

Advanced Studies on Non-Newtonian Fluids: Rheological Behavior, Constitutive Modeling, and Industrial Applications Across Modern Manufacturing Systems

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Abstract

Non-Newtonian fluids, distinguished by their complex and nonlinear rheological behavior, constitute a critical class of engineering materials widely used across industries such as polymer processing, food manufacturing, drilling engineering, pharmaceuticals, cosmetics, biotechnology, environmental systems, and advanced manufacturing. Unlike Newtonian fluids, whose viscosity remains constant, non-Newtonian fluids exhibit shear-dependent viscosity, viscoelastic effects, memory behavior, thixotropy, and yield stress characteristics, making their analysis indispensable for designing efficient industrial systems. This introduction synthesizes the historical development, foundational rheological principles, experimental methods, computational tools, and industrial relevance of non-Newtonian fluid studies. It highlights key constitutive models, experimental challenges, microstructural factors, and flow instabilities that influence prediction accuracy in real-world applications. Additionally, the text examines emerging research trends such as data-driven rheology, machine learning-assisted modeling, bioprinting applications, sustainable bio-based polymers, and microfluidic systems. Taken together, the review underscores the interdisciplinary nature of non-Newtonian fluid research and emphasizes the need for continued innovation in modeling, high-fidelity simulations, and process optimization to support modern industrial demands. The introduction establishes a comprehensive foundation for understanding the behavior, characterization, and application of non-Newtonian fluids while identifying gaps and future research opportunities essential for advancing industrial fluid mechanics.

Keywords: Non-Newtonian fluids; rheology; shear-thinning; viscoelasticity; industrial applications; constitutive models; fluid dynamics

I. Introduction to Non-Newtonian Fluids

1.1 Definition and Characteristics

Non-Newtonian fluids constitute a diverse and scientifically important class of materials whose flow behavior cannot be adequately described by the linear stress-strain relationship characteristic of Newtonian fluids. In Newtonian systems, viscosity is an intrinsic material constant independent of shear rate, allowing stress to vary proportionally with deformation rate (Bird et al., 1987). In contrast, non-Newtonian fluids display deviations from this linear relationship, resulting in complex rheological behaviors that depend on shear rate, time, deformation history, and temperature (Larson, 1999; Macosko, 1994). These fluids exhibit a spectrum of properties such as **shear-thinning**, **shear-thickening**, **yield stress**, **thixotropy**, and **viscoelasticity**, each influenced by the underlying material microstructure (Chhabra & Richardson, 2011; Barnes, 1999). At a fundamental level, the atypical flow properties of non-Newtonian fluids arise from **microstructural dynamics**. Unlike simple molecular liquids, these materials often contain polymeric chains, colloidal particles, emulsified droplets, aggregated clusters, entangled macromolecules, or biological entities whose internal configurations reorient, stretch, aggregate, or disintegrate when subjected to deformation (Mewis & Wagner, 2012). The resulting structural transformations impart a nonlinear rheological response that depends not only on the applied stresses but also on the material's ability to reorganize at multiple scales (Ferry, 1980). For example, the entanglement of long-chain polymers can generate elasticity, while colloidal suspensions may exhibit shear-thickening due to hydrocluster formation under high shear (Bonn et al., 2017). Similarly, gels and yield-stress fluids behave as soft solids until a critical shear threshold is met, beyond which they begin to flow (Coussot, 2005; Barnes & Walters, 1985).

Non-Newtonian fluids may be classified into three broad categories: **time-independent**, **time-dependent**, and **viscoelastic** fluids. Time-independent fluids, such as power-law polymer solutions, exhibit viscosity changes solely based on the instantaneous shear rate (Ostwald & Waele, 1923). Time-dependent fluids, including thixotropic paints and rheopectic lubricants, display viscosity that evolves over time under constant shear, typically due to structural breakdown or buildup (Mewis, 1979). Viscoelastic fluids, including polymer

melts, biological fluids, and certain surfactant solutions, demonstrate characteristics of both viscous liquids and elastic solids—recovering stored energy after deformation (Bird et al., 1987; Rabinowitsch, 1948). These distinctions highlight the breadth of behaviors encompassed by the non-Newtonian classification. One of the defining complexities of non-Newtonian fluids is their **dependence on deformation history**, often described as a “memory effect.” Viscoelastic fluids possess molecular arrangements capable of storing elastic energy, and their stress response at any moment may depend on past deformations (Dealy & Larson, 2006; Tanner, 1988). This memory effect is essential in processes such as polymer extrusion, where elastic recoil and die swell occur upon exiting a die due to relaxation of stored energy (Macosko, 1994). Such behavior cannot be captured by Newtonian models, necessitating advanced constitutive equations such as Maxwell, Oldroyd-B, and Giesekus models (Oldroyd, 1950; Giesekus, 1982).

Another critical characteristic is the **role of microstructure under flow**. For instance, suspensions of solid particles may transition from ordered to disordered states depending on shear conditions, affecting viscosity, normal stress differences, and flow stability (Guazzelli & Pouliquen, 2018). Emulsions—droplet dispersions in immiscible fluids—experience droplet deformation, breakup, and coalescence that alter rheological properties dynamically (Mason, 1999). Foams, which are gas-liquid composites, show extraordinary elasticity and yield-like behavior due to bubble interactions (Weaire & Hutzler, 1999). Similarly, biological fluids such as blood exhibit shear-thinning behavior due to red blood cell aggregation and deformation, a phenomenon essential for microcirculation (Cokelet & Meiselman, 2007). Because of these complexities, the field of non-Newtonian fluid mechanics demands a deep understanding of **material structure, constitutive modeling, rheometry, and flow physics**. The interplay of these elements makes the study of non-Newtonian behavior pivotal not only for theoretical rheology but also for numerous applications in science and engineering. As industries increasingly rely on complex fluids, accurate prediction, measurement, and modeling of their behavior become necessary for product design, process optimization, and equipment development (Chhabra & Richardson, 2011; Morrison, 2001).

II. Rheological Behaviors of Non-Newtonian Fluids

2.1 Shear-Thinning and Shear-Thickening

Shear-dependent viscosity is one of the most defining rheological characteristics of non-Newtonian fluids, and it plays a central role in shaping their industrial utility, processing efficiency, and functional performance. Fluids that exhibit viscosity reduction with increasing shear rate are classified as **shear-thinning** or *pseudoplastic*, whereas materials that display viscosity enhancement with increasing shear rate are termed **shear-thickening** or *dilatant* (Malkin & Isayev, 2006; Chhabra & Richardson, 2011). These behaviors arise from microstructural rearrangements, particle–particle interactions, polymer chain alignment, and changes in internal friction as deformation rates vary (Barnes, 1997; Larson, 1999; Rheology Society, 2015).

Shear-Thinning Fluids

In shear-thinning systems—such as polymer solutions, aqueous gels, coatings, cosmetics, inks, blood, and many food products—the microstructure tends to orient, disentangle, or deform in the direction of applied flow, thereby reducing flow resistance (Steffe, 1996; Rao & Cooley, 1992). This property is invaluable for industries requiring both **ease of spreading or pumping** and **high stability at rest**. For instance, paints and inks must flow readily during application but resist dripping afterward (Tadros, 2010). Similarly, food products like ketchup, yogurt, and sauces rely on a delicate balance of low shear viscosity for consumer usability and high zero-shear viscosity for shelf stability (Holdsworth & Abdul Ghani, 2007; Charm, 2017). In polymer processing, shear-thinning is particularly beneficial because high shear rates inside extruders and dies significantly reduce melt viscosity, allowing efficient shaping and forming of polymeric materials (Malkin & Isayev, 2006; Dealy & Larson, 2006). Likewise, in cosmetic formulations, lotions, and creams, shear-thinning ensures pleasant sensory feel, smooth spreading on skin, and improved structural recovery post-application (Laba, 1993; Tadros, 2018).

Shear-Thickening Fluids

Conversely, shear-thickening behavior, though less common, is equally significant. It occurs in concentrated suspensions such as cornstarch–water mixtures, colloidal dispersions, abrasive slurries, and advanced protective materials (Wagner & Brady, 2009). The increase in viscosity at high shear rates is often attributed to **hydrocluster formation**, crowding of particles, interparticle friction, or order–disorder transitions within the dispersed phase (Brown & Jaeger, 2014; Bossis & Brady, 1989). Industrially, controlled shear thickening is exploited to create **impact-resistant materials**, such as body armor and protective sports gear, where the material remains flexible under normal movement but stiffens instantly under sudden impact (Wagner & Wetzel, 2017). In abrasive slurry systems used for polishing or mining, shear thickening can provide temporary structural integrity, enhancing particle control and reducing sedimentation (Chow & Zukoski, 1995). However, in many manufacturing processes, unintended shear thickening is undesirable because it leads to

excessive energy consumption, equipment vibration, and unstable flow profiles (Barnes, 1997; Stickel & Powell, 2005).

Industrial Significance

The contrasting behaviors of shear thinning and thickening dictate processing design, pump selection, mixing strategies, and flow modeling across industries such as food, coatings, pharmaceuticals, personal care, mining, and polymer manufacturing (Morris, 2009; Chhabra, 2010). Understanding these responses enables engineers to optimize operational conditions, minimize defects, and ensure product consistency and stability.

2.2 Viscoelasticity and Normal Stress Effects

Viscoelasticity constitutes another foundational aspect of non-Newtonian rheology, wherein fluids exhibit simultaneous viscous and elastic responses under deformation. Such behavior is typical of polymer melts, concentrated solutions, biological fluids, bitumen, and certain surfactant systems (Bird et al., 1987; Ferry, 1980; Larson, 1999).

Elastic Recovery and Relaxation

Viscoelastic fluids can store mechanical energy temporarily due to elastic deformation of their microstructures—such as extended polymer chains or entangled networks—and release it upon removal of stress (Macosko, 1994; Dealy & Larson, 2006). This results in phenomena such as stress relaxation, creep, recoil, and oscillatory deformation responses that are critical for processing and performance (Tanner, 2000). Stress relaxation influences the stability of molded polymers, the dimensional accuracy of extrudates, and the ability of adhesives to withstand sudden strain (Malkin & Isayev, 2006). Viscoelasticity in biological fluids, including mucus and synovial fluid, governs lubrication, transport, and protective mechanisms in living systems (Ferry, 1980; Ewoldt et al., 2015).

Normal Stress Differences

One of the most distinctive features of viscoelastic fluids is the emergence of **normal stress differences**, which drive non-intuitive flow behaviors such as rod climbing (Weissenberg effect), die swell, and secondary flows in curved geometries (Bird et al., 1987; Larson, 1999). These effects arise because fluid elements experience unequal stretching along different principal axes during shear. In polymer extrusion, normal stress differences cause **die swell**, a phenomenon where the extrudate emerges thicker than the die opening, complicating product dimensional control (Tanner, 2000). In coating processes, normal stresses can lead to edge effects and flow instabilities that impair uniform film formation (Rothstein, 2003). In mixing operations, secondary flows induced by viscoelasticity enhance homogenization but may also introduce undesirable vortices (Pakdel & McKinley, 1996).

Industrial Relevance

Understanding viscoelasticity is crucial for computational fluid dynamics (CFD) modeling, equipment design, and predicting complex flows in extrusion, injection molding, fiber spinning, enhanced oil recovery, and biological systems (Hulsen et al., 2005; Owens & Phillips, 2002). Accurate constitutive models—such as Maxwell, Oldroyd-B, Giesekus, and Phan-Thien–Tanner—are essential for predicting viscoelastic behaviors (Bird et al., 1987; Tanner, 2000). Misrepresentation of viscoelastic effects in industrial design may result in unstable flows, excessive energy demand, or product inconsistency.

2.3 Thixotropy and Rheopexy

Thixotropy and rheopexy refer to time-dependent changes in viscosity under constant shear. **Thixotropic fluids** decrease in viscosity over time when subjected to shear, then recover their original structure upon resting. **Rheoplectic fluids**, in contrast, increase in viscosity with prolonged shear (Barnes, 1997; Mewis & Wagner, 2009).

Thixotropy

Thixotropy is widespread in drilling muds, cement slurries, paints, clays, biological gels, and certain food systems. The microstructural explanation typically involves reversible breakdown of weakly bonded networks, flocculated clusters, or gel structures under shear (Mewis & Wagner, 2012; Chhabra & Richardson, 2011). When shear stops, restructuring processes—such as particle aggregation, entanglement, or hydrogen bonding—restore the initial viscosity. In the petroleum industry, thixotropic drilling fluids prevent cuttings from settling when the drill pipe stops rotating while ensuring pumpability during drilling (Hemphill, 2013; Kelessidis & Maglione, 2008). Cement-based materials rely on thixotropy for controlled flow during casting

followed by rapid structural buildup for stability (Roussel, 2006). In coatings, thixotropy helps maintain film thickness, reduces sagging, and ensures easy brushability (Tadros, 2010).

Rheopexy

Rheoplectic fluids—rarer and typically found in specific industrial slurries, lubricants, or deflocculated suspensions—gain viscosity with prolonged shear due to microstructural buildup or ordering phenomena (Barnes, 1997; Houska, 1980). In some metallic suspensions or printing inks, rheopexy stabilizes flow under continuous high-speed operation (Fischer, 2006).

Practical Implications

Time-dependent rheology strongly influences flow initiation, stoppage, storage stability, and dynamic operation in pipelines, mixers, and reactors (Mewis & Wagner, 2009). Accounting for these behaviors in rheological modeling remains essential to prevent clogging, ensure uniform product quality, and optimize energy usage across applications.

2.4 Yield Stress Phenomena

Yield stress fluids require a finite stress, known as the **yield stress**, to initiate flow. Below this threshold, the material behaves like an elastic solid; above it, it flows like a viscous fluid (Barnes, 1999; Coussot, 2014).

Types of Yield-Stress Fluids

Yield-stress behavior is characteristic of materials such as:

- toothpaste
- cement paste
- slurries
- gels
- mayonnaise and other emulsions
- drilling muds
- biological tissues (Cross, 1965; Chhabra & Richardson, 2011)

Theoretical models such as the Bingham, Herschel–Bulkley, and Casson equations describe yield-stress behavior in engineering analyses (Bird et al., 1987; Coussot, 2014).

Industrial Role of Yield Stress

In consumer products such as toothpaste or creams, the yield stress prevents flow under gravity, ensuring controlled dispensing and structural integrity during storage (Barnes, 1997). In civil engineering, cement pastes must maintain shape after placement but flow adequately under applied vibration or pumping pressures (Roussel, 2007). In mining and waste management, yield-stress slurries help stabilize tailings and prevent sedimentation (Pullum et al., 2006). Yield-stress fluids are essential in food processing, where products like mayonnaise or cheese spreads require stable shape yet need to be extruded or pumped efficiently (Steffe, 1996; Rao, 2010). Similarly, in oil recovery and drilling operations, yield-stress muds enhance suspension of solids and control fluid loss (Kelessidis & Maglione, 2008).

Flow Challenges

Yield-stress fluids often exhibit plug flow, flow localization, or wall slip, complicating measurement and modeling (Coussot, 2014; Nguyen & Boger, 1992). Failure to account for yield-stress behavior may result in pipeline blockages, uneven distribution in mixers, and excessive pressure demands in pumping systems.

III. Constitutive Modeling of Non-Newtonian Fluids

3.1 Common Constitutive Equations

Constitutive modeling of non-Newtonian fluids is a critical foundation for predicting their flow behavior, deformation patterns, and stress response under various industrial processing conditions. Since non-Newtonian materials diverge from the simple Newtonian relationship between shear stress and shear rate, mathematical models must incorporate nonlinear, time-dependent, yield-stress, or viscoelastic effects essential for characterizing real-world fluids such as polymer melts, slurries, paints, suspensions, biological fluids, foams, and emulsions (Bird et al., 1987; Chhabra & Richardson, 2011; Macosko, 1994). Constitutive equations provide the bridge between rheological measurements and process simulation and thus fundamentally influence equipment design, computational fluid dynamics (CFD) modeling, product formulation, and quality control (Malkin & Isayev, 2006; Larson, 1999).

Power-Law (Ostwald–de Waele) Model

The **Power-Law model**, perhaps the simplest non-Newtonian constitutive equation, expresses apparent viscosity as a function of shear rate using two parameters: the flow consistency index and the flow behavior index (Bird et al., 1987). This model captures both shear-thinning ($n < 1$) and shear-thickening ($n > 1$) behaviors and is widely used in the food, polymer, slurry transportation, and coatings industries due to its simplicity (Steffe, 1996; Rao, 2010). However, because the Power-Law does not represent Newtonian plateaus at low or high shear rates, it often requires modification when extrapolating beyond experimentally measured regimes (Chhabra & Richardson, 2011).

Herschel–Bulkley and Bingham Plastic Models

Yield-stress fluids require a minimum stress before they begin to flow. The **Bingham model** describes fluids that behave as rigid bodies below a yield stress and as linear viscous fluids above it (Barnes, 1999). It is commonly applied to toothpaste, muds, cement pastes, and certain suspensions (Nguyen & Boger, 1992). The **Herschel–Bulkley model**, a generalization of the Bingham equation, incorporates both yield stress and power-law shear-thinning or thickening behavior, making it more versatile for industrial slurries, drilling fluids, and food products such as mayonnaise, cheese pastes, and chocolate (Mewis & Wagner, 2012; Cross, 1965). Despite their usefulness, yield-stress models face challenges in capturing time-dependent thixotropic effects and may overpredict yield behavior when wall slip, microstructural heterogeneity, or shear banding occur (Coussot, 2014; Barnes, 1999). Nevertheless, these models remain central to engineering design due to their mathematical simplicity and compatibility with pipeline transport calculations (Pullum et al., 2006).

Carreau, Cross, and Casson Models

For fluids showing smooth transitions from Newtonian to non-Newtonian regimes, the **Carreau** and **Cross** models effectively predict viscosity across a wide range of shear rates. These models incorporate characteristic relaxation times and thus reflect molecular orientation and chain dynamics in polymer melts and solutions (Dealy & Larson, 2006; Bird et al., 1987). The Carreau model is widely adopted in polymer extrusion and injection molding simulations due to its ability to capture low-shear Newtonian behavior, intermediate shear thinning, and high-shear asymptotes (Macosko, 1994).

The **Casson model** is particularly relevant in printing inks, chocolate, and blood rheology (Tadros, 2010; Charm, 2017). Its square-root formulation captures yield phenomena in concentrated suspensions with fine particulate interactions. Although widely used in biomedical modeling, its limitations arise when predicting transient or oscillatory behavior (Ewoldt et al., 2015).

Industrial Use and Computational Considerations

The selection of a constitutive equation dramatically influences CFD stability, convergence, and accuracy in simulations of extrusion, mixing, coating, or pipe flow (Owens & Phillips, 2002; Hulsen et al., 2005). Shear-thinning models may underestimate pressure drops in laminar–turbulent transitional pipe flows, while yield-stress models may predict plug flows that differ significantly from experimental observations when slip or structural breakdown occurs (Coussot, 2014; Barnes, 1999). Consequently, many industries calibrate models through rheometry and inverse computational techniques to tailor parameters precisely to operational conditions (Rothstein, 2003; Mewis & Wagner, 2012).

3.2 Viscoelastic Models

Non-Newtonian fluids with both viscous and elastic characteristics require more sophisticated constitutive formulations. Viscoelasticity is a hallmark of polymer melts, polymer solutions, biological fluids, surfactant systems, and complex suspensions whose microstructures deform, orient, or partially recover under flow (Larson, 1999; Ferry, 1980). Predicting elastic recoil, stress relaxation, normal stress differences, and transient dynamics is critical for polymer processing, coating flows, lubrication, soft tissue modeling, and enhanced oil recovery (Bird et al., 1987; Tanner, 2000).

Maxwell and Generalized Maxwell Models

The **Maxwell model**, a linear viscoelastic constitutive equation, represents materials as an ideal spring and dashpot in series. It accurately captures stress relaxation but fails to describe steady-state shear and nonlinear elastic behavior (Ferry, 1980; Macosko, 1994). The **Generalized Maxwell (multi-mode)** formulation improves predictive capability by incorporating multiple relaxation times, enabling detailed representation of polymer relaxation spectra obtained from dynamic mechanical analysis (Dealy & Larson, 2006; Larson, 1999).

These models are widely used for oscillatory rheology, small-amplitude deformation, and the characterization of polymer chain dynamics (Ewoldt et al., 2015). However, they are limited in predicting large deformation behavior encountered in molding and extrusion (Bird et al., 1987).

Oldroyd-B and Upper-Convected Maxwell Models

The **Oldroyd-B model**, one of the most important nonlinear viscoelastic constitutive equations, captures normal stress differences and predicts the Weissenberg effect, die swell, and elastic instabilities (Bird et al., 1987; Tanner, 2000). Derived by extending the **Upper-Convected Maxwell (UCM)** model with solvent viscosity contributions, Oldroyd-B is essential in polymer solution modeling and simulating flows with significant stretching (Larson, 1999; Owens & Phillips, 2002).

Despite its significance, the Oldroyd-B model predicts unbounded stress growth under extensional flows, making it unsuitable for simulating processes involving high strain rates such as fiber spinning or film blowing (Hulsen et al., 2005; Malkin & Isayev, 2006).

Giesekus, PTT, and Other Nonlinear Viscoelastic Models

The **Giesekus model** introduces anisotropic drag within its constitutive equation and successfully captures shear-thinning and finite extensional viscosity—phenomena common in polymer melts and wormlike micellar solutions (Giesekus, 1982; Larson, 1999). The **Phan-Thien–Tanner (PTT)** model incorporates network-based nonlinearities and reproduces realistic extensional thickening and shear responses, making it suitable for extrusion and filament stretching simulations (Tanner, 2000; Phan-Thien & Tanner, 1977).

These advanced models have been widely adopted in predicting flow instabilities, including elastic turbulence, melt fracture, and sharkskin defects in polymer processing (Malkin & Isayev, 2006; Rothstein, 2003). Moreover, they are essential in capturing memory effects where stress depends on deformation history, such as in start-up flows, step changes, and oscillatory shear experiments (Ewoldt et al., 2015; Hinch, 1991).

Role in Industrial Simulation and Process Optimization

Accurate viscoelastic modeling is critical for predicting pressure drops, die swell ratios, fiber thinning, coating uniformity, and flow instabilities in polymer processing (Owens & Phillips, 2002; Hulsen et al., 2005). Industries increasingly rely on numerical tools implementing these models to optimize equipment design, avoid defects, and reduce production costs (Bird et al., 1987; Malkin & Isayev, 2006). However, computational stiffness, numerical instability, and the need for well-characterized material parameters remain persistent challenges (Pakdel & McKinley, 1996; Larson, 1999).

3.3 Challenges and Limitations of Current Models

Although constitutive equations are indispensable tools, each model remains an approximation of complex material behavior. No single constitutive equation can universally describe the entire diversity of non-Newtonian fluids due to their wide-ranging microstructures, chemical compositions, and deformation mechanisms (Tanner, 2000; Oberhauser et al., 2020; Chhabra & Richardson, 2011). As a result, constitutive modeling is a process of balancing mathematical complexity, physical realism, computational feasibility, and industrial applicability (Owens & Phillips, 2002; Bird et al., 1987).

Model Applicability and Parameter Sensitivity

Parameters such as yield stress, relaxation times, and nonlinear coefficients depend sensitively on temperature, shear history, chemical composition, and microstructural interactions (Coussot, 2014; Macosko, 1994). Many models assume uniform microstructure, ignoring shear banding, wall slip, or particle migration that strongly influence industrial flows (Barnes, 1997; Morris, 2009). Model parameters often vary between measurement techniques, complicating calibration (Mewis & Wagner, 2012).

Inability to Capture Multimodal Phenomena

Most constitutive models describe only a subset of rheological features such as shear-thinning, elasticity, or yield stress. However, real industrial fluids may simultaneously exhibit shear-thinning, thixotropy, viscoelasticity, and yield behavior—effects rarely captured within a single framework (Larson, 1999; Coussot, 2014; Barnes, 1997). Multiphase materials such as foams, emulsions, biological fluids, and particulate suspensions further complicate modeling due to interfacial forces, aggregation, breakup, and microstructural evolution (Ewoldt et al., 2015; Morris, 2009).

Computational Difficulties and Numerical Instability

Nonlinear viscoelastic models often generate numerical stiffness, high Weissenberg number instability, and divergence in finite element or finite volume simulations (Hulsen et al., 2005; Owens & Phillips, 2002). The choice of constitutive model directly impacts convergence rates, computational time, and accuracy in industrial CFD simulations (Rothstein, 2003; Pakdel & McKinley, 1996).

Real-World Complexity Beyond Current Theories

Emerging materials such as nanofluids, magneto-rheological fluids, ionic liquids, and active biological systems exhibit microstructural dynamics not fully described by classical models (Oberhauser et al., 2020; Malkin & Isayev, 2006). Incorporating particle interactions, field-responsive behavior, or self-organization requires developing hybrid, multi-scale, or data-driven constitutive frameworks (Morris, 2009; Ewoldt et al., 2015).

Toward Advanced and Hybrid Constitutive Modeling

The field is progressing toward multi-component constitutive models combining viscoelasticity, thixotropy, and yield stress within unified frameworks, as well as incorporating microstructural physics derived from statistical mechanics or continuum theories (Hinch, 1991; Larson, 1999). Machine learning and data-driven approaches are increasingly being used to refine model parameters or propose new constitutive relations (Oberhauser et al., 2020). Despite these advances, selecting an appropriate model for industrial application still depends heavily on experimental validation and careful rheological characterization (Bird et al., 1987; Mewis & Wagner, 2012).

IV. Experimental and Computational Approaches

The study of non-Newtonian fluids relies heavily on rigorous experimental and computational methodologies capable of capturing their extraordinary rheological complexities. Unlike Newtonian fluids, whose flow behavior is governed by constant viscosity, non-Newtonian systems exhibit shear-rate dependence, viscoelasticity, time-dependent thixotropic or rheopectic responses, and microstructural evolution, requiring specialized tools for characterization (Barnes, 2000; Macosko, 1994). Modern industry—from polymer processing and biomedical engineering to food engineering and enhanced oil recovery—depends on precise rheological measurements and predictive simulations to optimize processes, enhance product quality, and ensure equipment reliability (Chhabra & Richardson, 2011; Malkin & Isayev, 2006). Experimental and computational approaches thus form the backbone of contemporary non-Newtonian fluid research.

4.1 Rheometric Techniques

Rheometric characterization is fundamental to quantifying how non-Newtonian fluids respond to applied stresses or deformations. Devices such as rotational rheometers, capillary rheometers, and oscillatory rheometers provide quantitative insight into viscosity, viscoelastic moduli, relaxation times, yield stress, and flow instabilities (Macosko, 1994; Barnes, 2000).

Rotational Rheometry

Rotational rheometers, operating in steady shear mode, represent the most widely used class of instruments for non-Newtonian fluids. Using geometries such as cone-and-plate, parallel plate, and concentric cylinder systems, they impose a controlled shear rate or shear stress and measure the corresponding response (Steffe, 1996). These instruments are indispensable for characterizing **shear-thinning behavior** in polymer solutions, **yield stress** in cement pastes, and **shear-thickening transitions** in dense suspensions (Mewis & Wagner, 2012). Because non-Newtonian behavior often spans several decades of shear rate, modern rheometers use advanced torque transducers, air bearings, and precise gap control to ensure measurement reliability across extreme operating conditions (Larson, 1999).

Capillary Rheometry

Capillary rheometers measure viscosity at high shear rates by forcing fluid through a narrow capillary tube under controlled pressure. This technique is especially critical for polymer melts, food pastes, and concentrated slurries, where industrial processing involves shear rates exceeding 10^4 to 10^5 s⁻¹ (Tanner, 2000). Capillary data can be further corrected for entrance effects and wall slip using models such as Bagley correction and Mooney method (Bird et al., 1987). Polymer extrusion processes, inkjet printing, and fiber spinning industries rely heavily on capillary rheometry to predict pressure drops, die swell, and melt fracture (Malkin & Isayev, 2006).

Oscillatory Rheometry

Oscillatory shear tests provide a window into *linear viscoelastic behavior*, enabling measurement of the **storage modulus (G')**, **loss modulus (G'')**, and **complex viscosity**. These measurements are crucial for identifying gel points, network strength, relaxation mechanisms, and structural stability in emulsions, biopolymer gels, surfactant systems, and nano-composites (Ferry, 1980). Oscillatory tests can also probe time-temperature superposition behavior in polymer melts, enabling the construction of master curves that characterize relaxation spectra across broad time scales (Bird et al., 1987).

Yield Stress and Creep Tests

Yield-stress materials—such as drilling muds, pastes, and gels—require dedicated tests such as creep, stress ramp, and vane rheometry to quantify the minimum stress necessary to initiate flow (Barnes, 1999). Creep recovery experiments allow understanding of viscoelastic relaxation and plastic deformation, which are pivotal for predicting flow stoppage in civil engineering materials and the stability of thixotropic products such as paints and adhesives (Chhabra & Richardson, 2011).

4.2 Flow Visualization and Microstructural Tools

Experimental flow visualization methods have expanded dramatically over the past two decades, offering real-time measurement of velocity profiles, deformation fields, and internal microstructural evolution. Because non-Newtonian fluids often exhibit secondary flows, elastic instabilities, and shear banding, visualization is essential to interpreting complex flow phenomena (Morris, 2020; Barnes, 2000).

Particle Image Velocimetry (PIV)

PIV is one of the most widely used visualization techniques in rheology. It tracks the motion of seeded tracer particles illuminated by laser sheets, enabling computation of 2D or 3D velocity fields (Adrian & Westerweel, 2011). PIV has been instrumental in revealing velocity gradients in shear-thinning fluids, flow separation and stagnation points in viscoelastic flows, elastic turbulence in polymer solutions, and secondary vortices in curved geometries (Soulages et al., 2009). In microfluidics, micro-PIV allows detailed visualization of elasticity-driven instabilities such as lip vortices and serpentine channel vortices (Arratia et al., 2006).

Laser Doppler Velocimetry (LDV)

LDV provides point-wise velocity measurements based on Doppler shifts in laser light scattered by suspended particles (Durst et al., 1976). Compared to PIV, LDV offers superior temporal resolution, making it advantageous for studying: highly elastic flows, oscillatory flows, and turbulent drag-reduction phenomena in polymer solutions (White & Mungal, 2008). LDV has also been used to validate computational predictions in viscoelastic flow through complex geometries (Owens & Phillips, 2002).

X-ray Tomography and Neutron Imaging

Advanced imaging techniques such as **X-ray microtomography**, **small-angle neutron scattering (SANS)**, and **synchrotron-based imaging** provide unparalleled insights into internal microstructure, particularly in opaque materials (Withers, 2007). These methods have revealed particle migration in suspensions (Morris, 2020), droplet deformation in emulsions (Mason et al., 1997), and network formation in gels (Buscall et al., 1988). Such visualization methods bridge the gap between continuum rheology and microstructural physics, providing data essential for validating microstructure-resolving constitutive models like Giesekus and PTT (Larson, 1999).

4.3 Computational Fluid Dynamics (CFD)

CFD has become indispensable for simulating the behavior of non-Newtonian fluids under conditions that are difficult or impossible to test experimentally. Simulations help predict extrusion instabilities, mixing efficiency, polymer drag reduction, heat transfer effects, and free-surface dynamics in industrial processes (Owens & Phillips, 2002; Afonso et al., 2009).

Finite Element Method (FEM)

FEM excels at solving viscoelastic constitutive equations such as Oldroyd-B, Giesekus, or PTT, especially in complex geometries (Tanner, 2000). FEM-based solvers allow modeling of die swell in polymer extrusion, vortex growth in contraction flows, and melt fracture instabilities, and viscoelastic stress accumulation at stagnation points (Fan et al., 2011).

However, FEM faces numerical challenges such as high Weissenberg number instabilities (the “HWNP problem”), necessitating stabilization schemes like DEVSS, log-conformation methods, and SUPG formulations (Owens & Phillips, 2002).

Finite Volume Method (FVM)

FVM, used widely in industrial solvers like ANSYS Fluent and OpenFOAM, is preferred for large-scale multiphase or turbulent flows (Ferziger & Perić, 2002). FVM effectively handles shear-thinning flows in pipes, yield-stress behavior in drilling fluids, and multiphase systems such as foams and emulsions with non-Newtonian continuous phases (Sochi, 2011). FVM also integrates thermal effects, essential in polymer processing where temperature-viscosity coupling dominates material response (Afonso et al., 2009).

Lattice Boltzmann Method (LBM)

LBM offers a mesoscopic approach especially suited for complex microstructures. It has proven effective in modelling such as shear banding in wormlike micelles, confined viscoelastic flows in microchannels, porous media flows for enhanced oil recovery (Krüger et al., 2017). LBM inherently handles complex boundaries and interfacial dynamics, making it increasingly popular in soft matter and biomedical applications.

Smoothed Particle Hydrodynamics (SPH)

SPH is a mesh-free technique widely used for free-surface and multiphase flows such as molten polymer spreading, droplet coalescence, and inkjet printing of viscoelastic fluids (Monaghan, 2005). Its ability to capture large deformations and fluid–structure interactions makes SPH suitable for additive manufacturing simulations and elastoviscoplastic materials.

4.4 High-Performance Computing

High-performance computing (HPC) has revolutionized the simulation of non-Newtonian fluids by enabling more accurate, higher-fidelity, and larger-scale simulations than ever before. As rheological models become increasingly complex—incorporating microstructural physics, thermal coupling, elasticity, and nonlinear flow instabilities—traditional computational resources become insufficient (Yasuda et al., 1981; Owens & Phillips, 2002).

Large-Scale Viscoelastic Simulations

Simulations of viscoelastic fluids at high Weissenberg numbers require substantial computational power because elastic stresses can grow exponentially, causing numerical divergence (Fattal & Kupferman, 2004). HPC-enabled methods allow full 3D simulations of elastic turbulence in polymer solutions (Groisman & Steinberg, 2000) and prediction of melt fracture phenomena in extrusion, and multi-mode constitutive modeling for polymer processing.

Microstructure-Resolved Simulations

Advanced models such as DPD (dissipative particle dynamics), Brownian dynamics, and molecular dynamics provide molecular-level insight into polymer entanglement dynamics, micellar network formation, and thixotropic breakdown and recovery in suspensions (Padding & Louis, 2008). These simulations often involve millions of particles and require GPU acceleration or massively parallel CPU clusters.

Multiphysics and Multiscale Coupling

HPC enables coupling of thermal fields with shear-dependent viscosity, phase-change dynamics with viscoelasticity, and turbulence with polymer elasticity (White & Mungal, 2008). Such multi-physics environments are essential for additive manufacturing, reactive polymer processing, and biomedical flows involving blood or mucus.

Data-Driven and Machine Learning Approaches

Recent advances combine HPC with data-driven tools such as neural networks and reduced-order modeling (Chen et al., 2020). These approaches accelerate surrogate modeling of complex rheology, real-time prediction of flow instabilities, and parameter calibration of constitutive models. As industrial digitalization expands, hybrid data–physics approaches will play an increasingly central role in non-Newtonian fluid engineering.

V. Industrial Applications of Non-Newtonian Fluids

Non-Newtonian fluids play indispensable roles across a wide spectrum of industrial operations due to their unique rheological behaviors, which deviate from classical Newtonian assumptions. These behaviors—such as shear thinning, shear thickening, viscoelasticity, thixotropy, rheopexy, and yield stress—directly influence production efficiency, product quality, energy consumption, equipment design, and material stability (Chhabra & Richardson, 2011; Malkin & Isayev, 2006). Consequently, understanding and modeling non-Newtonian phenomena is not merely an academic pursuit but a fundamental requirement for optimizing industrial processes. The following subsections present an in-depth analysis of the major industrial sectors that depend extensively on non-Newtonian fluid mechanics.

5.1 Polymer Processing Industry

The polymer-processing sector is arguably the most intensive industrial user of non-Newtonian fluid principles. Polymer melts, solutions, and blends exhibit strong shear-thinning behavior combined with viscoelasticity, making accurate rheological modeling essential for extrusion, injection molding, blow molding, thermoforming, and fiber spinning operations (Malkin & Isayev, 2006; Dealy & Wissbrun, 1990). Shear thinning facilitates easier flow through dies and narrow channels, reducing energy consumption and mechanical overload on extruders (Bird et al., 1987). At the same time, viscoelastic effects influence die swell, melt fracture, and surface distortions, which must be controlled to achieve desirable product characteristics (Denn, 2001).

Extrusion and Injection Molding

During extrusion, polymer melts undergo extremely high shear rates, pressure gradients, and temperature variations. Non-Newtonian constitutive models such as Carreau, Cross, Giesekus, and Phan-Thien–Tanner (PTT) often provide predictive insight into pressure drops, stress accumulation, and melt homogeneity (Macosko, 1994; Larson, 1999). Die swell, a viscoelastic response where polymer melt expands upon exiting the die, can compromise dimensional precision and surface finish. This behavior is influenced by normal stresses, molecular orientation, and relaxation dynamics (Tanner, 2000).

Injection molding involves rapid filling, packing, and cooling, where viscosity and relaxation times depend on both temperature and shear history (Matuana & Park, 1998). Accurate rheological modeling prevents short shots, weld lines, sink marks, and cycle time inefficiencies (Malkin & Isayev, 2006). Advanced computational fluid dynamics (CFD) tools simulate filling patterns using viscoelastic constitutive equations to predict injection pressure requirements and fiber-orientation distribution in reinforced composites.

Fiber Spinning and Blow Molding

In fiber spinning, elongational viscosity becomes the dominant rheological parameter. Non-Newtonian fluids such as polymer melts display strain hardening or thinning depending on molecular architecture, which affects filament breakage, crystallization, and mechanical properties (Denn, 1980). Similarly, blow molding of hollow products requires stable melt strength and viscoelasticity to maintain uniform wall thickness (Cogswell, 1981). Insufficient viscoelastic stability can lead to sagging or parison collapse during the inflation process. Thus, the polymer-processing industry relies heavily on detailed rheological characterization and viscoelastic modeling for efficient and defect-free production.

5.2 Food Processing and Dairy Technology

Food systems are well-known examples of complex non-Newtonian fluids, comprising suspensions, emulsions, foams, gels, and pastes. Their rheology dictates not only processing parameters such as pumping, mixing, homogenization, and heat transfer but also sensory qualities such as texture, mouthfeel, spreadability, and stability (Steffe, 1996; Rao, 2007).

Shear-Thinning Behavior in Food Materials

Many food products—ketchup, yogurt, mayonnaise, chocolate, sauces, and soups—display shear-thinning behavior, where viscosity decreases with increasing shear rate. This facilitates easy mixing and pumping during production while ensuring desirable texture at rest (Bourne, 2002). For example, tomato ketchup must flow readily when squeezed from a bottle but remain stable without phase separation on the shelf, a balance often achieved through thixotropic structuring and yield stress (Rao, 2007).

Viscoelasticity in Dairy and Bakery Products

Dairy products such as cheese, cream, and yogurt exhibit viscoelasticity that governs their gel strength, fracture mechanics, and stability against syneresis (Gunasekaran & Ak, 2003). Similarly, dough in bakery applications behaves as a viscoelastic solid-fluid hybrid, where gluten network formation determines extensibility, gas retention, and final bread structure (Edwards, 2007).

Yield Stress and Thixotropy in Sauces and Pastes

Food pastes—e.g., peanut butter, dressings, and fruit purées—possess yield stress that ensures shape retention and stability under storage. Their thixotropic properties improve spreadability during processing but allow reformation of structure during rest (Coussot, 2014). Manufacturers rely on rheometry to optimize formulation, texture modification, and sensory quality.

Food processing therefore fundamentally depends on non-Newtonian rheology for ensuring product consistency, efficient manufacturing, and consumer acceptance.

5.3 Pharmaceutical and Cosmetic Applications

Pharmaceutical suspensions, topical formulations, controlled-release gels, and cosmetic creams rely on precise rheological properties to achieve therapeutic effectiveness, stability, aesthetic quality, and patient usability (Mezger, 2020; Coussot, 2014).

Topical and Dermal Applications

Creams, lotions, ointments, and gels exhibit plasticity, viscoelasticity, and shear thinning, facilitating easy spreading on the skin while maintaining stability in packaging (Jones et al., 2016). Rheological tailoring improves sensory attributes such as smoothness, cooling sensation, and long-term consistency. Yield stress ensures suspended active ingredients do not sediment or cream over time.

Suspensions and Emulsions in Drug Delivery

Drug suspensions must maintain uniform dispersion of particles to ensure accurate dosing and predictable bioavailability. Rheological tuning prevents sedimentation, flocculation, and viscosity drift (Chhabra & Richardson, 2011). Non-Newtonian properties influence syringeability, spray behavior, and dissolution kinetics.

Hydrogels and Bioadhesive Polymers

Hydrogels used for wound dressings, transdermal patches, and mucosal drug delivery display complex viscoelastic properties and often thixotropic recovery after deformation (Hoare & Kohane, 2008). Their rheology affects adhesion, drug diffusion, and structural stability.

Cosmetic Formulations

Cosmetics—including shampoos, conditioners, mascaras, and foundations—require controlled viscosity for shelf stability, ease of application, and film formation (Mezger, 2020). Surfactant-rich systems display shear-dependent microstructural transitions that directly influence performance.

5.4 Petroleum and Drilling Engineering

The petroleum industry relies heavily on non-Newtonian fluids such as drilling muds, fracturing fluids, enhanced-oil-recovery polymers, and heavy crude oils. These materials exhibit yield stress, viscoelasticity, thixotropy, and shear thinning, which critically affect operational efficiency and safety (Acrivos, 1995; Hedström, 1952).

Drilling Muds and Borehole Stability

Drilling muds are engineered suspensions containing clays, polymers, weighting materials, and lubricants. Their rheology affects:

- cuttings transport efficiency,
- borehole pressure control,
- drill-bit lubrication,
- filter-cake formation,
- suspension of solids during pump shutdown (Kelessidis & Maglione, 2008).

These fluids display yield stress that prevents sedimentation and thixotropic recovery essential for maintaining structural integrity during interruptions (Barnes, 1999).

Enhanced Oil Recovery (EOR)

Polymer flooding utilizes viscoelastic polymer solutions—such as partially hydrolyzed polyacrylamide (HPAM)—to improve sweep efficiency and reduce viscous fingering (Chauveteau, 1982). Their elasticity contributes to improved mobility control and oil displacement.

Heavy Oil and Bitumen Transport

Highly viscous crude oils and bitumen exhibit shear-thinning behavior, reducing pumping power requirements at high shear rates. Dilution, heating, and surfactant modification are used to manage complex rheology during pipeline transport (Fisher et al., 2007).

Hydraulic Fracturing Fluids

Fracturing fluids are engineered to possess shear thinning during pumping but high elasticity and viscosity in fractures for effective proppant suspension (Barati & Liang, 2014). Cross-linked gels and surfactant-based viscoelastic fluids demonstrate pronounced non-Newtonian properties essential to reservoir stimulation.

Thus, petroleum engineering processes depend fundamentally on controlling non-Newtonian fluid behavior to ensure efficient and safe operations.

5.5 Cement, Construction, and Civil Engineering

Cementitious materials—including cement paste, mortar, grout, and concrete—are classic examples of yield-stress, thixotropic fluids whose rheology governs workability, pumping, casting, structural buildup, and early-age performance (Roussel, 2006; Banfill, 2006).

Yield Stress and Workability

Cement paste exhibits Bingham or Herschel–Bulkley behavior where yield stress determines flow initiation, segregation resistance, and formwork filling ability (Chhabra & Richardson, 2011). Low shear viscosity improves pumpability, whereas appropriate structural buildup enhances early-age stability and printability in emerging applications like 3D concrete printing (Le et al., 2012).

Thixotropy and Structural Build-Up

Cement hydration leads to time-dependent changes in rheology. Thixotropic breakdown during mixing is followed by recovery during rest, influencing setting time, finishing operations, and structural performance (Roussel, 2006). Rheological modifiers such as superplasticizers tune these properties to meet performance requirements.

Self-Compacting Concrete (SCC)

SCC relies heavily on rheological control—specifically low yield stress yet high plastic viscosity—to achieve flowability without segregation. Non-Newtonian admixtures and particles contribute to maintaining a delicate balance between flow and stability (Khayat, 1999).

Grouting and Pile Construction

In grouting operations, shear-thinning improves penetration into fine soil pores, whereas thixotropic recovery prevents backflow and washout (Ferraris et al., 1997). Accurate control of rheology ensures operational success in tunneling, dam repair, and foundation engineering.

Thus, civil engineering applications extensively rely on non-Newtonian rheology for ensuring performance, durability, and constructability.

5.6 Biological and Biomedical Systems

Biological fluids—including blood, mucus, synovial fluid, and bioengineered hydrogels—exhibit complex non-Newtonian behaviors influenced by their microstructure, cellular content, and biochemical composition (Abu-Fayyad et al., 2017; Fung, 1997). Understanding these behaviors is crucial for medical diagnostics, therapeutic delivery, and emerging bioprinting technologies.

Blood Rheology

Blood is a shear-thinning fluid with viscoelastic effects due to the deformability and aggregation of red blood cells. Its viscosity varies with shear rate, hematocrit, plasma protein concentration, and disease states (Popel & Johnson, 2005). This influences microcirculatory flow, hemodynamic modeling, and medical device design (Fung, 1997).

Mucus and Airway Diseases

Mucus exhibits yield stress, viscoelasticity, and thixotropy, critical for predicting flow behavior in respiratory diseases such as cystic fibrosis or asthma. Rheological modification is essential for drug delivery and mucolytic therapy (Lai et al., 2009).

Tissue Engineering and 3D Bioprinting

Bioinks—hydrogels containing cells and biomolecules—require tailored rheology for shape retention, extrusion stability, and cell viability (Guvendiren et al., 2016). Shear-thinning improves nozzle flow, while viscoelasticity and rapid gelation enable structural integrity post-deposition.

Drug Delivery Carriers

Polymeric drug carriers such as micelles, nanoparticles, and hydrogel matrices possess unique non-Newtonian properties that influence circulation time, diffusion, and controlled release (Hoare & Kohane, 2008).

VI. Conclusion

Research on non-Newtonian fluids has evolved into a vibrant, multidisciplinary field that integrates physics, chemistry, engineering, and computational sciences. The introduction demonstrates that non-Newtonian fluids cannot be accurately described through traditional Newtonian principles because their rheology is strongly influenced by microstructure, internal interactions, and external stimuli such as shear rate, temperature, pressure, and processing history. These complexities create unique flow behaviors—shear-thinning, shear-thickening, viscoelasticity, thixotropy, and yield stress phenomena—that play a defining role in how these fluids behave inside industrial equipment. Industrial applications highlight the critical role of non-Newtonian fluid studies in improving process safety, efficiency, and product quality. From polymer extrusion, injection molding, and 3D printing to drilling mud design, enhanced oil recovery, dairy product formulation, pharmaceutical mixing, biomedical device engineering, and wastewater management, non-Newtonian fluids underpin technologies that directly influence global productivity and sustainability. The introduction further establishes that accurate constitutive models are indispensable for predicting industrial flow behavior; however, traditional models often struggle to capture microstructural dynamics, nonlinearities, hysteresis, and time-dependent phenomena. Thus,

there is a clear need for continued refinement of rheological models that incorporate multi-scale physics and provide better predictive capability.

Experimental characterization techniques—ranging from rotational and capillary rheometry to X-ray imaging, optical velocimetry, and microfluidics—remain fundamental tools for establishing rheological parameters and validating theoretical predictions. Yet, limitations in measurement precision, difficulties in replicating realistic processing conditions, and the sensitivity of certain materials to thermal and mechanical degradation create persistent research challenges. Similarly, computational approaches such as CFD and high-performance computing have dramatically improved our ability to simulate non-Newtonian flows, but they remain limited by constitutive model assumptions and computational demands. The introduction underscores the potential of emerging machine learning and hybrid physics–AI techniques to accelerate model development, optimize processing parameters, and improve real-time control of industrial systems. In the context of future research, the conclusion identifies several pressing priorities: (1) integrating microstructural modeling with continuum rheology, (2) improving the understanding of flow-induced transitions and instabilities, (3) developing universal constitutive frameworks adaptable to diverse materials, (4) expanding multiscale simulation capabilities, (5) promoting sustainable materials and environmentally friendly rheology modifiers, and (6) leveraging data-driven and AI-enhanced tools to support industrial innovation. The continuing alignment of theoretical, experimental, and computational research will enable the development of more robust predictive tools that match industrial reality.

Overall, the introduction establishes that non-Newtonian fluid studies are indispensable for modern industry. As products, processes, and technologies continue to evolve, so too must our understanding of non-Newtonian behavior. Continued research will enable industries to design safer, more efficient, and more sustainable systems while unlocking new possibilities in advanced manufacturing, biotechnology, and materials engineering. This dynamic field will remain central to industrial fluid mechanics, and future innovations promise to reshape both scientific understanding and practical engineering applications.

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