

## **Electroencephalography Signal Classification based on Sub-Band Common Spatial Pattern (SBCSP)**

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**Abstract:** Brain-computer interface (BCI) is a communication pathway between brain and an external device. It translates human thought into commands to control the external devices. Electroencephalography (EEG) is cost effective and easier way to implement the BCI. This paper presents a novel method for classifying EEG during motor imagery by the combination of common spatial pattern (CSP) and linear discriminant analysis (LDA). In the proposed method, the EEG signal is bandpass-filtered into multiple frequency bands. The CSP features are then extracted from each of these bands. The LDA classifier is subsequently used to classify the CSP features. In this paper, experimental results are presented on a publicly available BCI competition dataset and the performance is compared with existing approaches. The experimental result shows that the proposed method yields comparatively superior cross validation accuracies compared to prevailing methods.

**Keywords:** brain computer interface, electroencephalography, sub-band common spatial pattern.

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### **I. Introduction**

Brain-computer interface (BCI) is a communicating system between a brain and a device that enables signals from the brain to direct some external devices, such as a computer, wheelchairs [1], robotic arms, prostheses [2] etc. The interface translates human thoughts into command to control the external devices. The key target of the BCI is to restore or repair useful function to people disabled by neuromuscular disorders such as Amyotrophic Lateral Sclerosis (ALS), cerebral palsy, stroke, or spinal cord injury. Although people may become totally paralyzed through these types of disorders their minds are un-affected. Considering this issue brain computer interface translates human thoughts directly to the external world [3]. An electroencephalography (EEG) is the recorded electrical activity generated in the brain which is recorded by the electrodes placing on the scalp.

Common Spatial Patterns (CSP) is an algorithm commonly used in BCI systems to preprocess the electroencephalogram (EEG) signals [4, 5, 6]. The algorithm finds optimal spatial filters that are functional in discriminating two classes of EEG signals in motor imagery based BCI. The effectiveness of the spatial filters depends on its subject specific frequency band. If the EEG signals are unfiltered or have been filtered with badly chosen frequency range then the classification of that signals using CSP shows poor accuracies [7]. Consequently, subject specific frequency bands are generally used with the CSP algorithm [8].

To overcome the limitation of manually selecting the subject specific frequency bands for the CSP, the Common Spatio-Spectral Pattern (CSSP) algorithm has been proposed where simple filters are optimized together with the CSP algorithm [9]. The Common Sparse Spectral Spatial Pattern (CSSSP) algorithm improves the performance of CSSP algorithm. It allows concurrent optimization of an arbitrary Finite Impulse Response (FIR) filter within CSP analysis [8]. Another approach called Spectrally weighted Common Spatial Pattern (SPEC-CSP) algorithm [10] optimizes the temporal filter in the frequency domain and after that the spatial filter in an iterative method [11]. However, due to the inherent nature of optimization problem, the solution of filter coefficients depends significantly on the selection of initial parameters [7].

Sub-band Common Spatial Pattern (SBCSP) method [7] was alternatively proposed and has been shown better classification accuracy compared against CSSP and CSSSP. In this method publicly available dataset from BCI competition III in 2005 has been used. As a substitute of temporal FIR filter within the CSP algorithm, SBCSP uses a filter bank that decomposes the EEG signals into sub-bands. The CSP algorithm is then employed on each of these sub-bands to obtain sub-band scores. To fuse the sub-band score two fusion methods namely Recursive Band Elimination (RBE) and Meta-Classifer (MC) are used. An additional classifier is then used to classify the fused sub-band scores. In [7] comparative study of using different sub-band score fusion techniques and classification algorithms are not available [12].

The Filter Bank Common Spatial Pattern (FBCSP) algorithm [12] was proposed to classify EEG signals. In the algorithm, the EEG signals are bandpass-filtered into some frequency bands and CSP features are extracted from each of these bands. Finally, a classification algorithm is used to classify the selected CSP features. The FBCSP algorithm used the typical estimation of multivariate covariance matrices from the

EEG signals for a filter bank of CSP [12]. Usually EEG signals are contaminated with artifacts or different types of noise sources. Due to the contamination the normal pattern of the majority of the EEG data are differed [13]. In the case of large amount of contamination, the multivariate covariance estimates typically varies significantly from the estimate without the contamination [13]. Therefore, the FBCSP algorithm is sensitive to artifacts in the training data [14].

A Robust Filter Bank Common Spatial Pattern (RFBCSP) algorithm was proposed [14] where the Minimum Covariance Determinant (MCD) estimator is used to estimate the covariance matrices. Likewise, to estimate the variance of the projected EEG signals the Median Absolute Deviation (MAD) is used. The classification performance of the RFBCSP is better in some specific subjects but the overall results are not statistically significant.

In this paper, a novel approach is proposed for EEG signal classification in motor imagery-based BCI. The proposed approach is subdivided into the following three stages. In the first stage, the EEG signal is divided into multiple frequency bands using bandpass filter. In the second stage, CSP features are extracted from each of these frequency bands. A classification algorithm is used to classify the CSP features in the third stage. In the third stage, the classification of each band is done by three steps: finding Linear Discriminant Analysis (LDA) scores, blending LDA scores and classifying based on the LDA scores.

The paper is organized as follows— Section II discusses a feature extraction technique called Common Spatial Pattern (CSP), the Linear Discriminant Analysis (LDA) technique is explained in section III, section IV contains the description of the proposed method, the experimental results are illustrated in section V and the section VI includes some concluding remarks.

## II. Common Spatial Pattern

Common Spatial Pattern (CSP) is a feature extraction technique used in signal processing for separating a multivariate signal into additive subcomponents. The technique used to design spatial filters such that the variance of the filtered data from one class is maximized while the variance of the filtered data from the other class is minimized. Thus, the resulting feature vectors increase the discriminability between the two classes by means of minimize the intra class variance and maximize the inter class variance [15]. This property builds CSP as one of the most effective spatial filters for EEG signal processing. The method of CSP was first introduced to EEG analysis for detection of abnormal EEG [16] and effectively applied on movement-related EEG for the classification purpose [4, 6]. The target of the CSP is to project the multichannel EEG data into low dimensional spatial subspace with a projection matrix using linear transformation [17].

For details explanation of the CSP algorithm, assume the original EEG data matrix  $E_k^i$  from trial  $i$  for class  $k$ . The dimension of each  $E_k^i$  is  $N \times T$ , where  $N$  is the number of channels and  $T$  is the number of samples per channel. For the explanation, the EEG data of a single trial ( $i = 1$ ) is represented as  $E_{k \in \{l,r\}}$  where  $l$  denotes left hand and  $r$  denotes right hand movement. The normalized spatial covariance of the EEG for the left hand movement,  $C_l$  and for the right hand movement,  $C_r$  can be calculated as:

$$C_l = \frac{E_l E_l^T}{S_l}, \quad C_r = \frac{E_r E_r^T}{S_r} \quad (1)$$

Where  $E_l$  and  $E_r$  represent the original EEG matrices for left hand and right hand movement respectively,  $E_l^T$  is the transpose of  $E_l$  and  $E_r^T$  is the transpose of  $E_r$ . The  $S_l = \text{trace}(E_l E_l^T)$  and  $S_r = \text{trace}(E_r E_r^T)$  are the sum of the diagonal elements of  $E_l E_l^T$  and  $E_r E_r^T$  respectively. The composite spatial covariance,  $C$  is the sum of the averaged normalized spatial covariance  $\bar{C}_l$  and  $\bar{C}_r$ . The  $\bar{C}_l$  and  $\bar{C}_r$  are estimated by averaging over all the trials of each class. The composite spatial covariance,  $C$  is calculated as

$$C = \bar{C}_l + \bar{C}_r = M_e \delta_e M_e^T \quad (2)$$

Where  $M_e$  is the matrix of eigenvectors,  $M_e^T$  is the transpose of  $M_e$  and  $\delta_e$  is the diagonal matrix of eigenvalues.

The averaged normalized spatial covariance  $\bar{C}_l$  and  $\bar{C}_r$  are transformed as

$$J_l = X \bar{C}_l X^T \text{ and } J_r = X \bar{C}_r X^T \quad (3)$$

Where  $X = M_e^T / \sqrt{\delta_e}$  is the whitening transformation matrix and its transpose is  $X^T$ .  $J_l$  and  $J_r$  share common eigenvectors and the sum of corresponding eigenvalues for the two matrices will always be one. If  $J_l = Y\Lambda_l Y^T$  and  $J_r = Y\Lambda_r Y^T$  then  $\Lambda_l + \Lambda_r = I$ , where  $I$  is the identity matrix. Since the sum of two corresponding eigenvalues is always one, a high eigenvalue for  $J_l$  means that a high variance for EEG in left hand movement and a low variance for the EEG in right hand movement (low eigenvalue for  $J_r$ ) and vice versa. The classification operation is done based on this property. The projection of whitened EEG onto the eigenvectors  $Y$  corresponding to the largest  $\Lambda_l$  and  $\Lambda_r$  will give feature vectors that significantly enhance the discrimination ability.

The goal of the CSP is to find  $F$  spatial filters to create a projection matrix  $W$  of dimension  $N \times F$  (each column is a spatial filter). The projection matrix  $W$  is represented as

$$W = Y^T X \quad (4)$$

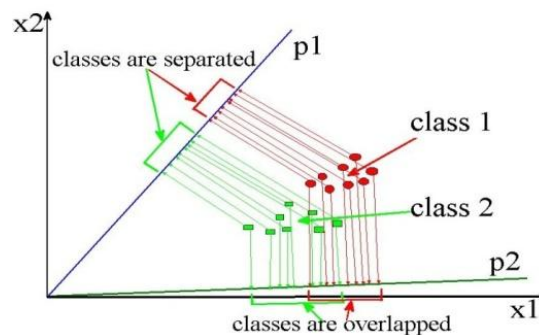
The projection matrix  $W$  linearly transforms the original EEG into uncorrelated components according to:

$$Z = WE \quad (5)$$

The original EEG,  $E$  can be reconstructed by  $E = W^{-1}Z$  where  $W^{-1}$  is the inverse matrix of  $W$ . The columns of  $W^{-1}$  are spatial patterns that describe the variance of the EEG. The first and last columns contain the most discriminatory spatial patterns that explain the high variance of one class and the low variance of the other.

### III. Linear Discriminant Analysis

Linear Discriminant Analysis (LDA), also known as Fisher's linear discriminant analysis is a technique used to find a linear combination of features that separates two or more classes of data. It is typically used as a dimensionality reduction step before classification [18]. It reduces dimensionality but at the same time preserves as much of the class discriminatory information as possible. The goal of the LDA is to use a separating hyperplane that maximally separate the data representing the different classes. The hyperplane is found by selecting the projection where the same classes are projected very close to each other and the distance between the two classes means is as maximum as possible [19]. An example of a selection of data projection is shown in Fig. 1. As shown in Fig. 1 projection p1 is a better line where class 1 and class 2 are well separated whereas projection p2 line is unable to separate the two classes.



**Figure 1:** An example of a selection of data projection. Projection p1 maximize the separation of data compare to projection p2

Let as assume that we have  $K$  classes, each containing  $N$  observations  $x_i$ . The within-class scatter,  $\tilde{S}_w$  for all  $K$  classes can be calculated as:

$$\tilde{S}_w = \sum_{k=1}^K f_k S_w^k \quad (6)$$

Where the within-class covariance matrix  $S_w^k$ , the fraction of data  $f_k$  and the mean vector  $\mu_k$  of class  $k$  are calculated according to the following formulas:

$$S_w^k = \sum_{i=1}^{N_k} (x_i^k - \mu^k)(x_i^k - \mu^k)^T \quad (7)$$

$$f_k = \frac{N_k}{\sum_{j=1}^K N_j}, \mu_k = \frac{1}{N_k} \sum_{i=1}^{N_k} x_i^k \quad (8)$$

The between class scatter  $\tilde{S}_b$  for all K classes can be calculated as:

$$\tilde{S}_b = \sum_{k=1}^K f_k S_b^k \quad (9)$$

Where the between class covariance matrix,  $S_b^k$  for the mean of all observations  $x_i$  for all K classes,  $\mu$  can be estimated as

$$S_b^k = \sum_{i=1}^K (\mu^k - \mu)(\mu^k - \mu)^T \quad (10)$$

The main objective of LDA is to find a projection matrix that maximizes the ratio of the determinant of  $\tilde{S}_b$  to the determinant of  $\tilde{S}_w$ . The projections that providing the best class separation are eigenvectors with the highest eigenvalues of matrix P [18]:

$$P = \frac{\tilde{S}_b}{\tilde{S}_w} \quad (11)$$

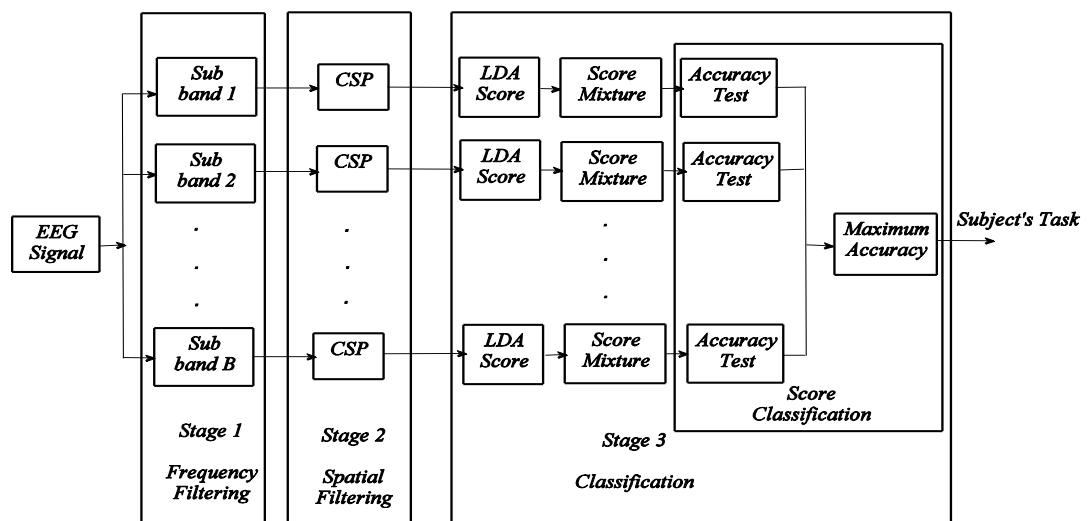


Figure 2: Block diagram of the proposed EEG signal classification approach

Since the matrix P is asymmetric, the calculation of eigenvectors can be difficult. This difficulty can be minimized by using generalized eigenvalue problem [20]. Now, the aim of the LDA is to seek (K-1) projections  $[y_1, y_2, y_3, \dots, y_{K-1}]$  by means of (K-1) projection vectors. The transformed data set y is obtained as a linear combination of all input features x with weights W.

$$y = x^T W \quad (12)$$

Where  $W = [w_1, w_2, w_3, \dots, w_D]$  is a matrix form with the D eigenvectors of matrix P associated with the highest eigenvalues. The LDA reduces the original feature space dimension to D. The LDA performs well when the discriminatory information of data depends on the mean of the data. But it does not work for the variance depended discriminatory informative data. Also, the performance of the LDA is not good for nonlinear classification.

#### IV. Proposed Approach

The proposed EEG signal classification approach is illustrated in Fig. 2. This approach is subdivided into three stages for EEG signal processing and machine learning. In the first stage, the EEG signal frequency is filtered into multiple pass bands using bandpass filter. In the second stage, CSP features are extracted from each of these frequency bands. In the third stage, the classification operation is performed by finding LDA scores, blending and classifying the scores. A detail of each stage is described in below.

**Frequency filtering:** the first stage filters the EEG signal into multiple frequency passbands. The digital Butterworth bandpass filter is used to filter the EEG signal. Here, the most dominating rhythmic components alpha and beta (8-32Hz) are selected. A total of six bandpass filters 8-12Hz, 12-16Hz, 16-20Hz, 20-24Hz, 24-28Hz and 28-32Hz are used. The filtered six subbands are used individually for the classification.

**Spatial filtering:** in this stage the CSP algorithm is used to perform the spatial filtering operation. The spatial filter produces CSP features for the particular frequency range of each of the sub bands. **Classification:** In the third stage, classification algorithms called LDA classifier is used to model and classify the selected CSP features. Each sub band feature is passed separately through the classifier. To validate the classification,  $P \times Q$  cross validation is used. At the first step of the classification stage, the LDA classifier computed LDA scores for every value of P and Q. In the score mixture step, the LDA scores are mixed up according to

$$\Psi = \frac{1}{Q} \sum_{i=1}^Q \Phi \quad (13)$$

Where,  $\Psi$  and  $\Phi$  denote the mixed LDA scores and the LDA scores computed by the LDA classifier respectively. The mixed LDA scores are converted to predicted classes. The accuracy,  $\Gamma_{j \times Q}$  is tested based on the predicted classes in the score classification step where  $j=1,2,\dots,P$ . After  $P \times Q$  cross validation, the classification accuracy for each subband is estimated by the following formula:

$$A_b = \frac{1}{P} \sum_{j=1}^P \Gamma_{j \times Q} \quad (14)$$

Where,  $A_b$  is the classification accuracy for subband  $b(b = 1,2,\dots,B)$ . Finally, the classification rate (CR) of the EEG signal is calculated according to (15)

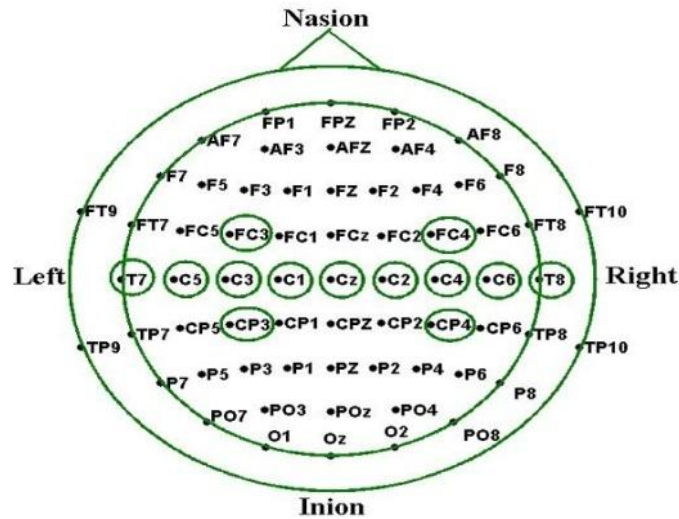
$$CR = \max(A_b) \times 100\% \quad (15)$$

#### V. Experimental Results

The performance of the proposed method is evaluated by classifying EEG during imagined movement. The proposed approach is applied to the publicly available BCI competition dataset. A filter bank is used in this method that covers alpha and beta rhythmic components (8-32Hz). The filter bank comprises six bandpass filters namely 8-12Hz, 12-16Hz, 16-20Hz, 20-24Hz, 24-28Hz and 28-32Hz. A fourth-order Butterworth filter is used to subband the EEG data. To extract features from the data, the CSP algorithm with  $m=2$  is used in this experiment.

**Dataset:** To evaluate the performance of the proposed method, the dataset IVa from the publicly available BCI competition III 2005 [21] is used in this experiment. This dataset contains data from the four initial sessions without feedback. The dataset is recorded from five healthy subjects (labelled ‘aa’, ‘al’, ‘av’, ‘aw’, ‘ay’) who performed right hand and right foot movement imagination [22]. The data for each subject comprises 280 trials from 118 EEG channels and 140 trials in each class. The visual cues at each trial last for 3.5 seconds. The sampling rate of the data is 100 Hz. In this experiment, the data between 0.5 seconds and 2.5 seconds from the visual cue (i.e. 200 time points at each trial) is extracted.

**Channel selection:** The motor imagery response of brain is more active in its central part [23]. In this experiment, out of the 118 EEG channels, from the central area 13 are selected for classification. The selected EEG channels are “FC3”, “FC4”, “Cz”, “C1”, “C2”, “C3”, “C4”, “C5”, “C6”, “T7”, “T8”, “CP3”, and “CP4”. The spatial distribution of the channels on the scalp in 10/20 EEG system is illustrated in Fig. 3. The letters specify the spatial location as A = ear lobe; C = central; P = parietal; F = frontal; T = temporal; O = occipital; FP = frontal polar; FC = between frontal and central; CP = between central and parietal; FT = between frontal and temporal; PO = between parietal and occipital and AF = intermediate between frontal polar and frontal. The channels used in this experiment are indicated by the circle of bold line in Fig. 3.



**Figure 3:** Location and nomenclature of the intermediate 10% electrodes (10/20 EEG system), as standardized by the American EEG society.

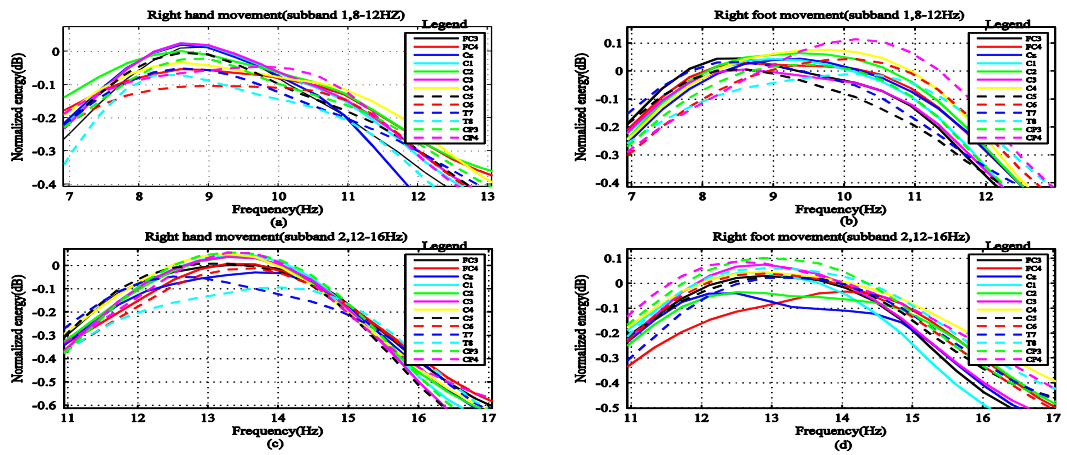


Figure 4: The spectrum of (a-b) subband 1(8-12Hz) and (c-d) subband 2(12-16Hz) components for right hand and right foot movement respectively.

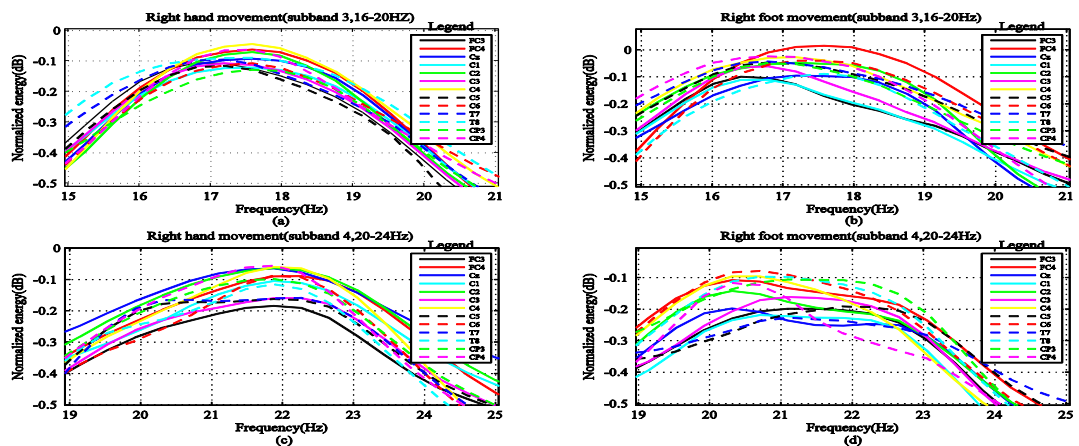


Figure 5: The spectrum of (a-b) subband3(16-20Hz) and (c-d) subband4(20-24Hz) components for right hand and right foot movement respectively.

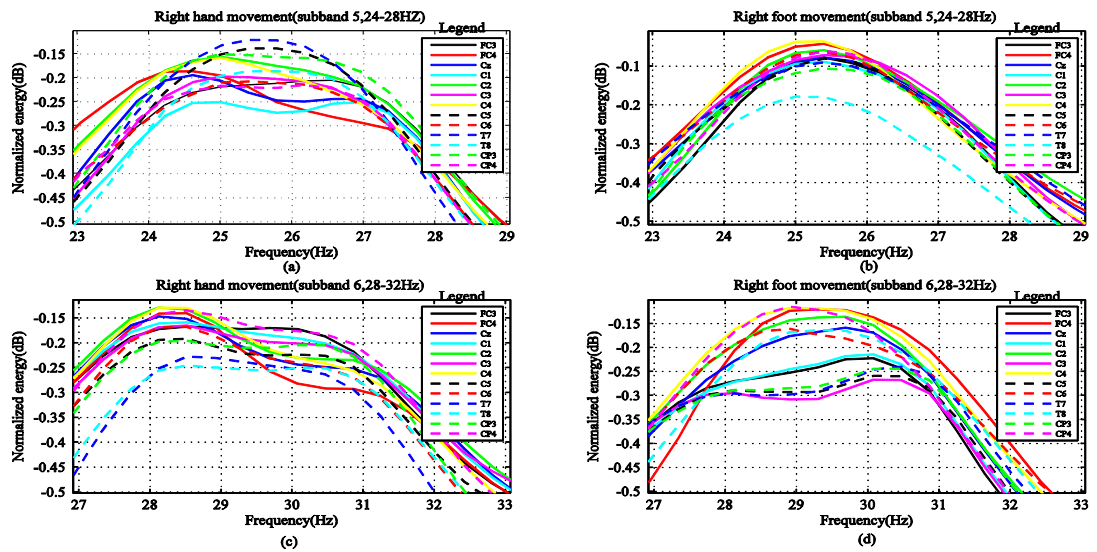


Figure 6: The spectrum of (a-b) subband5(24-28Hz) and (c-d) subband6(28-32Hz) components for right hand and right foot movement respectively.

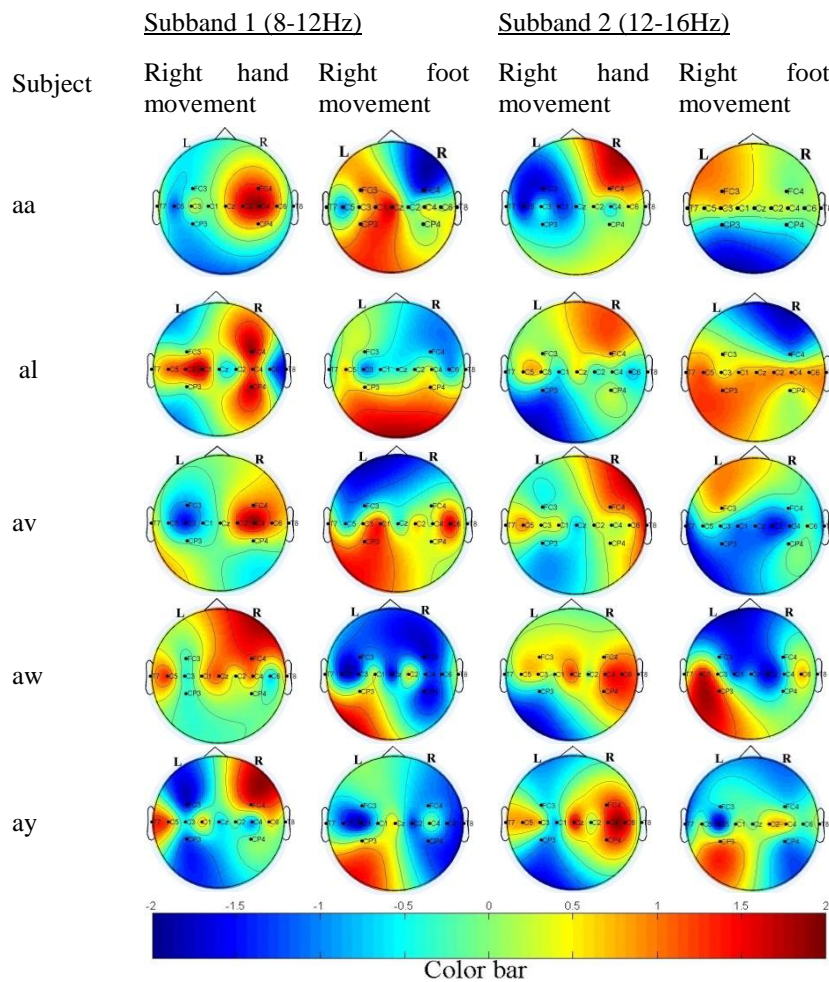


Figure 7: Topographical map of brain for subband 1(8-12Hz) and subband 2(12-16Hz) of five subjects; first trace: right hand, second trace: right foot movement of subband 1; third trace: right hand, fourth trace: right foot movement of subband 2.



**Table I:** Classification accuracy (%)

Method	subject					
	aa	al	av	aw	ay	average
CSP1	71.3	88.4	48.6	89.9	79.9	75.6
CSP2	65.3	90.2	63.7	80.3	87.3	77.4
EMD-CSP	68.4	89.6	64.1	82.5	86.9	78.3
MEMD1-CSP	68.8	90.0	68.8	76.3	87.5	78.3
MEMD2-CSP	60.3	82.9	55.3	60.7	74.0	66.6
FBCSP <sub>w</sub>	93.3	98.5	66.8	93.8	93.6	89.2
FBCSP <sub>f</sub>	86.0	97.9	76.8	96.8	94.0	90.3
<b>Proposed Method</b>	<b>94.4</b>	<b>98.7</b>	<b>81.2</b>	<b>98.2</b>	<b>96.8</b>	<b>93.9</b>

In Fig. 4-6, the individual color line indicates the energy (normalized) contributed by the different subbands to the selected channels. Fig. 4 (a) and Fig. 4 (b) show the spectrums of the subband 1 (8-12Hz) component for right hand and right foot movement respectively. The spectrums of the subband 2 (12-16Hz) for right hand and right foot movement are depicts in Fig. 4 (c) and Fig. 4 (d) respectively. Each color trace in Fig. 5 represents the spectrum of the activity of subband 3 (16-20 Hz) and subband 4 (20-24Hz) for both right hand and right foot movement. The spectrums of the higher frequency subbands, subband 5 (24-28Hz) and subband 6 (28-32Hz) are shown in Fig. 6. From Fig. 4-6, the overall observation is that for right hand movement channels C2, C3, Cz, C4, FC4 and T8 shows comparatively more energy than other channels. On the other hand, channels C3, C4, CP3, PC3, FC4 and T7 shows comparatively more energy than rest of the channels for the right foot movement.

The topographical brain maps for subband 1 and subband 2 during imaginary right hand and right foot movement for the five subjects ('aa', 'al', 'av', 'aw', 'ay') are shown in Fig. 7. The most significant CSP of the two subbands are used for the topographical brain maps. The first and second trace (Fig. 7) shows the topographical brain maps of the subband 1 for imaginary right hand and foot movement respectively. The topographical brain maps of subband 2 for the imaginary right hand and foot are shown in third and fourth trace (Fig. 7) respectively. From Fig. 7 we observed that for right hand movement the electrodes of right hemisphere of the head scalp are more active whereas the electrodes of left hemisphere of the head scalp are more active for the right foot movement.

**Classification results:**In this paper, we found the classification results of the EEG during imagined right hand and right foot movement using the proposed method. Table I shows the classification accuracy of unbiased 10×10-fold cross validations performed. We compare the performance of the proposed method to that of the other methods (CSP, EMD-CSP and MEMD-CSP) proposed in [24] and methods (FBCSP<sub>w</sub>, FBCSP<sub>f</sub>) proposed in [12].Table I show that our proposed method yields superior result than all other methods.

## VI. Conclusion

A novel method to classify EEG during imagined right hand and right foot movement is introduced in this paper. In this method the EEG is filtered into multiple sub ands for the purpose of selecting an appropriate operational frequency band. The discriminative CSP features are then extracted from each of these subbands. To classify the extracted features a classification algorithm, LDA is used. LDA score is produced for every fold cross validation. The LDA scores are mixed up and the mixed scores are converted to predicted class. The classification accuracy is tested based on the predicted class. The experimental results show that the proposed method yields superior classification accuracy compared against existing methods CSP, EMD-CSP, MEMD-CSP, FBCSP<sub>w</sub> and FBCSP<sub>f</sub>.

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