IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 11, Issue 3 Ser. II (May. – June. 2019), PP 72-74
www.iosrjournals.org

Rewriting the Master Equation

Dr Spiros Koutandos, GREECE
Corresponding Author: Dr Spiros Koutandos

Abstract: *We revisit the master equation given in previous papers and reach at a new equation regarding the total time derivative of the velocity. The calculations are a bit lengthy but the result has a simple physical interpretation and is very symmetric.*

Date of Submission: 21-05-2019 Date of acceptance: 06-06-2019

Main Part

According to our gained knowledge so far, the velocity may be decomposed to:

$$\vec{v} = -i\hbar\nabla\psi = \hbar e^{i\phi}|\psi|\nabla\phi - i\hbar e^{i\phi}\nabla|\psi| = \psi\frac{d\vec{r}}{dt} + i\hbar e^{i\phi}\nabla|\psi| \quad (1)$$

The master equation on the other hand correctly reads:

$$\frac{i\hbar}{2m}\nabla\times\vec{\Omega} + \vec{\alpha} = \frac{i\hbar}{2m}\psi^*\Delta\vec{v} + (\vec{v}^*\cdot\nabla)\vec{v} + \frac{\hbar^2}{2m}\nabla\Delta P \quad (2)$$

In equation (2) alpha stands for:

$$\vec{\alpha} = |\psi|^2\nabla U \quad (3)$$

Next we are going to analyze the second term of the right member of equation (2):

$$(\vec{v}^*\cdot\nabla)\vec{v} = \left(\psi^*\frac{d\vec{r}}{dt} - ie^{-i\phi}\nabla|\psi|\right)\cdot\nabla\vec{v} \quad (4)$$

We are going to assume that:

$$\psi^*\frac{d\vec{v}}{dt} = \left(\left(\psi^*\frac{d\vec{r}}{dt}\right)\cdot\nabla\right)\vec{v} + \psi^*\frac{\partial\vec{v}}{\partial t} \quad (5)$$

For the second term of the right member of equation (5) we shall use the time dependent Schrodinger equation in Hamiltonian form:

$$\psi^*\frac{\partial\vec{v}}{\partial t} = \psi^*\nabla\left(-i\hbar\frac{\partial\psi}{\partial t}\right) = \psi^*\nabla\left(\frac{\hbar^2}{2m}\Delta\psi + U\psi\right) \quad (6)$$

Comparing equation (6) with equation (2) we see that the second term on the left side of equation (2) and the first term of the right member of equation (2) have been accounted for to extract this result. So we rewrite equation (2) :

$$\frac{i\hbar}{2m}\nabla\times\vec{\Omega} = \psi^*\frac{d\vec{v}}{dt} - \left(\left(ie^{-i\phi}\nabla|\psi|\right)\cdot\nabla\right)\vec{v} + \frac{\hbar^2}{2m}\nabla\Delta P \quad (7)$$

Now we are going to study the second term of the right member of equation (7)

$$\left(\left(ie^{-i\phi}\nabla|\psi|\right)\cdot\nabla\right)\vec{v} = \left(i\hbar e^{-i\phi}\nabla|\psi|\right)\cdot\nabla\left(e^{i\phi}|\psi|\nabla\phi - ie^{i\phi}\nabla|\psi|\right) \quad (8)$$

We see that by the operation of the gradient on the phase fact we have the following two terms which in steady state conditions are zero:

$$\left(ie^{-i\phi}\nabla|\psi|\cdot\nabla\left(ie^{i\phi}\right)\right)\nabla\phi - \left(ie^{-i\phi}\right)\left(\nabla|\psi|\cdot\nabla\left(ie^{i\phi}\right)\right)\nabla|\psi| \quad (9)$$

The sum of (9) equals to:

$$-i\hbar\left(\nabla|\psi|\cdot\nabla\phi\right)\left(\nabla|\psi|-\nabla\phi\right)=0 \quad (10)$$

On the overall we may rewrite equation (7) like:

$$\frac{i\hbar}{2m}\nabla\times\vec{\Omega}=\psi*\frac{d\vec{v}}{dt}-2\left(i\hbar\left(\nabla\frac{\left(|\psi|^2\right)}{2}\right)\cdot\nabla\right)\nabla\phi-2i\left(\left(\nabla|\psi|\right)^2\nabla\phi\right)-\nabla\left(\left(\nabla|\psi|\right)^2\right)+\frac{\hbar^2}{2m}\nabla\Delta P \quad (11)$$

It is easy to see that the second term and the third term of the right member of equation (11) belong to a different expansion of the rotation of vorticity which we will expose since it is imaginary.
In expanding the rotation of vorticity in a different way we are going to need the laplacian of the absolute value of psi:

$$\Delta|\psi|^2=2\nabla\cdot\left(|\psi|\nabla|\psi|\right)=2\left(\nabla|\psi|\right)^2+2|\psi|\Delta|\psi| \quad (12)$$

From a different expansion of the laplacian of the probability we know that:

$$\Delta|\psi|^2=-\frac{2m}{\hbar^2}|\psi|^2(E-U)+2\left(|\nabla\psi|^2\right) \quad (13)$$

Combining equations (12) and (13) gives the desired result:

$$2|\psi|\Delta|\psi|=-\frac{2m}{\hbar^2}|\psi|^2(E-U)+2|\psi|^2\left(\nabla\phi\right)^2 \quad (14)$$

Finally we expand the rotation of the vorticity:

$$i\nabla\times\vec{\Omega}=2i\nabla\times\left(|\psi|\nabla|\psi|\times\nabla\phi\right)=2i\nabla|\psi|\times\left(\nabla|\psi|\times\nabla\phi\right)+2i|\psi|\nabla\times\left(\nabla|\psi|\times\nabla\phi\right) \quad (15)$$

We are going to produce the usual vector identity so that the reader may be convinced that we already have a term cancelled:

$$\nabla|\psi|\times\left(\nabla|\psi|\times\nabla\phi\right)=\left(\nabla|\psi|\cdot\nabla\phi\right)\nabla|\psi|-\left(\nabla|\psi|\right)^2\nabla\phi \quad (16)$$

Combining equations (16), (15) and (11) the progress so far is revealed:

$$2i|\psi|\nabla\times\left(\nabla|\psi|\times\nabla\phi\right)=\psi*\frac{d\vec{v}}{dt}+2\left(i\hbar\left(\nabla|\psi|^2\right)\cdot\nabla\right)\nabla\phi-\nabla\left(\left(\nabla|\psi|\right)^2\right)+\frac{\hbar^2}{2m}\nabla\Delta P \quad (17)$$

Everyone who is acquainted with vector calculus recognizes that the second term of the right member of equation (17) belongs to the expansion of the left member of equation (18)
The rest is simply vector calculus. We are going use that

$$\nabla\left(\nabla|\psi|\cdot\nabla\phi\right)=0 \quad (18)$$

Also that:

$$\nabla\times\nabla|\psi|=0 \quad (19)$$

$$\Delta\phi=0 \quad (20)$$

The final result so far will be:

$$2i\hbar|\psi|\Delta|\psi|\nabla\phi=\psi*\frac{d\vec{v}}{dt}+\frac{\hbar^2}{2m}\nabla\left(|\psi|\Delta|\psi|\right) \quad (21)$$

CONCLUSION

The left member of equation (21) may be translated to a form of a current or a friction term since it has the imaginary role. The second term of the right member of (21) is accounted for some form of force density. We finally have:

$$\psi*\frac{d\vec{v}}{dt}=2i|\psi|\Delta|\psi|\frac{d\vec{r}}{dt}+\frac{\hbar^2}{2m}\nabla\left(|\psi|\Delta|\psi|\right) \quad (22)$$

References

[1]. A newly proposed model for the electron, SpirosKoutandos, IOSR , JAP
[2]. Regarding the total time derivative of the radius .SpirosKoutandos, IOSR, JAP

Dr Spiros Koutandos " Rewriting the Master Equation.” IOSR Journal of Applied Physics (IOSR-JAP) , vol. 11, no. 3, 2019, pp. 72-74.