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A Mathematical Model For Well Formation Around A Candle Wick

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Abstract: *In this paper, a mathematical model has been presented which can predict the size of the well formed around a burning candle wick.*
Keywords: *burning candle, well formation, heat transfer, Stefan-Boltzmann Law*

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I. Introduction

In this paper, the crater formed around the wick of a burning candle is studied. The aim is to develop a model which can predict the size of the well [1] that is formed around the burning wick given certain initial parameters. With such a model, we can effectively determine the length of time for which there will be no overflow of wax, given the dimensions of the candle.

II. Material And Methods

Materials:
1. Cylindrical candle with diameter = (10 ± 0.01) cm and height = (3.8 ± 0.01) cm
2. Stop watch
3. Thermocouple of type K. Type K thermocouple has been used to measure the temperature of the flame. Since the temperature of a candle flame usually lies in the range of -200 to 1350 Celsius, type K is appropriate.

Experimental Procedure:
Over short periods of time we measured the following:
1. The temperature of the candle flame (The average of the highest and lowest temperature taken at different instants.)
2. The width of the candle flame at different instants. This was done by dimming the lights in the room, holding a plain sheet of paper by the side of the flame and measuring the width of the bright region formed.

Finally, after each time period we blew out the candle briefly. A picture was taken of the crater that had formed. The height and width of the crater were determined later by pixel counting. Since the height and width of the crater is not uniform in an image, the average was taken. The candle was again burnt.
The mass of the melted wax can be weighed with a beam balance. In the absence of a beam balance, the difference in volume times the density gives the mass of the melted wax.

A mathematical model was developed and is presented here:

$$E_f = \epsilon_f \ \sigma \ T_f^4 (\text{Stefan} - \text{Boltzmann Law}) \quad \ldots(1)$$

$\mathbf{E_f}$ represents the energy radiated by the flame (per unit time, per unit area) as a function of its temperature($\mathbf{T_f}$)[2]

Total energy emitted is now expressed in terms of the area of the flame ($\mathbf{A_f}$) and time (**t**) the flame has been burning:

$$\text{Total energy emitted} = A_f E_f t \quad \ldots(2)$$

We define **α** as follows:

$$\alpha = \frac{\text{Energy absorbed per second}}{\text{Energy incident per second}} \qquad \ldots(3)$$

Finally, we arrive at this result:

$$m = \frac{\alpha A_f (E_f \sigma T_f^4) t}{C_w (MP - RT) + L} \qquad \ldots(4)$$

Here, **m** is the mass of the melted wax, **MP** and **RT** are the melting points of wax and the room temperature respectively, and **L** is the latent heat of wax

Simplifying Assumptions:

The effect of heat transfer by conduction and convection have been ignored as the model is used for a small time period.[3]This assumption is supported by the work done by Hammins and Bundy, where they establish that at small distances from the flame (around 40mm) radiative heat flux is the predominant mode of heat transfer. However, after long use of the candle, the diameter of the crater exceeds the limit.

While the energy radiated out depends on surrounding temperature gradient, we noticed that the room temperature is negligible in comparison to the flame temperature when they are measured in Kelvin and raised to the power of four. It has also been assumed that all radiated heat is used up in melting.

III. Result

Table no 1: Data collected for time gap, crater dimensions, and flame temperature

Time gap(±0.01s)	Diameter(±0.01cm)	Height(±0.01cm)	Height Change	Average Temperature(K)*
900	1.42	0.65	No	1268
1200	1.95	1.325	No	1327
900	2.16	1.725	No	1244
600	2.362	2.066	No	1283
420	2.425	2.45	No	1306
600	2.625	2.666	No	1281
900	2.74	3.2	No	1266
600	2.81	3.5	No	1274

*(Range of error of thermocouple is 0.1% of reading +/- 0.4 Celsius)

The predicted mass was calculated with the data in the table above and the actual mass of wax that had melted were compared.
The ratio(**r**) of the mass of the wax that has actually melted and predicted values is taken.

Table no 2: Records the ratio **r**

Time Period(Number)	Ratio(r)
2	0.944
3	0.927
4	0.941
5	0.983
6	1.028
7	1.077
8	1.064

IV. Conclusion

This model is quite accurate considering that only a few parameters, which could be readily measured, were used. The model, as seen from the last table, predicts values that are less than the actual mass of melted wax, after a sufficiently long period of time. This was not surprising as, with increasing separation between the flame and the wall of the crater, the effect of heat transfer through convection etc. become appreciable.

In the future, we will work towards incorporating the effect of heat transfer through convection and identifying the optimum time interval for using this model.

References

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