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Quantum and Generalized Special Relativistic Model for Electron Charge Quantization

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Abstract: *Explanation of electron self-energy and charge quantization is one of the challenging problems facing quantum electrodynamics. In this work one quantizes electron and elementary particles charges on the basis of electromagnetic Hamiltonian in a curved space-time at vacuum stage of the universe,using quantum spin angular momentum and Klein-Gordon equation beside generalized special relatively. Electron charge is found to be quantized and the electron self-energy is finite. The radius of the electron is also found.*

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I. Introduction

The universe building blocks are atoms. These atoms consist of electrons, protons and neutrons. The recent elementary particle models, like electroweak standard model, say that even these particles are consisted of smaller elementary particles called quarks and leptons [1,2], these particles have certain masses. The origin of these masses are explained by the so called Higgs mechanism [3,4].Recently Higgs particles which are thought to be responsible for mass generation was discovered experimentally in CERN [5]. This discovery comes as un-ultimate reword confirming the key predictions of the electroweak standard model (ESM) [6]. This mechanism states that when symmetry is broken at minimum potential masses are generated.The electron charge is responsible for electric field generation. The motion of electric field generates magnetic field [7]. Electromagnetic field shown to be generated by time changing electric and magnetic field as shown by Maxwell. Maxwell equations predicts that electromagnetic wave (E.M.W) can be produced by oscillating charges [7]. These (E.M.W) are shown to carry information, therefore they are widely used in telecommunication systems [8]. According to Maxwell equations (M.E) electromagnetic field and light behaves as waves. But the black body relation and other related phenomena, shows that (E.M.W) sometimesbehaves as particles called photons describe behavior of (E.M.W) and the atomic scale [9]. The theory which describes the behavior of photons is quantum electrodynamics [10]. This theory succeeded in describing a wide meaty of physical phenomena can caring (E.M.W) the atomic scale. However, there are some setbacks associated with quantum electrodynamics, one of these problems is quantization of electron charge and electron self-energy [11].

II. Electromagnetic Hamiltonian In A Curved Space Time And Vacuum Energy

Accordingto general relativity (GR) any energy form cause space to be curved. Thus electromagnetic field can cause space to be curved. According to GR, the time – component of the metric is gives by [12]

$$g_{00} = -\left(1 + \frac{2\phi_g}{c^2}\right) \qquad (1)$$

Where ϕ_g is the gravity potential per unit mass and is related to electric potential ϕ and electron charge e through the relation [13]

$$\phi_g = \frac{U}{m} = \frac{e\phi}{m} \qquad (2)$$

Thus equation (1) becomes

$$g_{00} = -\left(1 + \frac{2e\phi}{mc^2}\right) \qquad (3)$$

At early stages of the universe electric charge is generated due to the electromagnetic (e.m) field at vacuum stage. This requires minimizing the Hamiltonian (H) w.r.t electric potential ϕ to find the electric charge and see how it is generated. Since the Hamiltonian part representing charge itself can be neglected as for as they are independent of ϕ. The charge field interactions are neglected for simplicity. One also assumes electric charge to be at rest. This means that the magnetic field is not generated. Therefore

$$A_0 = \phi, \quad A_i = 0, \quad i = 1,2,3,... \qquad (4)$$

To find the Hamiltonian in curved space, one generalized the space one [14,15]

$$H = \eta^{002} \varepsilon_i \left(\partial_i A_0 - \partial_0 A_i\right)^2 \qquad (5)$$

To be written in a curved space in the form [16,17]

$$H = g_{00}^2\, \varepsilon_i \left(\partial_i A_0 - \partial_0 A_i\right)^2 \qquad (6)$$

From Equation (3) one gets

$$H = \left(1 + \frac{2e\phi}{mc^2}\right)^2 (\nabla\phi)^2 \qquad (7)$$

Thus minimization condition requires:

$$\frac{dH}{d\phi} = 2\left(1 + \frac{2e\phi}{mc^2}\right)\left(\frac{2e}{mc^2}\right)(\nabla\phi)^2 = 0$$

$$1 + \frac{2e\phi}{mc^2} = 0, \quad \phi = \frac{-mc^2}{2e} \qquad (8)$$

Assuming the mass energy to be resulting from electric field energy density $\mathrm{E_d}$ where $\mathrm{E_d} = \varepsilon_0 \mathrm{E}^2$

Inside electron of radius r_0, one gets

$$mc^2 = \mathrm{E_d} V = \varepsilon_0 \mathrm{E}^2 \frac{4}{3}\pi r_0^3 = \frac{\varepsilon_0 e^2}{16\pi^2 \varepsilon_0^2 r_0^4}\frac{4}{3}\pi r_0^3 = \frac{e^2}{12\pi\varepsilon_0 r_0} \qquad (9)$$

The vacuum energy potential which results from electric charge becomes

$$U_v = -e\phi = \frac{e^2}{12\pi\varepsilon_0 r_0} \qquad (10)$$

according to a vacuum energy potential which takes the form [18]

$$U_v = \rho_v \left[\left(\pi^2 n^2 / x_0^2 n_0^2\right) + \omega^2\right]^{-3} \qquad (11)$$

Thus combining Equations (10) and (11) yields

$$\frac{e^2}{12\pi\,\varepsilon_0 r_0} = \left[\left(\pi^2 n^2 / x_0^2 n_0^2\right) + \omega^2\right]^{-3}$$

Thus the electric charge is given by

$$e = \left[\left(\pi^2 n^2 / x_0^2 n_0^2\right) + \omega^2\right]^{-3/2} \left(12\pi\,\varepsilon_0 r_0\right)^{1/2} \qquad (12)$$

Setting □ to be equal to zero, for simplification. The electric charge is given by

$$e = (12\pi\varepsilon_0 r_0)^{1/2} \left(x_0 n_0 / n\,\pi\right)^3 \qquad (13)$$

r_0 is the electron radius and x_0 is the universe radius. Thus the electron radius can be found by assuming that the electron energy results from its spinning, where the spin angular momentum is given by

$$L_s = \hbar\left[s(s+1)\right]^{1/2} = \frac{\sqrt{3}\,\hbar}{2} \qquad (14)$$

Where for electron

$$s = \mp\frac{1}{2} \qquad (15)$$

At vacuum stage we choose minimums lower value.

$$L_s = \frac{1}{2}\hbar \qquad (16)$$

Assume that rest mass is neglected in relativistic expression to get

$$mc^2 = E = c\,p \qquad (17)$$

$$mc = p \qquad (18)$$

The same relation can hold for Newtonian mechanics by considering wave nature of electrons, where the maximum velocity v_m is related to the effective value v through the relations

$$v = \frac{v_m}{\sqrt{2}} \qquad (19)$$

By assuming

$$p = mv$$

Thus the Newtonian expression for free particle takes the form

$$E = \frac{1}{2} m v_m^2 = m v^2 = \frac{m^2 v^2}{m} = \frac{p^2}{m} \quad (20)$$

If one believes in relativistic energy mass relation, one gets

$$m c^2 = E = \frac{p^2}{m}$$

Thus one gets:

$$m^2 c^2 = p^2 , \qquad mc = p \qquad (21)$$

Since the momentum p is related to L according to the

$$p = mv = \frac{m v r_0}{r_0} = \frac{L_s}{r_0} \qquad (22)$$

It follows from equation (21) that

$$\frac{L_s}{r_0} = mc$$

Using equation (16) one gets

$$r_0 = \frac{L_s}{mc} = \frac{\hbar}{2mc} \qquad (23)$$

Substituting the values of h, m and c, the electron radius can be calculated. The electric charge is assumed to be born at very early stages of the universe where vacuum exist and the minimum radius is x_0 where [18]

$$x_0 = 26.635 \times 10^{-3} m$$

The electric charge is numerically given by $e = 1.6 \times 10^{-19} C$. It can be obtained by adjusting the quantum numbers n and n_0 to be

$$\frac{n}{n_0} = \frac{\pi}{x_0} \left[e \big/ \left(12 \pi \varepsilon_0 r \right)^2 \right]^{1/3} \quad (24)$$

Similarly, the charges of quarks and charged leptons can be found by adjusting the quantum numbers n and n_0

Equation (10) shows that vacuum energy is repulsive due to the existence of positive sign. This can form with cosmological models, which suggests repulsive vacuum energy. Inflation models suggest also very large vacuum energy. If one believes in this model, such that

$$\phi \approx \frac{U_v}{m_0} \to \frac{c^2}{2} \qquad (25)$$

in this case according to generalized special relativity model the electron mass is given by

$$m = m_0 \left(1 - 2 \phi_g / c^2 \right) \to \text{ large} \qquad (26)$$

Assume for simplicity

$$m = 10^{13} m_0 \approx 10^{13} \times 9 \times 10^{-31} = 9 \times 10^{-18} kg \quad (27)$$

From Equations (16), (21) and (22) the electron radius can be given to be

$$r_0 = \frac{\hbar}{2mc} = \frac{h}{4 \pi mc} = \frac{6.63 \times 10^{-34}}{4 \pi \times 9 \times 10^{-18} \times 3 \times 10^8}$$

$$r_0 = 1.954 \times 10^{-26} m \qquad (28)$$

Which is quite reasonable as far as nucleus or proton radius for very light atoms are

$$r_b \approx 10^{-14} \qquad r_p \approx 10^{-16}$$

III. Conclusion

In section(II) the Hamiltonian of free electromagnetic field in a curved space-time is minimized to get ϕ in terms of *c*, *m* and *e* at vacuum stage as shown by equation (8). Vacuum energy is obtained by minimizing ϕ and is equated with that obtained from electric energy density according to equations (9,10). The expression for classical angular momentum and quantum spin angular momentum are used to find electron radius. The electron charge is shown to be quantized according to equation (13) due to the existence of two quantum numbers n and n_0 which can be adjusted early to find the value of *e*. The radius of the electron can be found by using equations (23,28). The values obtained are very small compared to proton and nuclear radius which is quite reasonable. According to equations (9,28), the electron self-energy is finite.

References

[1]. Griffiths, David. Introduction to elementary particles. John Wiley & Sons, 2008

[2]. Glashow, Sheldon L. "Partial-symmetries of weak interactions." Nuclear Physics 22.4 (1961): 579-588; Weinberg, S. "WEINBERG 1967." Phys. Rev. Lett 19 (1967): 1264.

[3]. LEP, WG. "for Higgs boson searches, R. Barate et al." Phys. Lett. B 565 (2003): 61.

[4]. Bernstein, Jeremy. "Spontaneous symmetry breaking, gauge theories, the Higgs mechanism and all that." Reviews of modern physics 46.1 (1974): 7.

[5]. Consonni, Sofia Maria. "Higgs search at ATLAS." arXiv preprint arXiv:1305.3315 (2013).

[6]. Iliopoulos, Jean. "Introduction to the STANDARD MODEL of the Electro-Weak Interactions." arXiv preprint arXiv:1305.6779(2013).

[7]. Fitzpatrick, Richard. Maxwell's Equations and the Principles of Electromagnetism. Jones & Bartlett Publishers, 2008.

[8]. Lee, William CY. Mobile cellular telecommunications: analog and digital systems. McGraw-Hill Professional, 1995.

[9]. Phillips, Anthony C. Introduction to quantum mechanics. John Wiley & Sons, 2013.

[10]. Weisskopf, V. F. "Det Kgl Danske Videnskab Selskab Mat-Fys Medd 1936, 14, 1; Reprinted in Schwinger, J." Quantum Electrodynamics.

[11]. Blinder, S. M. "Structure and self- energy of the electron." International journal of quantum chemistry 90.1 (2002): 144-147.

[12]. Weinberg S. Gravitation and cosmology. principles and applications of the general theory of relativity (Wiley, 1972) (ISBN 0471925675) (685s)

[13]. Ford, L. H. "D3: QUANTUM FIELD THEORY IN CURVED SPACETIME." General Relativity and Gravitation. 2002. 490-493

[14]. Thidé, Bo. Electromagnetic field theory. Uppsala, Sweden: Upsilon Books, 2004.

[15]. Saad, Zoalnoon, et al. "Second Order Lagrangian." Journal of Scientific and Engineering Research 3.6 (2016): 70-74.

[16]. Lancaster, Tom, and Stephen J. Blundell. Quantum field theory for the gifted amateur. OUP Oxford, 2014.

[17]. Itzikson, C., and J. B. Zuber. "Quantum Field Theory, McGraw-HillBook Company." (1980).

[18]. TAYFOR, MONTASIR SALMAN ELFADEL. The Role of Vacuum Energy in the Universe and Generation of Fields and Matter. Diss. Sudan University of Science and Technology, 2014., URI: http://repository.sustech.edu/handle/123456789/9972

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