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A new insight into the physics of Lorentz invariance: Implications for the GZK and TeV-γ photon theories

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Abstract: *The main argument of the Greisen-Zatsepin-Kuzmin (GZK) and TeV-γ photon theories[1,2,3,4,5,6] is based on the concept of Lorentz invariance. In this paper, we revisit this concept and show that as the mathematical derivation of Lorentz transformations is based on a specific orientation in space of two inertial frames, it cannot be used to mathematically correlate coordinates measured from inertial frames in a different orientation. This built-in mathematical restriction on the applicability of Lorentz transformations naturally leads to new restrictions on the applicability of Lorentz invariance that invalidate the results of the GZK and TeV-γ photon theories.*

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I. Introduction

The results of the interaction between ultra-high-energy cosmic rays (UHECR) and some kind of universal photon background, as predicted by the GZK and TeV- γ photon theories[1,2,3,4,5,6], are proving challenging for the scientific community to defend because of the experimental detection of UHECR particles and multi TeV gamma photons on Earth at energy values above the thresholds defined by the GZK and TeV- γ photons theories[7,8,9]. The announcement[10] of the observation of the GZK cut-off in the HiRes experiment did not alter this situation, for it does not prove that super GZK particles do not exist and does not affect the status of the TeV- γ photon paradox. As several attempts[11,12] have been made over the last two decades to account for the presence of UHECR particles on Earth by proposing various models—some involving new physics and few the drastic assumption of a violation of Lorentz invariance—without success, it is likely that the predictions of the GZK and TeV- γ photon theories are incorrect and need to be re-examined.

The main argument of both the GZK and the TeV- γ photon theories is based on the concept of Lorentz invariance. In the GZK cut-off, for example, the relative motion of an incoming projectile—the UHECR proton (assuming the UHECR particle is a proton)—toward the CMBR photon leads to a collision that, when Lorentz-transformed to the projectile rest frame, is seen as a collision between a proton at rest and an energetic gamma ray photon and, thus, should result in photopion production. However, the detection of super GZK particles and multi-TeV-gamma photons on Earth suggests that the expected interactions did not occur because most likely the colliding particles did not behave as expected following the above mentioned special relativistic operation. This seemingly improbable line of approach is taken up in this paper as all other viable scenarios have already been examined over the last two decades. This motivates us to re-examine the physics underlying the main argument of these theories, and this approach leads to some startling results.

Following the above line of argument, the key objective of this study is to revisit and analyze the physics of the GZK and the TeV-gamma photon theories. This is based on ordinary Special Relativity in ways that remain unaffected by the fact that this analysis does not involve the mathematical formalisms of Hopf algebras used to construct new relativity (DSR) theories, which nevertheless has not been proven to be consistent with the physics of these theories[11].

The perspective described can help reveal new insights into the physics of Lorentz invariance, the derivation of which involves only deductive logic. As we shall see, this is based on the original work of Einstein, and is in agreement with work by Lorentz and Poincaré. Using this new revelation, the challenges facing high-energy particle physics at present are met while preserving the success achieved by applications of Lorentz invariants in particle physics in the past.

II. A specific orientation of inertial frames used in relativity theories

We start this analysis by pointing out the seemingly trivial, but actually significant, observation that the mathematical derivation of transformations in both Galilean Relativity and Special Relativity is based on a specific orientation in space of two coordinate systems in relative uniform translational motion. Its significance lies in the fact that this derivation restricts their applicability to only this particular orientation. To elaborate the point further, we consider this orientation, which was elaborately described by Einstein[13]. He begins with coordinate systems S and S' (Fig. 1) placed in standard configuration at time $t = t' = 0$, when their origins and all three axes coincide. A constant velocity *v* is then imparted to the origin of one of the two systems or frames (say S') along the positive *x* direction of the other stationary frame (*S*). If a point event can be defined with coordinates (*x*, *y*, *z*, *t*) and (x', y', z', t') relative to S and S', respectively, then, to find a mathematical relation between these sets, he introduced two postulates. Based on them, he obtained the following transformation equations

$$\begin{aligned} x' &= \gamma(x - vt), \\ y' &= y, \quad z' = z, \\ t' &= \gamma(t - vx/c^2). \end{aligned} \tag{1}$$

Lorentz used the same orientation of coordinate systems and arrived independently at (1), as noted by Poincaré[14], who claimed that "Lorentz's concept may be summarized thus: If a common translatory motion may be imparted to the entire system without any alteration of the observable phenomena, then the equations of an electromagnetic medium are unaltered by certain transformations, which we shall call Lorentz transformations. In this way two systems, of which one is fixed and the other is in translatory motion, become exact images of each other." Lastly, we note that one can obtain Galilean transformations for the same orientation of coordinate systems by substituting an infinitely large value for the velocity of light *c* in the above transformations.

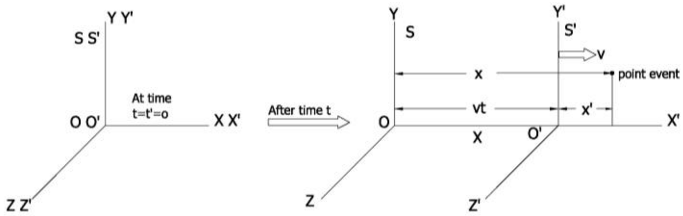

Fig. 1. The inertial frames S and S' in standard configuration at time $t = t' = 0$ before the commencement of uniform translational motion, and their position after time t when S' has moved distance *vt* along the positive x-axis of *S*. In the geometric relation, $x' = x - vt$, *vt* (a vector quantity) marks the relative location in space of the two inertial frames used to measure the coordinates of the point event.

After revisiting the derivation of the Lorentz transformations for two specifically oriented coordinate systems in space, we proceed to show that they cannot be used to mathematically correlate coordinates measured in coordinate systems having different relative orientations in space, unlike those shown in Fig. 1.To this end, we begin by examining the important term $x' = x - vt$ appearing in both the Galilean and the Lorentz transformations. It is important to note that this relation is geometrically based on the location of the point event as well as the relative location in space of the two inertial frames in translational motion and, therefore, can be easily recognized in Fig. 1. This is because the origins of both the inertial frames coincide at time $t = t' = 0$ and after time *t*, frame S' has covered distance vt along the positive *x* direction of *S*. It is important to note that this relation not only relates the two coordinates x' and x, but also contains information concerning the relative locations in space of the two frames from which the coordinates were measured, which is expressed by the vector term *vt* (Fig. 1).

Let us consider two other inertial frames K and K' in relative motion toward each other (Fig. 2), in contrast to the inertial frames S and S', which are moving away from each other. The same point event now has the coordinates (*x*, *y*, *z*, *t*) and $(-x', y', z', t')$ relative to K and K', respectively, if we choose to use the same symbols as in S and S'. On comparing the orientations, directions of motion, and relative locations in space of the inertial frames in Fig. 2 with those in Fig. 1, one can immediately see that the geometric relation

$x^{'} = x - vt$ does not hold in Fig. 2 as it is impossible to mark the distance *vt* in it. It follows that the Lorentz transformations cannot be used to relate coordinates measured from the frames in this orientation. The revelation of this mathematical restriction built into the Lorentz transformations is not as startling as it appears, for we are already aware of this situation as we presently show. However, we first note here that the *physical* significance of the relative motion of inertial frames in any orientation, e.g., $S - S^{'}$ and $K - K^{'}$, is the same because the direction of motion never enters into the formulated laws. That is why none of the consequences of the Lorentz transformations, viz., simultaneity, time dilation, and length contraction, are dependent on the direction of the moving inertial frame.

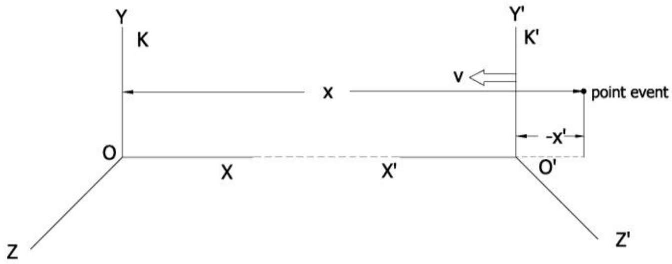

Fig. 2. In sharp contrast to Fig. 1, the inertial frame $K^{'}$ is moving towards K. In this case, the coordinates, x and $-x'$, of the same point event are not relatable by the geometric relation $x^{'} = x - vt$.

The above revelation also does not mean that Special Relativity cannot deal with inertial frames in orientations other than that shown in Fig. 1. We routinely do so by exploiting the fact that the direction of motion of the inertial frames does not enter into the laws of nature, as mentioned above. This gives us the freedom to choose the frame in motion according to convenience. To demonstrate this more clearly, we present a simple example from a commonly used textbook of physics for undergraduates[15]. In this example, a starship from Earth moves *toward* a planet that is an outpost, whereas an enemy has a base on the moon of this planet. The starship follows a straight-line course, first past the planet and then past the moon, i.e., the situation is similar to the relative motion of frames K and $K^{'}$. Now, according to the textbook, to tackle this situation in the framework of Lorentz transformations, the frame of reference of starship is assumed to be stationary and that of the planet–moon system as relatively in motion toward the right with respect to the velocity of the starship. The arrangement of the frames of reference is now like that of S and $S^{'}$ and, thus, allows us to deal with related problems by using Lorentz transformations. The manner in which this problem is dealt with in common physics textbooks highlights common knowledge of this restriction that, nonetheless, has never been explicitly stated.

III. On the practical use of Lorentz transformations in solving problems of particle physics

We routinely use relativistic mechanics to successfully solve particle collision problems, and it never forces us to differentiate between the $S - S^{'}$ and $K - K^{'}$ orientations of frames. Nonetheless, this fact itself leads us now to note an important point: that it is not the relativistic mechanics but the Lorentz transformations that mathematically differentiate between these orientations, implying that the use of relativistic mechanics in particle collision problems does not involve the use of Lorentz transformations.

To show the above unambiguously, which in turn can help us reveal the physics underlying the concept of Lorentz invariance and, thus, the principles behind it, viz., the relativity of frames and the equivalence principle, we take up the well-known case of a collision between an X-ray photon (K_{α} of Mo) and a loosely bound valence electron of a carbon target in Compton scattering. As Compton's equation can be obtained in many ways by using various mathematical formalisms of relativistic mechanics, we briefly discuss them below.

Compton's method

Compton[16] interpreted the scattering experiment in terms of energy and momentum transfers, via photons, between the incident photon and an electron. He used the *relativistic expression for kinetic energy* for the conservation of energy, which takes the form

$$hf_1 = hf_2 + m_0 c^2 (\gamma - 1)$$
$$\text{or} \quad \frac{h}{\lambda_1} = \frac{h}{\lambda_2} + m_0 c(\gamma - 1). \tag{2}$$

Subscripts 1 and 2 represent the incident and the scattered photon, respectively. The conservation of momentum (again, he used the *relativistic expression for momentum*) in a two-dimensional situation gives

$$\frac{h}{\lambda_1} = \frac{h}{\lambda_2} \cos\phi + \gamma m_0 v \cos\theta$$
$$0 = \frac{h}{\lambda_2} \sin\phi - \gamma m_0 v \sin\theta. \tag{3}$$

where ϕ is the scattering angle of the photon and θ the angle of recoil of the electron. The elimination of v and θ from Eqs. (2) and (3) yields the well-known result

$$\lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1 - \cos\phi). \tag{4}$$

It is clear that Compton arrived at this result by using only two relativistic expressions not involving the Lorentz transformations. For further clarity, we note that one does not need the Lorentz transformations to arrive at the relativistic expressions for momentum and energy.

Use of the energy–momentum relation to obtain Compton's equation

Compton's equation can also be obtained by using the energy–momentum relation, $E^2 - p^2c^2 = m_0^2c^4$. By using this, Eq. (2) of the conservation of energy takes the form

$$hf_1 + m_e c^2 = hf_2 + \sqrt{(p_e^2 c^2 + m_e^2 c^4)} \tag{5}$$
where, $m_e = m_0$ of electron.

The equation for the conservation of momentum can be rewritten as

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_e$$
$$\text{or} \quad (p_e c)^2 = (hf_1)^2 + (hf_2)^2 - 2h^2 f_1 f_2 \cos\phi. \tag{6}$$

Equations (5) and (6) also yield the same Compton Eq. (4). But the use of the energy–momentum relation, better known as a Lorentz invariant dispersion relation, tacitly suggests the involvement of the Lorentz transformations in the process, - in contrast to what has been shown in the previous subsection - and thus, this needs to be resolved. To address this problem, we focus on the use of the energy–momentum relation, and begin by pointing out that this relation can be obtained by simply blending the relativistic expressions for momentum and kinetic energy used in the previous subsection. The energy–momentum relation is thus a simple mathematical expression relating relativistic momentum to kinetic energy. It does not make a difference whether one uses the relativistic expressions for momentum and kinetic energy or a relation obtained by simply blending them to arrive at Eq. (4). This renders it irrational to accept that the mere use of the energy–momentum relation for the conservation of energy and momentum in arriving at Eq. (4) must be taken to imply the involvement of Lorentz transformations in the process, simply because the said relation *also happens to be a Lorentz invariant*. It follows that the energy–momentum relation plays two roles—in some situations as a Lorentz invariant relation and in others as a simple mathematical expression. To elaborate further, it is necessary to briefly revisit work in classical physics leading to Lorentz invariance and the Lorentz invariant dispersion relation.

On a property of Lorentz invariance: It is helpful to recall that the first postulate of Special Relativity defines the Lorentz symmetry of the universe by requiring the *form invariance* of the physical laws in inertial frames in uniform translatory motion relative to each other. Einstein[13], Lorentz and Poincaré[14] (see the quote in Section II) showed that Maxwell's equations of electromagnetism are *form invariant* with reference to the Lorentz transformations following the transition from S to S' and, thus, are *Lorentz invariant*. With this definition in mind, we revisit the above example of the Lorentz invariance in addition to two others from classical physics to show that it has a broader meaning than afforded by the customary perspective.

We first consider an example of Maxwell's equations of electromagnetism and present them as given in the relevant paper by Einstein[13]. This is because by using them, he established the very concept of the Lorentz invariance. He expressed these equations for a stationary inertial frame *S* as

$$\begin{aligned}
&\frac{1}{c}\frac{\partial X}{\partial t} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, \quad \frac{1}{c}\frac{\partial L}{\partial t} = \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}, \\
&\frac{1}{c}\frac{\partial Y}{\partial t} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}, \quad \frac{1}{c}\frac{\partial M}{\partial t} = \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}, \\
&\frac{1}{c}\frac{\partial Z}{\partial t} = \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}, \quad \frac{1}{c}\frac{\partial N}{\partial t} = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}.
\end{aligned} \tag{7}$$

and showed that they can be Lorentz-transformed from S to $S^{'}$ under Lorentz transformations, and can be expressed for the moving frame $S^{'}$ as

$$\begin{aligned}
&\frac{1}{c}\frac{\partial X^{'}}{\partial \tau} = \frac{\partial N^{'}}{\partial \eta} - \frac{\partial M^{'}}{\partial \zeta}, \quad \frac{1}{c}\frac{\partial L^{'}}{\partial \tau} = \frac{\partial Y^{'}}{\partial \zeta} - \frac{\partial Z^{'}}{\partial \eta}, \\
&\frac{1}{c}\frac{\partial Y^{'}}{\partial \tau} = \frac{\partial L^{'}}{\partial \zeta} - \frac{\partial N^{'}}{\partial \xi}, \quad \frac{1}{c}\frac{\partial M^{'}}{\partial \tau} = \frac{\partial Z^{'}}{\partial \xi} - \frac{\partial X^{'}}{\partial \zeta}, \\
&\frac{1}{c}\frac{\partial Z^{'}}{\partial \tau} = \frac{\partial M^{'}}{\partial \xi} - \frac{\partial L^{'}}{\partial \eta}, \quad \frac{1}{c}\frac{\partial N^{'}}{\partial \tau} = \frac{\partial X^{'}}{\partial \eta} - \frac{\partial Y^{'}}{\partial \xi}.
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
&X^{'} = X, \qquad\qquad\qquad L^{'} = L \\
&Y^{'} = \beta(Y - (v/c)N), \quad M^{'} = \beta(M + (v/c)Z) \\
&Z^{'} = \beta(Z + (v/c)M), \quad N^{'} = \beta(N - (v/c)Y).
\end{aligned} \tag{9}$$

In this way, he convincingly demonstrated that these equations remain *form invariant under Lorentz transformations* and, thus, are said to be *Lorentz invariant*. To move forward in our analysis, we need to verbally distinguish the types of 'form invariants'—Eqs. (7) and (8).We use the prefix 'unprimed' for the former and 'primed' for the latter equation. It is important to note that in this first and well-accepted *demonstrative* example of Lorentz invariance, the unprimed form of the equations are not mathematically equal to the primed form, i.e., $\text{Eqs. } 7 \neq \text{Eqs. } 8$, because of the relations expressed in Eq. (9). It should also be noted that the definition of Lorentz symmetry does not in any way convey that this must not be the case.

While interpreting these results, Einstein had concluded that as the electrical and magnetic forces are fundamentally related to each other, they can appear in different proportions to different inertial observers.

Lorentz invariant space–time interval and the dispersion relation: We now briefly review two other classical examples of Lorentz invariance that have profoundly shaped our understanding of this concept. Minkowski[17] found that on replacing the usual time variable *t* with an imaginary variable ict proportional to it, the Lorentz transformations satisfy the following condition:

$$s^2 = c^2t^2 - x^2 - y^2 - z^2 = c^2t^{'2} - x^{'2} - y^{'2} - z^{'2}. \tag{10}$$

This can be rewritten for the four coordinates of interest to us as

$$s^2 = c^2t^2 - x^2 = c^2t^{'2} - x^{'2}. \tag{11}$$

This means that the expression $c^2t^2 - x^2$ remains *form invariant* $c^2t'^2 - x'^2$ after the transition from inertial frame S to S' *under the Lorentz transformations,* as shown below, and therefore is Lorentz invariant per the definition:

$$\begin{aligned} c^2t'^2 - x'^2 &= c^2\gamma^2(t - vx/c^2)^2 - \gamma^2(x - vt)^2 \\ &= c^2t^2 - x^2 \end{aligned} \tag{12}$$

Recall that the ordered pair $(x,\ ct)$ directly leads us to another ordered pair $(cp,\ E)$ that has similar transformation-related properties and, thus, supplies us with the following analogous relation:

$$m_0^2c^4 = E^2 - p^2c^2 = E'^2 - p'^2c^2. \tag{13}$$

In this case as well, the expression $E'^2 - p'^2c^2$ remains *form invariant* $E^2 - p^2c^2$ following the transition from inertial frame S' *to* S *under the energy–momentum Lorentz transformations,* as shown below, and therefore is Lorentz invariant:

$$\begin{aligned} E'^2 - p'^2c^2 &= \gamma^2(E - \frac{v}{c}cp)^2 - \gamma^2(cp - \frac{v}{c}E)^2 \\ &= \gamma^2[(E^2 - p^2c^2) - \frac{v^2}{c^2}(E^2 - p^2c^2)] \\ &= E^2 - p^2c^2. \end{aligned} \tag{14}$$

It is important to note that these two examples based on Minkowski's formalism not only demonstrate the original feature of the picture of Lorentz invariance presented by Einstein, Lorentz, and Poincaré, viz., 'form invariance' with reference to the Lorentz transformations, but also show that unlike Maxwell's equations, unprimed form of the expressions in these cases are mathematically equal to the primed forms.

Years after showing the Lorentz invariance of Maxwell's equations, Einstein wrote[18] that "Maxwell's equations are the simplest Lorentz invariant field equations," implying that there are other similar Lorentz invariants. Furthermore, the quantities x, t, E, p remain *form invariant as* x', t', E', p' under Lorentz transformations following the transition from a stationary to a moving frame and should, therefore, be Lorentz invariant per the definition. But in this case, unlike in Eqs. (12) and (14), $x \neq x', t \neq t', E \neq E', p \neq p'$. It follows that the mathematical equality of the unprimed and primed forms of a law or quantity is not a necessary condition for it to be Lorentz invariant.

It is now clear that the extensive use of the relativistic mechanics based on Minkowski's formalism has led us to regard Lorentz invariance only as a *mathematical property* of inertial frames, whereas the crucial *physical property* of the Lorentz invariance of forms of physical laws has taken a back seat in our thinking. Consequently, the acceptance of the quantities x, t, E and p as Lorentz invariant may be resisted due to this prejudice, because of the relations $x \neq x', t \neq t', E \neq E', p \neq p'$, until it is realized that these are the examples of a physical property of the *Lorentz invariance of the forms*.

In addition to the above realization, the classical examples discussed force us to recognize two other simple facts. In all these cases, the Lorentz invariance has been shown for the transition between the inertial frames, S and S', mathematically relatable through the Lorentz transformations. In relativistic mechanics, on the contrary, we routinely use the 'invariance' of certain quantities/expressions between the frames in a different orientation ($K - K'$) not relatable through the Lorentz transformations. We show in the next subsection that this 'invariance' is not the same as the 'Lorentz invariance.'

Form-invariant and frame-independent quantities: We now introduce a new property of inertial frames, to be called the property of 'frame independence'—a familiar term that, however, is currently used as a synonym for 'form invariance' in the literature. A frame-independent quantity can be defined as one that remains the same in all frames, i.e., it does not have a primed version and, therefore, like 'form invariants,' does not depend on Lorentz transformations for the transition from one frame to another. For example, $s^2, m_0,\ c\ etc.$ Einstein[19] also described the quantity s^2 as defined independently of any particular choice of frame, although he did not distinguish it from Lorentz invariants. As the Lorentz transformations are not required, these quantities can be easily 'transformed' between the frames K - K'. This unique aspect has rendered them very useful in relativistic mechanics when dealing with particle collision problems.

We have thus far been treating form invariance and frame independence as identical because of the unique relation (Eq. 11) born out of the formalism introduced by Minkowski, which naturally led to another unique relation (Eq. 13). To elaborate on this, we rewrite the latter as

$$E^2 - p^2c^2 = m_0^2c^4. \tag{15}$$

It is important to note that the left-hand side in this relation is form invariant whereas the right-hand side is frame independent, which makes it unique :It can be used both as a form-invariant or Lorentz-invariant and as a frame-independent relation (double role). The former role has long been known, but to prove the latter, we argue that if the right-hand side in Eq. (15) is frame independent, so must the left-hand side be.

To differentiate between the usages of this relation as form invariant and frame independent, we note that as the former, it can be Lorentz-transformed between inertial frames S and S' by explicitly using the energy–momentum transformations as in Eq. (14), but not between K - K' because these frames do not fall under the direct purview of Lorentz transformations. However, as a frame-independent relation, it holds in all frames irrespective of their orientation and, therefore, can be 'transformed' between the frames K - K' like an ordinary mathematical relation *without using the energy–momentum Lorentz transformations*. In the absence of the use of these transformations, it cannot be said to have been Lorentz-transformed from one frame to another, but to have simply been 'transformed'—or, better, 'transferred,' as this causes the word 'transformed' to be reserved for the case of form invariants. While solving particle collision problems, we never Lorentz-transform this relation from the primed to the unprimed version, or vice versa, by using energy–momentum transformations. Instead, we transfer the variables *E* and *p* between the frames, or replace their actual values in the frames under consideration, and proceed as shown in the example below.

Consider a proton–proton collision in the lab frame, which results in three protons and an antiproton. If all four particles are at rest in the CE/CM frame, the initial energy of the protons in the lab frame is the minimum necessary to generate an antiproton, and is calculated following the calculation of the Lorentz factor by the following standard conservation law plan:

CE after *Lab before*

$$E^2 - p^2c^2 = (4m_0c^2)^2 - 0 = (m_0c^2 + \gamma m_0c^2)^2 - (\gamma m_0 v)^2c^2.$$

or $16m_0^2c^4 = 2m_0^2c^4(\gamma+1)\ or\ \ \gamma = 7.$

It is clear that in this example, the energy–momentum transformations are not used to Lorentz-transform the energy–momentum relation between the frames CE after and lab before in the standard manner used to arrive at Eq. (14). It follows that in particle collision problems, the property of the *frame independence* of the energy–momentum relation is exploited. This is confirmed by the fact that the frames involved in the above case are not in the same spatial orientation as in Fig. 1, which is an essential requirement, as has already been shown, to mathematically bring them under the purview of the Lorentz transformations. This makes it theoretically impermissible to Lorentz-transform the energy–momentum relation from one of these frames to another. Consequently, assertions to the effect that the relativity principle has been verified by similar experiments up to a certain Lorentz factor no longer have any theoretical basis, and are considered void.

On the Lorentz transformation of collisions to the projectile rest frame: Another reason to consider the example from the theory of Compton scattering in this study is to highlight the striking difference in the choice of frame of reference for action while dealing with the collision of a photon with a particle in the two theories—Compton scattering and GZK cut-off. In both these theories, an energetic particle moves in the lab/universal frame with relativistic speed toward another particle, and this may result in a collision. In Compton scattering the collision is considered in the lab frame itself, whereas in the GZK theory, it is claimed to have been Lorentz-transformed to the 'projectile rest frame.' To elaborate, let us consider what happens if an experimenter decides to consider the Compton collision, as in GZK theory, in the projectile rest frame. In this case, in its rest frame, a moving X-ray photon is seen as losing energy and behaving as a low-energy photon, whereas the carbon electron is considered the energetic particle. In this scenario, the low-energy photon is not scattered, and instead may be absorbed on collision by the energetic carbon electron. As a result, no Compton scattering is observed, which is contrary to experimental fact. Similarly, the result of the GZK/TeV-γ photon theory is not the same if the collision is considered in the universal frame. This is because as pointed out in the previous subsection, these frames are neither in the same spatial orientation nor in same state of relative motion as the frames S and S' of Fig. 1 and, hence, mathematically cannot be dealt with by the Lorentz transformations. This makes it theoretically impermissible to Lorentz-transform the collision from one of these frames to the other. Assertions of the type whereby a collision between a CMB photon and a UHE proton in the lab/universal frame is identical to one between an energetic photon (approximately 300 MeV) and a proton at rest, which is approximately a CM frame, thus have no theoretical basis.

It follows that in the above two cases, the collision occurs only in the lab/universal frame. This explains why in the GZK and TeV-γ photon theories, the expected collision in the projectile rest frame does not occur, and UHECR particles and multi TeV-γ photons are detected on Earth. This invalidates the results of these theories and, consequently, Special Relativity does not require modification or replacement[20,21,22,23,24,25] to account for the predictions of these theories.

Use of four- momenta in obtaining Compton's equation

We now briefly revisit the formalism of four-space, which is also used to derive Eq. (4) of Compton scattering. In this formalism, the conservation of four-momenta for Compton scattering can be written as

$$\mathbf{P}_1 + \mathbf{P}_2 = \mathbf{P}_1^f + \mathbf{P}_2^f \tag{16}$$

Then, we can proceed in the usual manner. Particle 1 is a photon and particle 2 is an electron, and *f* refers to their final state. Again restricting ourselves to our aim of showing that Lorentz transformations are not involved in this process, we note that as the use of the Lorentz invariant energy–momentum relation to obtain the Compton's equation led to a contrasting situation, the use of four- momenta also leads to the same situation because it is also Lorentz invariant. We, therefore, have no choice but to take a fresh look at the *physical basis* of this formalism to determine the meaning of the term 'Lorentz invariance of four- momentum.'

Minkowski [17] found a useful way of considering relativity by introducing the concept of the rotations of axes in 4-space. This elegant formalism significantly simplifies the mathematical transition from one frame to another. However, when interpreting concepts of the *physical theory of relativity*, we prefer to rely on translations in 3-space because this is easier. We show below why this is so.

The first postulate of Special Relativity requires the form-invariance of physical laws in all inertial frames *in uniform motion relative to one another*. In 4- space, this requirement is realized as

$$\begin{gathered}\text{Invariance under rotation of axes in } 4-\text{space}\\ \equiv \\ \text{Invariance under Lorentz transformations in } 3-\text{space}\end{gathered} \tag{17}$$

This is a crucial property of 4- space representation. According to it, if a law can be expressed as a 4-vector equation, it is invariant under rotations in 4-space, which means that it is Lorentz invariant. At the same time, it is important to note that the above arrangement does not account or compensate for *uniform relative motion*. To elaborate on this, we note that Eq. (17) is based on the following relations:

$$\begin{aligned}x' &= x\cos\phi - ict\sin\phi \Rightarrow x' = \gamma(x - vt)\\ ict' &= ict\cos\phi + x\sin\phi \Rightarrow t' = \gamma(t - (vx/c^2))\end{aligned} \tag{18}$$

These relations claim that the Lorentz transformations can be represented as a rotation of axes in 4 - space provided that $\cos\phi = \gamma$ and $\sin\phi = \gamma(iv/c)$. In this situation, as at $v = 0,\ \gamma = 1,$ for $v > 0$, $\gamma > 1$, i.e., $\cos\phi = \gamma > 1$. This is possible only if ϕ is imaginary. In other words, in the formalism of 4-space, *the concept of uniform relative motion* has no real representation and, consequently, the *concept of inertial frames* becomes redundant. Without the representation of these fundamental concepts of Special Relativity, the concept of Lorentz invariance loses its physical basis and becomes an abstract mathematical entity in imaginary 4 - space. It is thus impossible to determine the physical meaning of the Lorentz invariance of 4- momentum.

It is now clear that this formalism does not incorporate some of the basic physical concepts of Special Relativity. Nevertheless, there is no doubt that it has been successfully used as a versatile tool in relativistic mechanics to solve a variety of problems. It follows that the concepts of Special Relativity that it does not incorporate are not required in solving such problems, and this is consistent with findings in this paper.

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