

## Effects of Magnetized Plasma on Electromagnetic Wave Propagating Parallel to Its Magnetic Field

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**Abstract:** The study of waves in plasmas provides significant information on plasma properties and is very useful in plasma diagnostics. This research is on “Effects of Magnetized Plasma on Electromagnetic Wave Propagating parallel to its magnetic field”. Electromagnetic waves propagating parallel to magnetic field in magnetized plasma were analyzed. There are two modes parallel to the field, the L and R waves. Frequency values for the L-waves were obtained by substituting the values of wave number,  $k$  into its dispersion relation while the same frequency values were substituted to get the values of its refractive index ( $n^2$ ). The plasma frequency values obtained from Bohm-Gross’ formulae for electron plasma waves were substituted into the dispersion relation to obtain the frequency values with cut-off at  $\omega_L$  and  $\omega_R$ . These same frequency values were substituted into their related equations to obtain their respective refractive index. The results (tables & figures) show that properties of magnetized plasmas in the direction parallel to the magnetic field are different from those perpendicular to it. The motion of plasma parallel to the magnetic field lines is associated with dynamics of sound waves.

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### I. Introduction

#### Background to the study

Wave motion is the process of transferring a disturbance in the form of kinetic energy from one point to another in a medium without a net transfer of the particles of the medium. In other words, the particles of the medium in which the wave travels merely oscillate about their positions conveying the energy from particle to particle but without the particles themselves moving along. Particles of the medium vibrate if there is a wave motion in the medium. Waves do not transmit matter but only energy (Ike, 2009).

From a scientific point of view, matter in the known universe is often classified in terms of four states: solid, liquid, gas and plasma. The basic distinction between solids, liquids and gas lies in the difference between the strength of the bonds that hold their constituent particles together. (Bittencourt, 1986).

The term plasma is used to describe a wide variety of macroscopically neutral substances containing many interacting free electrons and ionized atoms or molecules, which exhibit collective behavior due to the long-range coulomb forces. Not all media containing charged particles, however, can be classified as plasmas (Bittencourt, 1986). For an ionized gas to become plasma, it must:

- have high temperature of approximately  $10^3$  keV.
- exhibit collective behavior.
- be quasi-neutral (i.e. ion density approximately equal to electron density).

The study of waves in plasmas provides significant information on plasma properties and is very useful in plasma diagnostics (Bittencourt, 1986). This research is on “Effects of Magnetized Plasma on Electromagnetic wave propagating parallel to its magnetic field”. Electromagnetic wave propagating parallel to magnetized plasma will be analyzed.

#### 1.1 Electromagnetic wave propagation

When an electromagnetic wave impinges upon the atoms of a material, the energy of that wave is absorbed. The absorption of energy causes the electrons within the atoms to undergo vibrations. After a short period of vibrational motion, the vibrating electrons create a new electromagnetic wave with the same frequency as the first electromagnetic wave. While these vibrations occur for only a very short time, they delay the motion of the wave through the medium. Once the energy of the electromagnetic wave is reemitted by an atom, it travels through a small region of space between atoms. Once it reaches the next atom, the electromagnetic wave is absorbed, transformed into electron vibrations and then reemitted as an electromagnetic wave. While the

electromagnetic wave will travel at a speed of light,  $c$  ( $3.0 \times 10^8$  m/s) through the vacuum of inter-atomic space, the absorption and reemission process causes the net speed of the electromagnetic wave to be less than the speed of light,  $c$ . The actual speed of an electromagnetic wave through a material medium is dependent upon the optical density of that medium. Different materials cause a different amount of delay due to the absorption and reemission process (NASA, 2010).

Radio signals exist as a form of electromagnetic wave. These radio signals are the same form of radiation as light, ultra-violet, Infra-red, etc. differing only in the wavelength or frequency of radiation. Like other forms of EM wave, radio signals can be reflected, refracted and undergo diffraction.

Medically, some of the neurological diseases such as brain injury, cervical spinal stenosis, Cytomegalic inclusion body disease (CIBD), Diabetic neuropathy, Epilepsy, Familial spastic paralysis, Fetal alcohol syndromes, Generalized anxiety disorder, sensory processing disorder, sleeping sickness etc. can be treated using bio-electromagnetic therapy in which certain tissues such as bladder, blood, Bone cortical and Vitreous Humor of human body are stimulated by electromagnetic fields (propagation).

### 1.2 Magnetized Plasma

Plasma with magnetic field strong enough to influence the motion of charged particles is said to be magnetized. A common quantitative criterion is that a particle on average completes at least one gyration around the magnetic field before making a collision, i.e.  $\omega_{ce}/v_{coll} > 1$ , where  $\omega_{ce}$  is the "electron gyrofrequency and  $v_{coll}$  is the "electron collision rate". It is often the case that the electrons are magnetized while the ions are not. (Richard, 2011).

Magnetized plasmas are anisotropic, meaning that their properties in the direction parallel to the magnetic field are different from those perpendicular to it. While electric fields in plasmas are usually small due to the high conductivity (Wiesenberger *et al*, 2014).

### 1.3 Statement of the problem

Magnetised plasma do influence electromagnetic wave propagation. For example, Ionosphere occasionally absorbs and distorts radio waves. The earth's magnetic field causes waves with different polarizations to propagate at different velocities, an effect which can give rise to "ghost signals". The questions that arise which this research is to answer are:

what are the effects of magnetized plasma on electromagnetic wave propagating parallel to its magnetic field and of what kind are these effects?

### 1.4 Aim of the study

This research is aimed at analysing the effect(s) of magnetized plasma on electromagnetic wave propagating parallel to its magnetic field.

### 1.5 Objectives of the study

To achieve the above aim, the following objectives have been drawn:

- a To analyse the influence of magnetized plasma on electromagnetic wave Propagating parallel to its magnetic field.
- b. Make graphical interpretation of the dispersion relations associated to EM waves propagating in magnetized plasma.

### 1.6 Significance of the study

The study of waves in plasma provides significant information on plasma properties and is very useful in plasma diagnostics (Bittencourt, 1986). This work will provide useful pieces of information on the influence of magnetized plasma on electromagnetic wave propagating in it. This will enable individuals and organizations to appreciate the existing methods or think of alternative strategies which could yield better results in the application(s) of electromagnetic wave propagation in magnetized plasma.

## II. Theoretical/Conceptual Frame Work

### 2.1 EM wave propagation parallel to magnetic field, $\vec{B}_0$

For parallel waves travelling along,  $\vec{B}_0$  only three basic equations are needed. Ampere's law, Faraday's law and the electron force equation (ignoring electron temperature and ion motion). These are Fourier transforming and linearizing immediately,

$$i\vec{k} \times \vec{E}_1 = \frac{i\omega}{c} \vec{B}_1 \quad (2.10)$$

$$i\vec{k} \times \vec{B}_1 = \frac{-4\pi n_0 e}{c} \vec{V} - \frac{i\omega}{c} \vec{E}_1 \quad (2.11)$$

$$\text{and } i\omega m_e \vec{V} = -e\vec{E}_1 - \frac{e}{c} \vec{V} \times \vec{B}_0 \quad (2.12)$$

By taken  $\vec{E}_1 = (\vec{E}_x, \vec{E}_y, 0)$ ,  $\vec{B}_1 = (\vec{B}_x, \vec{B}_y, 0)$  and  $\vec{V} = (\vec{V}_x, \vec{V}_y, 0)$ , Eqns.(2.10) to (2.12) yield

$$-ik\vec{E}_y = \frac{i\omega}{c} \vec{B}_x \quad (2.13)$$

$$ik\vec{E}_x = \frac{i\omega}{c} \vec{B}_y \quad (2.14)$$

$$-i\vec{k}\vec{B}_y = \frac{-4\pi n_0 e}{c} \vec{V}_x - \frac{i\omega}{c} \vec{E}_x \quad (2.15)$$

$$i\vec{k}\vec{B}_x = \frac{-4\pi n_0 e}{c} \vec{V}_y - \frac{i\omega}{c} \vec{E}_y \quad (2.16)$$

$$-i\omega m_e \vec{V}_x = -e\vec{E}_x - \frac{e}{c} \vec{B}_0 \vec{V}_y \quad (2.17)$$

$$\text{and } -i\omega m_e \vec{V}_y = -e\vec{E}_y + \frac{e}{c} \vec{B}_0 \vec{V}_x \quad (2.18)$$

Inserting (2.13) and (2.14) for  $\vec{B}_x, \vec{B}_y$  in (2.15) and (2.16), give

$$\vec{V}_x = \frac{\left(\frac{-ik^2c}{\omega} + \frac{i\omega}{c}\right)}{-4\pi n_0 e/c} \vec{E}_x \quad (2.19)$$

$$\vec{V}_y = \frac{\left(\frac{-ik^2c}{\omega} + \frac{i\omega}{c}\right)}{-4\pi n_0 e/c} \vec{E}_y \quad (2.20)$$

Inserting (2.19), (2.20) in (2.17), (2.18), the matrix equation for  $\vec{E}_x, \vec{E}_y$  is obtain

$$\begin{bmatrix} -i\omega m_e \frac{\left(\frac{-ik^2c}{\omega} + \frac{i\omega}{c}\right)}{-4\pi n_0 e/c} + e & \frac{eB_0}{c} \frac{\left(\frac{-ik^2c}{\omega} + \frac{i\omega}{c}\right)}{-4\pi n_0 e/c} \\ -\frac{eB_0}{c} \frac{\left(\frac{-ik^2c}{\omega} + \frac{i\omega}{c}\right)}{-4\pi n_0 e/c} & -i\omega m_e \frac{\left(\frac{-ik^2c}{\omega} + \frac{i\omega}{c}\right)}{-4\pi n_0 e/c} + e \end{bmatrix} \begin{pmatrix} \vec{E}_x \\ \vec{E}_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.21)$$

Setting the determinant of the coefficient equal to zero, yield

$$\left(1 + \frac{k^2c^2}{\omega_e^2} - \frac{\omega^2}{\omega_e^2}\right)^2 = \frac{\Omega_e^2}{\omega_e^4} \left(\omega - \frac{k^2c^2}{\omega}\right)^2 \quad (2.22)$$

By taking the square root of (2.22), retaining both signs,

$$1 - \frac{\omega}{\omega_e^2} \left(\omega - \frac{k^2c^2}{\omega}\right) = \pm \frac{\Omega_e}{\omega_e^2} \left(\omega - \frac{k^2c^2}{\omega}\right) \quad (2.23)$$

$$\text{or } 1 = \left(\frac{\omega}{\omega_e^2} \pm \frac{\Omega_e}{\omega_e^2}\right) \left(\omega - \frac{k^2c^2}{\omega}\right) \quad (2.24)$$

$$\text{orn}^2 \equiv \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_e^2/\omega^2}{1 \pm \Omega_e/\omega} \quad (2.25)$$

which is the index of refraction for electromagnetic waves traveling along the magnetic field.

The cutoffs are obtained by setting  $k = 0$  in (2.25);

$$\omega_{(R)}^{(L)} = \pm \frac{\Omega_e}{2} + \sqrt{\omega_e^2 + (\Omega_e^2/4)} \quad (2.26)$$

As the cut-offs for the R-wave and L-wave. These are precisely the cutoffs found for the extraordinary mode (EM wave propagating perpendicular to magnetic field), and this is why they are called left and right cutoffs.

**Table 2.1** Table of values for wave number (k) and frequency ( $\omega$ ) for L-wave ( $\vec{E} \parallel \vec{B}$ ) – (eqn.2.27).

Wave number, k	Frequency, $\omega$ (GHz)
0.00	0.87
1.92	1.15
5.05	1.99
9.88	3.40

**Table 2.1.1** Table of values for refractive index ( $n^2$ ) and wave frequency ( $\omega$ ) for L-wave ( $\vec{E} \parallel \vec{B}$ ) – (eqns. 2.26 and 2.27).

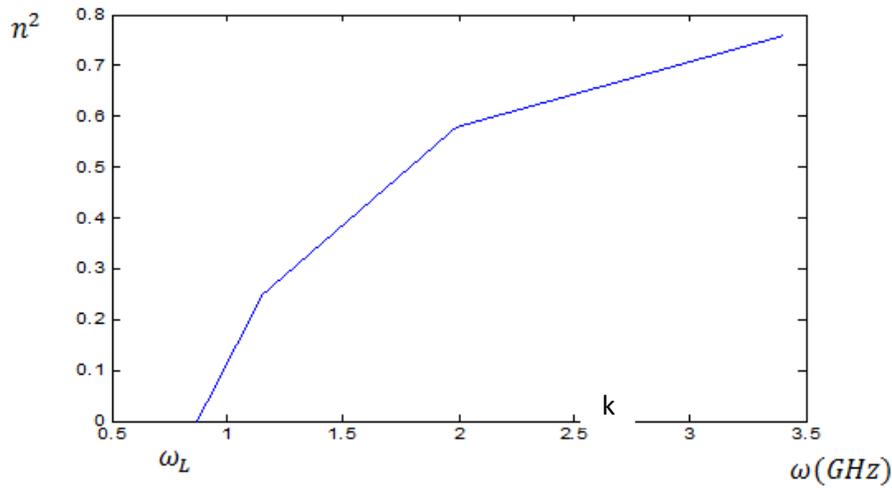
Frequency, $\omega$ (GHz)	Refractive index, $n^2$
0.87	0.00
1.15	0.25
1.99	0.58
3.40	0.76

**Table 2.2** Table of values for wave number ( $k$ ) and frequency ( $\omega$ ) for R-wave ( $\vec{E}||\vec{B}$ ) – (eqn.2.27).

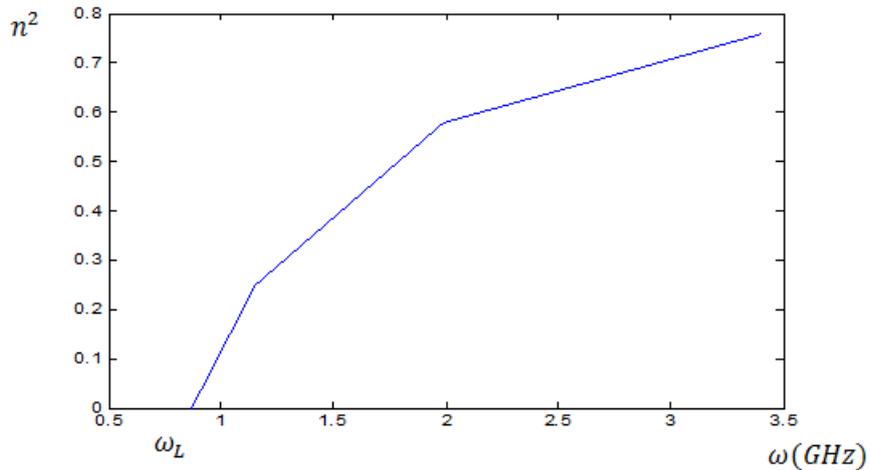
Wave number, $k$	Frequency, $\omega$ (GHz)
0.00	36.00
60.47	36.28
94.23	37.12
111.97	38.53

**Table 2.2.1** Table of values for refractive index ( $n^2$ ) and wave frequency ( $\omega$ ) for R-wave ( $\vec{E}||\vec{B}$ ) – (eqns.2.26 and 2.27).

Frequency, $\omega$ (GHz)	Refractive index, $n^2$
36.00	0.00
36.28	0.25
37.12	0.58
x38.53	0.76



**Figure 2.1** Sketch of dispersion relation  $\omega = \omega(k)$  for L-waves with cut-off at  $\omega_L$  – (Table 2.1).



**Figure 2.1.1** Dispersion diagram for L-waves with cut-off at  $\omega_L$  – (Table 2.1.1).

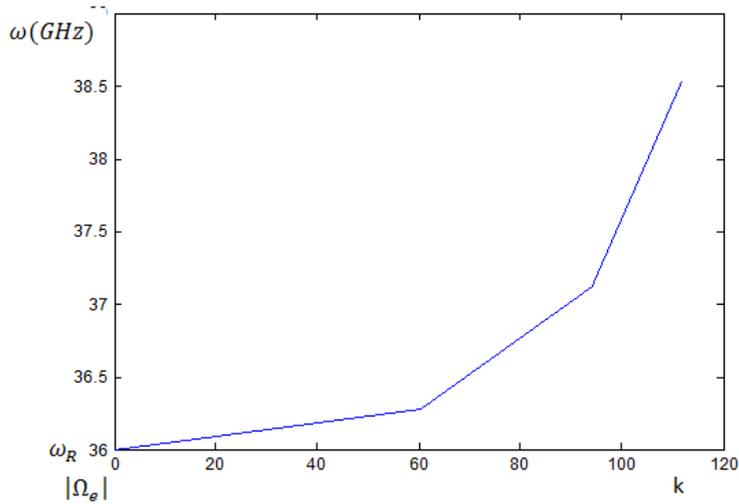


Figure 2.2 Sketch of dispersion relation  $\omega = \omega(k)$  for R-waves with cut-off at  $\omega_R$  – (Table 2.2).

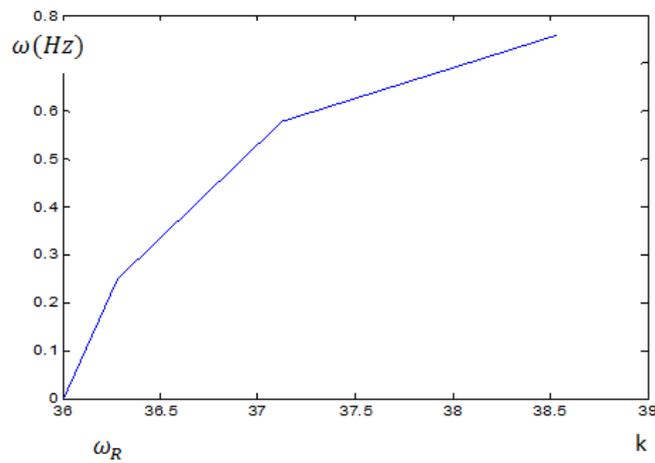


Figure 2.2.1 Dispersion diagram for R-waves with cut-off at  $\omega_R$  – (Table 2.2.1).

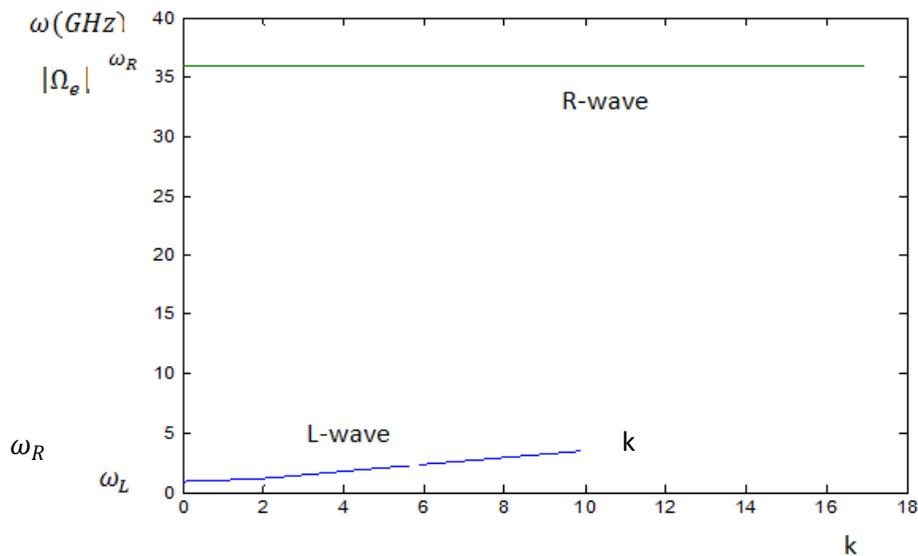


Figure 2.3 Sketch of Dispersion diagram for L-wave and R-wave ( $\vec{E} \parallel \vec{B}$ ) – (Tables 2.1 & 2.2).

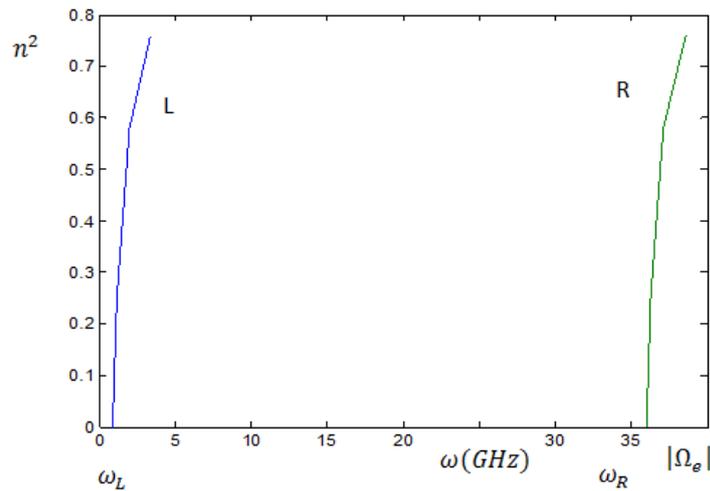


Figure 2.3.1 Dispersion diagram for L-wave and R-wave( $\vec{E} \parallel \vec{B}$ ) – (Tables 2.1.1 & 2.2.1).

### 2.2 Discussion of results

The left and right circularly polarized waves (i.e. L-wave and R-wave) come from the rotation of the electric field vector as the wave propagates (the right-hand rule places the thumb along  $\vec{k}$  and the fingers in the direction of the  $\vec{E}$  rotation for the R-wave and opposite for L-wave). This situation is cylindrically symmetric about  $\vec{B}_0$ , the  $\vec{E}_1$  vector describe a circle rather than an ellipse as in the X-mode case. The direction of rotation of the R-wave corresponds to the direction of gyration of electrons.

By taking the wave number,  $k = \frac{2\pi}{\lambda} (n_x^2 + n_y^2 + n_z^2)$ , the values of k where obtained (table 2.1). While, speed of light,  $c = 3 \times 10^8$  m/s and electron frequency,  $\omega_e = e\vec{B}/2\pi m_e$  ( $\vec{B} = 0.2T$ ) are constants. The Bohm-Gross' formula for electron plasma waves,  $\omega^2 = \omega_{pe}^2 + k^2 \gamma V_t^2$  (where  $\gamma = 3$  and  $V_t = c$ ) was used to obtain plasma frequency values as 5.59GHz, 6.47GHz, 8.60GHz and 11.29GHz which were substituted into equation (2.25) to obtain the values of refractive indexes for EM waves propagating parallel (table 2.2.1) to magnetic field. The same plasma frequency values were substituted into equation (2.26) to get the frequency values for L-waves and R-waves respectively.

The dispersion relation  $\omega = \omega(k)$  associated to EM waves propagating parallel to magnetic field in magnetized plasma are shown in figs (2.1) and figs (2.2). Where  $\omega_L = 0.87GHz$ ,  $|\Omega_e| = 35.13GHz$  and  $\omega_R = 36.00GHz$ . In this case, R-wave has two “pass bands”,  $0 < \omega < |\Omega_e|$  and  $\omega > \omega_R$ , separated by a “stop band”.

The L-wave exists only for  $\omega > \omega_L$ . Both high frequency branches asymptote to  $\omega \approx kc$  at high frequencies. The locations of the pass and stop bands are clearly seen by drawing  $n^2 = k^2 c^2 / \omega^2$  versus  $\omega$  and  $\omega$  versus  $k$  for EM waves propagating parallel to magnetic field as shown in figures 2.3 and 2.3.1 respectively. The low frequency branch of the R-wave is called the electron-cyclotron wave. The “stop bands” occur when  $n^2 < 0$  and the “pass bands” occur when  $n^2 > 0$ . For the low density plasma,  $\omega_L < |\Omega_e|$ . The low frequency branch of the R-wave is called the electron-cyclotron wave. The electron-cyclotron wave has a portion where  $V_g \equiv d\omega/dk$  increases as  $\omega$  increases. This is called the whistler wave, because the high frequency components of a wave packet travel faster than its low frequency components. An observer some distance away from a source (a lightning stroke, for example) will then hear a whistle starting at high frequencies and descending to lower frequencies (Bastian, 2005).

L-wave with cut-off at  $\omega_L$  has 0.87GHz as its cut-off frequency when the wave number is zero. At frequency of 3.40GHz,  $k = 9.88$  for L-wave as its square of refractive index is 0.76. R-wave attain its cut-off,  $\omega_R$  at frequency of 36.00GHz. R-wave with wave number of 111.97 has square of refractive index as 0.76. In the dispersion diagrams (fig 2.3 and fig 2.3.1), R-wave at very high frequencies is seen to have a higher phase speed than the L-wave. Thus, if a plane wave is incident on a plasma along the magnetic field,  $\vec{B}_0$ , its two normal mode components, R and L, travel at different speeds, and the plane of polarization of the plane wave rotates as it travels. This is known as Faraday rotation and is useful in measuring plasma densities in laboratory plasma and in interstellar space. The R-wave has a cut-off at  $\omega_R$  (hence the designation of this frequency) and a resonance at  $\Omega_e$ . While, L-wave has a cut-off at  $\omega_L$  and no resonance. R-waves at frequencies below  $\Omega_e/2$  are known as whistler modes (Chen, 1984).

### III. Conclusion

Mechanical and electromagnetic waves are two important ways that energy is transported in the world around us. The actual speed of an electromagnetic wave through a material medium is dependent upon the optical density of that medium. Different materials cause a different amount of delay due to the absorption and reemission process. Properties of magnetized plasmas in the direction parallel to the magnetic field are different from those perpendicular to it. The motion of plasma parallel to magnetic field-lines is associated with dynamics of sound waves. Radio signals (which are the same form of radiation as light, ultra-violet, infra-red etc.) exist as a form of electromagnetic wave.

There are two modes associated with EM waves propagating parallel to magnetic field-lines, the left and right circularly polarized waves (L- and R-modes). From the dispersion diagrams shown, R-wave at very high frequencies is seen to have a higher phase speed than the L-wave. Thus, if a plane wave is incident on plasma along its magnetic field ( $\vec{B}_0$ ), its two normal mode components, R and L, travel at different speeds, and the plane of polarization of the plane wave rotates as it travels. This is known as Faraday rotation, and is useful in measuring plasma densities in laboratory plasma and in interstellar space.

Some of the Neurological diseases such as Brain injury, Cervical Spinal Stereopsis, Diabetic Neuropathy, Epilepsy etc. can be treated using Bio-electromagnetic therapy in which certain tissues such as bladder, blood, bone cortical and vitreous humor of human body are stimulated by electromagnetic fields. However, Magnetized plasma do influence electromagnetic wave propagation. For instance, earth's ionosphere (a layer of partially ionized gas in the upper atmosphere) which reflects radio waves and is responsible for the fact that radio signals can be received when the transmitter is over the horizon also occasionally absorbs and distorts radio waves. Likewise, earth's magnetic field causes waves with different polarizations (relative to the orientation of the magnetic field) to propagate at different velocities. These effects can give rise to "ghost signals" (i.e., signals which arrive a little before or a little after the main signal).

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