowledge of content and how to teach that particular content. PCK was defined as the knowledge of, the rationale behind, planning for, and the act of teaching a piece of subject matter using specific methods for specific students to promote student learning (Gess-Newsome & Carlson, 2014). In addition, according to Abbit (2011) that PCK is knowledge of pedagogy, learning practices and lesson planning, as well as the appropriate methods to teach the material. According to Subanji (2015) PCK be the main thing for the development of teacher competence. By mastering pedagogical content at the same time, teachers will be easy to make students learning achievement maximum. This can happen because the teacher will understand how the process of knowledge construction by students.

The material system of linear equations in two variables (SLETV) given in class X Senior High School. In this material the students often have misconceptions and difficulties. For example, students are given about SLETV with infinitely many solutions, the next question is settled by the methods of elimination to produce the form 0 = 0. Students feel confusion over the meanings of these results. many say 0 = 0 indicates linear equations in two variables do not have a solution. Likewise, when given SLETV who do not have a solution, then resolved with substitution methods produce the form 0 = 1. Although students may find out that it does not have a solution but students confusion explains the meaning 0 = 1.

Some opinions are mentioned on PCK components are as follows. Shulman (1986) mentions three components of PCK: (1) knowledge of topics regularly taught in one’s subject area, (2) knowledge of forms of representation of those ideas, and (3) knowledge of students’ understanding of the topics. Grossman’s (1990) mentions the construct of PCK includes four central components: (1) conception of teaching purposes – knowledge and beliefs about the purposes for teaching a subject at different grade levels; (2) knowledge of students, including students’ understanding, conceptions, and misconceptions of particular topics in a subject matter; (3) curricular knowledge, which includes knowledge of curriculum materials available for teaching particular subject matter and knowledge about both the horizontal and vertical curricula for a subject; as well as

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(4) knowledge of instructional strategies and representations for teaching particular topics. While Rollnick et al. (2008) consider the PCK an amalgam of four areas of the knowledge base for teaching. They are: a) Content Knowledge; b) Knowledge of Students; c) General Pedagogical Knowledge; d) Context Knowledge. In this study, researchers used the opinion Rollnick et al. (2008) and focus on the three components of the first with the little change in terms is for general pedagogical knowledge into the knowledge of teaching.

There are some opinions that present a framework for analyzing the characteristics of PCK teachers based on certain levels, which are as follows. Ebert (1994) states that there are three levels of PCK: Level 0: inadequate, Level 1: good, Level 2: strong. Thompson (1994) says there are three levels in the PCK, namely: Level 0, Level 1 and Level 2. Lindgren (1996) also mention there are three levels in the PCK, namely: Level 0: rules and routines (RR), Level 1: discussion and games (DG), level 2: Open Approach (OA). While Karahasan (2010) combines three such frameworks into the new framework. The description of each component and the level presented in Table 1 as follows.

<table>
<thead>
<tr>
<th>Components of PCK</th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of Teaching</td>
<td>- are seen as knowledge providers and demonstrators for the students</td>
<td>- not only provide necessary rules and procedures but also help students to develop meaning and understanding</td>
<td>- facilitate and guide students rather than provide answers and explanations</td>
</tr>
<tr>
<td></td>
<td>- introduce procedures after concepts</td>
<td>- view their role as one of advising, appraising, and admonishing</td>
<td>- value student understanding and extend that understanding by questioning further mathematical knowledge</td>
</tr>
<tr>
<td></td>
<td>- dominate the flow of information that is a path between the teacher and student</td>
<td>- still dominate the flow of information which is a path between teacher to the student</td>
<td>- value student-to-student interactions</td>
</tr>
<tr>
<td></td>
<td>- have problems sequencing the topics and problems during teaching/lesson planning</td>
<td>- only have problems sequencing the problems during teaching/lesson planning</td>
<td>- allow and encourage students to construct mathematical knowledge through mathematical inquiry</td>
</tr>
<tr>
<td></td>
<td>- have difficulty in controlling the class to have a democratic teaching environment</td>
<td>- sometimes controls the class to have a democratic teaching environment</td>
<td>- sequence the topics and problems in an appropriate way</td>
</tr>
<tr>
<td>Knowledge of Learners</td>
<td>- have difficulty in diagnosing errors of the students</td>
<td>- diagnosing some of the student errors and even if they address the error they focus on the surface futures of the error</td>
<td>- easily diagnose student errors and address students difficulties</td>
</tr>
<tr>
<td></td>
<td>- view responding to students misconceptions as an opportunity for them to tell the student the direct rule or procedure</td>
<td>- solve similar numerical examples, practice problems but also appreciate the importance of discussion</td>
<td>- guide and facilitate students rather than providing answers and explanations</td>
</tr>
<tr>
<td></td>
<td>- have difficulty in realizing students needs for understanding</td>
<td>- from time to time realize students’ needs for understanding and prepare learning environments.</td>
<td>- aware of students’ needs for understanding and accordingly able to create rich learning environments.</td>
</tr>
<tr>
<td>Content Knowledge</td>
<td>- unable to express definitions correctly</td>
<td>- express definitions correctly</td>
<td>- express definitions correctly</td>
</tr>
<tr>
<td></td>
<td>- unable to use appropriate notation sensibly</td>
<td>- use appropriate notation sensibly</td>
<td>- use appropriate notation sensibly</td>
</tr>
<tr>
<td></td>
<td>- use only declarative and/or procedural questions</td>
<td>- still use declarative and/or procedural questions</td>
<td>- use all type of questions (declarative, procedural, and conditional) in an appropriate positions</td>
</tr>
<tr>
<td></td>
<td>- unable to interpret and use different representations easily</td>
<td>- interpret and use graphical and other representations</td>
<td>- interpret and use graphical and other representations sensibly</td>
</tr>
<tr>
<td></td>
<td>- face difficulty when there is a need to see connections between different topics/subunits</td>
<td>- see connections between different topics/subunits</td>
<td>- see connections between different topic/subunits and move among them smoothly</td>
</tr>
</tbody>
</table>

II. Research Method

This study used descriptive qualitative approach. Participants consisted of 2 novice mathematics teachers with details: subject 1 (S1) is a mathematics teacher with 3 years teaching experience and has been a certified educators through professional education of teachers (PPG) and subject 2 (S2) is a mathematics teacher with 3 years teaching experience and do not have a teaching certificate (S2).

There are two main type of instruments used, main and auxiliary instruments. The main instruments are the researchers themselves who act as planners, data collectors, data analysis, interpreters, and reporters of research results. The auxiliary instrument used in this study is vignette and interview guides.

Vignettes are scenarios including student comments, questions, and/or solutions, and are generally used for searching PCK of teachers. There are 3 vignettes used in this study are as follows:

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The procedures in this study include: subject fill the vignette and then interview to clarify the subject written response to the vignette. Activity in qualitative data analysis performed interactively and runs continuously until complete, so that the data is already saturated. Activities in the data analysis, namely data reduction, data presentation, and verification/conclusion.

III. Result And Discussion

Here is presented the results of research on the pedagogical content knowledge of beginning teachers on the material system of linear equations in two variables.
Subject 1 (S1)

Vignette case 1

Here is the response of S1 in a vignette case 1

<table>
<thead>
<tr>
<th>Response</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. made less appropriate analogy, it should be</td>
<td></td>
</tr>
<tr>
<td>2. set of solution of the problem has not been written</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4** Response of S1 in vignette case 1

The response given is seen that S1 considers analogy of the student is still not quite right, but the analogy is correct according to the S1 also still not right (as seen in the response above). If \( x \) = the number of a notebook and \( y \) = the number of a pencil, the solution is given not answer the questions given. On the matter of the question is the price of a notebook and a pencil. So the more appropriate analogy is \( x \) = the price of a notebook and \( y \) = the price a pencil. S1 response given is also reinforced by the results of the interview excerpts below:

**Figure 5** Response of S1 in vignette case 2

From an interview can be seen that the S1 still believes that the analogy is written in a comment in the vignette is correct. Although he was given the opportunity to re-examine the analogy made, S1 remains convinced that the analogy made appropriate. This indicates that S1 is still less careful in understanding this one case. From the excerpts of the interview can also be seen that the S1 can judge that the completion of the written student (Ghildan) is correct, it is seen from the final settlement obtained already meet SPL given. Here S1 did not provide a reason why the settlement is correct.

Vignette case 2

In case 2, S1 responded as follows:

**Figure 6** Below is the response of S1 in a vignette case 3. Based on these responses can be seen that the S1 can give one alternative way to explain to students what if the elimination method generates the form of \( 0 = 1 \) using the graphical method.

**Figure 7** Response of S1 in vignette case 3

The responses indicate that S1 could provide an alternative way to explain to students how if substitution methods produce the form \( 0 = 1 \), ie with the help of graphs SPL. S1 also provides conclusions from the completion of the SPL. Nevertheless, S1 has not provided an alternative way other than using a graph. So that when the researchers tried to interview to clarify the comments above S1 through excerpts of the interview follows:

**Figure 8** Response of S1 in vignette case 4

From an interview can be seen that the S1 only able to give an explanation of the results \( 0 = 1 \) using the graphical method.

Vignette case 3

Figure 6 Below is the response of S1 in a vignette case 3. Based on these responses can be seen that the S1 can give one alternative way to explain to students what if the elimination method generates the form of \( 0 = 1 \), ie with the help of graphs SLET.
When students complete the elimination method and the results 0 = 0, then the students are invited to draw a graph of the second SLETV because there are several ways to determine the solve SLETV. From the graph will be seen that both of two lines are coincides line, which means that all point located on the first line is also located on the second line so that the two lines have infinite number of solution.

Figure 6 Response of S1 in vignette case 3

As for a fuller explanation stated that if the SPL is drawn, it will get two lines are coincident. S1 also provides conclusions from the completion of the SPL. Nevertheless, S1 has not provided an alternative way other than using a graph. At the time of the interview the researchers tried to clarify comments S1, the previous discussions with researchers S1 could eventually provide an explanation other than the use of graphs, and following an interview which showed that:

\[ R: \text{You may be able to give an other explanation for this cases 3 than the with charts?} \]
\[ S1: \text{Look, we write down the equation } 0x + 0y = 0, \text{ then we determine the values of } x \text{ and } y \text{ are hits. Here any values of } x \text{ and } y \text{ certainly meets Sir.} \]
\[ R: \text{How many values of } x \text{ and } y \text{ that satisfy?} \]
\[ S1: \text{Infinity Sir. Eg } x = 1 \text{ and } y = 2. \]

From the excerpts can be seen that the S1 has been trying to provide an explanation regarding the case 3 by means other than graphics, but this explanation still needs to be clarified, because the DA to give a statement "here any values of } x \text{ and } y \text{ certainly meets Sir". In fact, if returned to SPLnya solution are not any values of } x \text{ and } y. Therefore, researchers clarified through the interview excerpts below:

\[ R: \text{Try checking again, it is a value that satisfies } 0 x + 0y = 0, \text{ but whether it meets the SPL supplied?} \]
\[ S1: \text{(Counting a while) no sir.} \]

From the excerpts of the interview can be seen that the S1 is still not fully understood form 0 = 0 from the elimination. Supposedly when adjusted for the procedure elimination method, when two equations are eliminated, it will be wasted one variable and leave one other variable. In these excerpts, S1 finally realized his mistake after re-checking. Thus, with a brief discussion S1 can write the form 0 = 0 as 0x = 0, which means that there are infinitely many } x \text{ that satisfies the equation, one of them for example } x = 1. Having obtained } x = 1, \text{ then substituted into one of the expressions of the SPL and obtained value of } y = 14/3.

Based on the presentation of data above related to the components pedagogical content knowledge can be as discussed as follows.

**Knowledge of Teaching**

In the component knowledge of teaching, S1 is still putting the procedure rather than the concept but had tried to help build understanding for students. For example in cases 2 and 3, S1 is using graphics rather than interpret the deeper form 0 = 1 and 0 = 0. As related to his role as assessor and a reminder, the subject of DA has been able to assess the results of the students' work, although not consistently. Based on the description then generally it can be said that the knowledge taught S1 is at "level 1".

**Knowledge of Learners**

S1 showed good knowledge in facilitating students to solve problems has been demonstrated quite well. For example, to guide students in solving SLETV, if it produces a 0 = 1 or a 0 = 0 when eliminated or substituted then it is suggested to use graphic. But in diagnosing errors S1 students still need to be improved. For example, in the case of 1, S1 has not been able to demonstrate an error in the analogy of the student. Therefore in general, it can be concluded that the knowledge of the students S1 is at "level 1".

**Content Knowledge**

S1 is less precise in doing analogy about the story into a variable, so that in case 1 when students write analogy } x = \text{ notebooks} \text{ and } y = \text{ and pencils subject considers the analogy is appropriate. However, S1 has been quite good at using graphic representations, namely to provide an explanation for the case 0 = 1 approach charts. So in general knowledge of the content S1 is at "level 1" as well.

Summary results of analysis of the S1 PCK shown in Table 2 below.
Table 2 Summary Analysis of PCK S1

<table>
<thead>
<tr>
<th>Component of PCK</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of Teaching</td>
<td>Level 1</td>
</tr>
<tr>
<td>Knowledge of Learners</td>
<td>Level 1</td>
</tr>
<tr>
<td>Content Knowledge</td>
<td>Level 1</td>
</tr>
</tbody>
</table>

Although S1 already attended training programs on teacher professionalism (PPG), turns S1 has not shown good pedagogical content knowledge. It's likely because S1 teaching experience is still lacking. According to Gatbonton (2008) a group of experienced teachers have the pedagogical knowledge that is more detailed, particularly in regards attitudes and behavior of students. Gatbonton found that pedagogical knowledge was similar between the two groups, but seems to have a group of experienced teachers pedagogical knowledge that is more detailed, particularly in regards attitudes and behavior of students. This study showed that college programs and the field are very helpful in developing the pedagogical knowledge of teachers, but a few years of experience will help build that knowledge to make it more specific and useful (Gatbonton, 2008).

Subject 2 (S2)
Vignette case 1

S1 elicits responses in case 1 as follows:

**Figure 7 Response of S2 in vignette case 1**

Seen that S2 directly comment on how the settlement is carried Ghildan, without commenting on the first step of analogy. In fact, in this analogy goes wrong steps are a pretty basic concept. S2 stated that the method is not written Ghildan substitution method because it is not in accordance with the procedures substitution method. According to the method used Ghildan S2 are closer to elimination method, although it cannot be said of the methods of elimination because the procedure used was also not as usual elimination method. Associated with the completion of the first step Ghildan, that analogy, the researchers tried to find out the response S2 through the interview as follows:

<table>
<thead>
<tr>
<th>R</th>
<th>From the story about the student named Ghildan write analogy, x = book, y = pencil. Do you think the analogy is right or not?</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2</td>
<td>Has not Sir, I think the opposite. That is, as you know: the story sounds notebooks and pencils, then the analogy are notebooks = x, and pencil = y.</td>
</tr>
<tr>
<td>R</td>
<td>In your opinion, x or y (variable) in the SPL result is what?</td>
</tr>
<tr>
<td>S2</td>
<td>Numbers Sir.</td>
</tr>
<tr>
<td>R</td>
<td>Now, if x is a number or rather the real numbers, the analogy does not match what is written Ghildan it?</td>
</tr>
<tr>
<td>S2</td>
<td>O yes, it means less suitable Sir, should x = price of the book, y = price of the pencil.</td>
</tr>
</tbody>
</table>

From an interview can be seen that the S2 did not realize where the mistakes were written Ghildan analogy in the vignette. S2 begin to realize there is something wrong after receiving questions from the researcher about the value of a variable in the form of real numbers, not the states of matter such as books and pencils.

While the completion of the work associated with the process Ghildan S2 has not provided an explanation, whether the job is right or not in accordance with the existing procedures in algebra. Therefore, through interviews of researchers trying to explore the opinions or comments of S2 as the following excerpt:

| S2 | In my opinion, the methods used are not clear, Sir. If elimination, why not eliminate one of the variables. If the substitution, really begins with steps like elimination. So the method is less precise. |
| R | What about the results? |
| S2 | The result is correct sir. Already I check it was true. But his methods are less precise. |
According to S2, Ghildan work has actually been true, but the procedure used instead of using the method of substitution or elimination, as the first step Ghildan summing two equations and produce a new equation which variables are still two, and it is not found in elimination method. So that researchers warned that such a move slightly according to the equation elementary operations, which allow two equation variables are added or subtracted although still 2.

**Vignette case 2**

In cases 2 S2 provides the following response:

- **Vignette case 2**

  ![Figure 8 Response of S2 in vignette case 2](image)

  When getting results 0 = 1, we can manipulate algebraic in the previous step 3y - 3y = 1 is not written as 0 = 1 but with still write the variable y with coefficients 0 following
  
  \[\begin{align*}
  3y - 3y &= 1 \\
  0y &= 1 \\
  y &= \frac{1}{0}
  \end{align*}\]

  In order to obtain \(y = 1/0\). We can give an example \(6/2 = 3\) is equivalent to \(3 \times 2 = 6\), then in the same way students are asked to find how many numbers that when multiplied by 0 produces 1, that’s the solution. From here the students are expected to understand that \(1/0\) had no settlement, because no number is multiplied by 0 produces 1.

**Vignette case 3**

Here is the response S2 in case 3:

- **Vignette case 3**

  ![Figure 9 Response of S2 in vignette case 3](image)

  The explanation given by S2 in the case 3 is similar to the case 2. S2 concluded that the SPL in this case has not up to much solution, but there is no explanation of how the settlement in question. It also can be
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seen S2 wrote the equation $0y = 0$ is equivalent to $y = 0/0$. According S1 it can be obtained by dividing both sides by 0. Therefore, researchers try to explore further knowledge of the subject S2 as excerpts of the interview follows:

<table>
<thead>
<tr>
<th>S2</th>
<th>In my opinion are equivalent Sir, because as I wrote it $0y = 0$ if both sides divided by 0, will obtain $y = 0/0$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Is $0/0 = 1$?</td>
</tr>
<tr>
<td>S2</td>
<td>(Seem to think) O yes Sir, $0/0$ that value can be 1, can be 2, and so forth, means not equivalent.</td>
</tr>
</tbody>
</table>

From the excerpts shown that S2 regard both sides divide by zero is undefined allowed and $0/0$. But when asked how much the value of $0/0$, S2 responded that its worth a lot. S2 also commented that in addition to using an analytical approach as above, to give a description of the SPL cases 2 and 3 can use the chart, that if the graph SPL are intersected, the SPL has exactly one solution, if overlaps have infinitely many solutions, and if parallel does not have a solution.

Based on the presentation of data above related to the components pedagogical content knowledge can be as discussed as follows.

Knowledge of teaching
S2 had tried to help students construct meaning and understanding. For example in the case of 2 and 3 to understand the meaning $0 = 1$, and $0 = 0$ on the outcome of the elimination or substitution of the students are asked to write down the results of the example as $0y = 1$ and $0y = 0$. From the form of the expected students can find relevant conclusions SLETV settlement. While related to his role as assessor and a reminder, S2 have been able to assess the results of student work properly. Based on the description then generally it can be said that the knowledge of teaching of S2 is at "level 2".

Knowledge of learners
S2 less able to diagnose the students' mistakes. This is shown by not showing errors analogy of the student in case 1. But in facilitating the students to solve problems, S2 demonstrated well, for example to guide students in solving SLETV, if it produces a $0 = 1$ or a $0 = 0$ at the time eliminated or substituted then students are asked to write down the results, for example as $0y = 1$ and $0y = 0$. So in general it can be concluded that knowledge of learners of S2 is at "level 2".

Content knowledge
S2 assume analogy $x =$ notebook and $y =$ a a pencil in case one is not quite right, but the subject let the student make the analogy because it is not a meaningful error. S2 also considers $0y = 1$ is equivalent to $y = 1/0$ and $0y = 0$ is equivalent to $y = 0/0$. But in the delivery of explanation, S1 is able to demonstrate knowledge of conceptual and procedural pretty good. So in general knowledge of the subject content is still at the "level 1".

In summary the general conclusion of the analysis PCK S2 are presented in Table 3 below.

<table>
<thead>
<tr>
<th>Table 3 Summary Analysis of PCK S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component of PCK</td>
</tr>
<tr>
<td>Knowledge of Teaching</td>
</tr>
<tr>
<td>Knowledge of Learners</td>
</tr>
<tr>
<td>Content Knowledge</td>
</tr>
</tbody>
</table>

Although S2 has only 3 years of teaching experience, but because the neighborhood / school where the teacher is teaching is one of the featured schools, it turns S1 able to demonstrate good pedagogical content knowledge. This is due to the environment (context) as it is, S1 must adapt so much to learn and actively pursue professional teacher training. Knowledge of context was mentioned by many researchers as an essential component of pedagogical content knowledge (Abd Rahman & Scaife, 2005; Grossman, 1990; Marks, 1990; Veal & MaKinster, 1999). Because of that, awareness of knowledge of context was investigated through the study.

IV. Conclusion
A general description pedagogical content knowledge of the subject 1 is the knowledge of teaching, the knowledge of learners, and the content knowledge they are all on level 1. As for the subject 2, the knowledge of teaching and the knowledge of learners are at level 2 and content knowledge is at level 1. Factors affecting pedagogical content knowledge of the teacher include teaching experience, educational background, professional training have been followed, and the environment in which teachers teach.
References


