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Critical behavior of the Ising model on non-local directed Small-World networks

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Abstract: *We investigate the critical properties of the Ising modelin two dimensions on non-local directed small-world networks* ***.****The disordered system is simulated by applying forMonte Carlo updates heat bath and Wolff algorithms***.***We have calculated the critical temperature, as well as the criticalexponents* , γ/ν,β/ν*and* $1/\nu$ *for several values of the rewiring probabilityP, We find that this system does not belong to the same universalityclass as the regular two-dimensional ferromagnetic model. TheIsing model on non-local directed} small-world lattices presents in fact asecond-order phase transition with new critical exponentsdependent on p (0<p<1).*

Keywords: *Ising Model, Small World, probability, Magnetization, susctibility*.

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I. Introduction

The Harris criterion [1, 2] establishes that the relevance of the effect of quenched random disorder on the critical behavior of a physical system can be classified solely observing the specific heat exponent of the pure system, α_{pure}. This criterion asserts that for $\alpha_{pure} > 0$ the quenched random disorder is a relevant perturbation, leading to different critical behavior than in the pure case. On the other hand, for $\alpha_{pure} < 0$ disorder is irrelevant and, in the marginal case $\alpha_{pure} = 0$ (like the d = 2 Ising model), no prediction can be made. There- fore, for the case of the Potts Model with 3 states in two dimensions, where $\alpha_{pure} > 0$, one would expect a universality class different from the non-random case. Pekalski, Herrero, and others authors [3{9] showed that the Ising model on a small-world (SW) network presents a phase transition well defined at a finite temperature. Silva et al. [10] presented simulations for the Potts model with q = 3 and 4 states on directed small-world networks (DSWN). The Potts model with q = 3 presented a second-order phase for rewiring probability p = 0.1, with exponent ratios $\beta/\nu = 0.24(5)$and $\gamma/\nu = 1.5(1)$. These exponents are different from the Potts model on a regular lattice, where $\alpha_{pure} > 0$, and for p = 0:9 they found a first-order phase transition. For q = 4 they have found a first-order phase transition for values of the rewiring probability p = 0.1 and = 0.9 that agree with the Harris criterion. Lima and Plascak [11] applying the Monte Carlo method with heat bath update algorithm investigated the critical properties of the two-dimensional spin-1/2 Ising and spin-1 Blume-Capel (BC) model on directed small- world lattices with quenched connectivity disorder. For both models, the critical temperature and the critical exponents were obtained for various values of the rewiringprobability p. They found that these systems do not belong to the same universality class as two-dimensional ferromagnetic models on regular lattices. In BC model, with zero crystal field (Δ= 0) interaction, on a directed small-world lattice presents a second-order phase transition for $p < p_c$, and a first-order phase transition for $p > p_c$, where $p_c \approx 0.25$. For the spin-1/2Ising model, where $\alpha_{pure} = 0$, Fernandeset al. [12] showed that the exponents do no change in the undirected Small-World-Voronoi-Delaunay (SWVD) random lattice, but for directed SWVD random lattice the situation is quite different. For $p < p_c$, we have a second- order phase transition and for $p > p_c$a first-order phase transition , where $p_c \approx 0.35$. In addition, the calculated critical exponents for $p < p_c$do not belong to the same universality class as the regular two-dimensional ferromagnetic model. Therefore both undirected and directed cases agree with the Harris criterion for $\alpha_{pure} = 0$. Recently, Ferraz et. al [13] studied the non-local directed small-world disorder effects in the three-state Potts model. Their results are similar to those of Silva et al. [10] for the Potts model with q = 3. In the present work, we studied the spin-1/2 Isingmodel on DSWN with non-local interactions. We have calculated the critical temperature and the critical exponents γ/ν, $\beta/\nu$and $\mathbf{1/\nu}$ for several values of the rewiring probability p. In the next section we present the model and the simulation. The results and conclusions are discussed in the last section.

TABLE I. The critical exponents, for spin-1/2 on non-localDSWN with probability p. γ/v^{max} are the results from the maximum of the magnetic susceptibility. Error bars are statistical only.

P	$1/v$	β/v	γ/v	γ/v^{max}
0.01	0:98(2)	0:352(20)	1:372(42)	1:371(34)
0.05	1:09(5)	0:388(24)	1:205(8)	1:237(25)
0.1	0:97(4)	0:448(7)	1:100(11)	1:153(22)
0.3	1:06(3)	0:443(18)	1:118(16)	1:123(11)
0.5	0:97(3)	0:440(7)	1:169(22)	1:167(8)

II. Model And Simulation

We consider the ferromagnetic spin-1/2 Ising model onDSWN by a set of spins variables Si = ±1 situated on every site i of a square lattice with N = LxLsites were L is the side of square cluster. In this lattice, similar to Sanchez et al. [14], we start from a two-dimensional lattice consisting of sites linked to their 4 nearest neighbors by both outgoing and incoming links. Then, with probability p, we replace nearest-neighbor outgoing links by outgoing links to different sites chosen at random and provided the different site is neither the site itself nor any of its 4 nearest neighbors (local interaction). Afterrepeating this process for every outgoing link, we are left with a network with a density p of DSWN directed links. Therefore, with this procedure, every site will have the old number of outgoing links and a different random number of incoming links. The evolution in time of these systems is given by a heat bath andWolff dynamics with a probability Pi given by:

$$Pi = 1/[1 + exp(2E_i/k_B T)], \tag{1}$$

where T is the temperature, kB is the Boltzmann constant, and Eiis the energy of the configuration obtained from the Hamiltonian:

$$H = -J \sum_{\langle i,j \rangle} S_i S_j \ , \tag{2}$$

where the summation runs over all neighbor pairs of sites(including the nearest-neighbor and the non-local ones determined by the probability p) and the spin-1/2 variables Si assume values ±1. In the above Hamiltonian J is the exchange coupling. The spin-1=2 case on square lattices is well known in the literature [15, 16]. The simulations have been performed on different DSWN lattice sizes L = 10, 20, 30, 40, 80, 120 and 160 comprising a number N = L^2sites. For all system sizes quenched averages over the connectivity disorder are calculated by averaging over R = 50 independent DSWN lattices. For all simulations, we have started with a uniform configuration of spins. We ran $3x10^5$ Monte Carlo steps (MCS)per spin with $1.5x10^5$ configurations dis-carded for thermalization using a random-number generator [17].Here, we have employed the heat bath and Wolff algorithms, where for every 10 MCS of heat bath, we apply 1step of Wolff, and for every 10 MCS the energy and themagnetization per spin, e = E/N and $m = \sum_i S_i/N_i$,were measured, respectively. From the energymeasurements we can calculate the average energy, specific heatand energetic fourth-order parameter, given respectivelyby:

$$u(T) = [\langle E \rangle]_{av}/N \ , \tag{3}$$

$$C(T) = T^2 N[\langle e^2 \rangle - \langle e \rangle^2]_{av} \ , \tag{4}$$

$$\mathrm{B(T)} = 1 - \left[\frac{\langle \mathrm{e}^4 \rangle}{3\langle \mathrm{e}^2 \rangle^2}\right]_{\mathrm{av}} , \tag{5}$$

In the above equations <….> represents the averageof a time series and $[\ldots]_{av}$represents the quench average. Similarly, we can calculate from the magnetization measurements the average magnetization, the susceptibility, and the fourth-order magnetic cumulant,

$$m(T) = [\langle |m| \rangle]_{av}, \tag{6}$$

$$\chi = \ \mathrm{TN}\,[\langle \mathrm{m}^2 \rangle - \langle \mathrm{m} \rangle^2]_{\mathrm{av}} \tag{7}$$

$$U_4(T) = 1 - \left[\frac{\langle m^4 \rangle}{3\langle |m| \rangle^4}\right]_{av} , \tag{8}$$

In order to calculate the exponent ratios of this model,we apply finite-size scaling (FSS) theory. We then expect, for large system sizes, an asymptotic FSS behavior of the form:

$$C = C_{reg} + L^{\alpha/v} f_C(x)[1 + \cdots], \quad (9)$$
$$[\langle|m|\rangle]_{av=L^{-\beta/v} f_m}(x)[1 + \cdots], \quad (10)$$
$$\chi = L^{\gamma/v} f_\chi(x)[1 + \cdots], \quad (11)$$

where C_{reg} is a regular background term, v, α, β and γ are the usual critical exponents, and $f_i(x)$ are FSS functions with:

$$x = (T - T_c)L^{1/v} \quad (12)$$

being the scaling variable. The dots in the brackets[1 + …] indicate corrections-to-scaling terms. We have calculated the error bars from the fluctuations among the different realizations of.

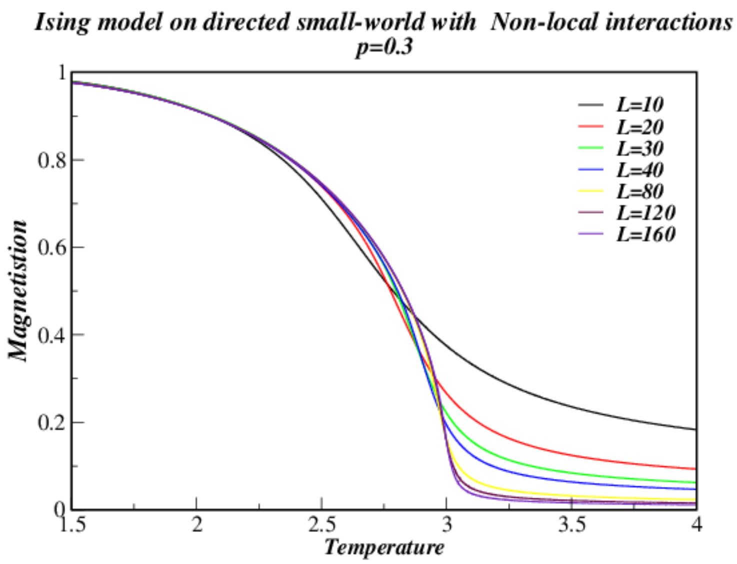

FIG. 1. (color online) Magnetization as a function of temperature for various lattice sizes with N = 100, 400, 900, 1; 600,6; 400, 14; 400 and 25; 600 and rewiring probability p = 0:3.

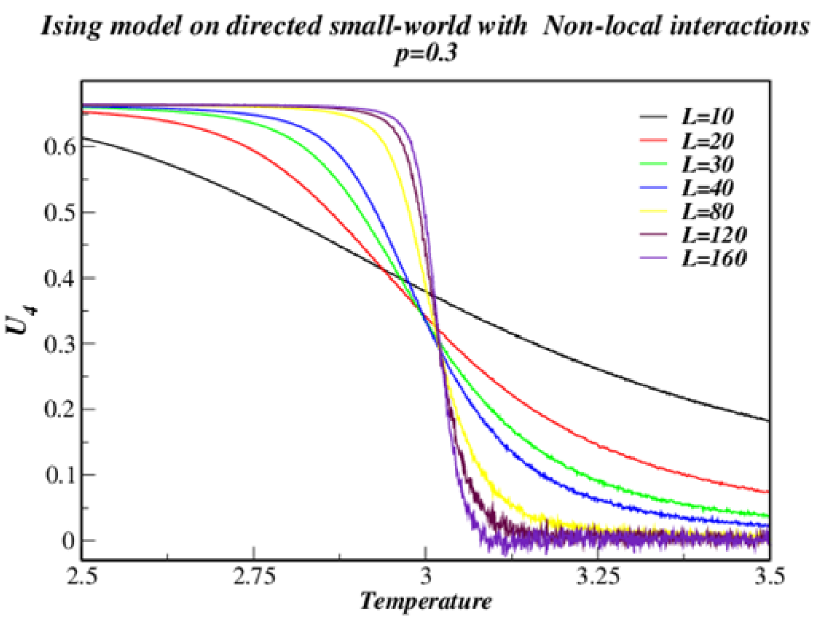

FIG. 2. (color online) The same as Figure 1 for order Binder cumulant as a function of temperature.

III. Results And Discussion

By applying the standard heat bath and Wol_ algorithm to each of the R energy data we determine the temperature dependence of Ci(T), χ_i(T),..., i= 1,...,R. After the temperature dependence is determined for each realization, we can calculate the disorder average, e.g., C(T) = PTi=1 Ci(T)=R, and then determine the maxima of the averaged quantities, e.g., Cmax(Tmax) =maxTC(T). The variable R (= 50) represents the number of replicas in our simulations.

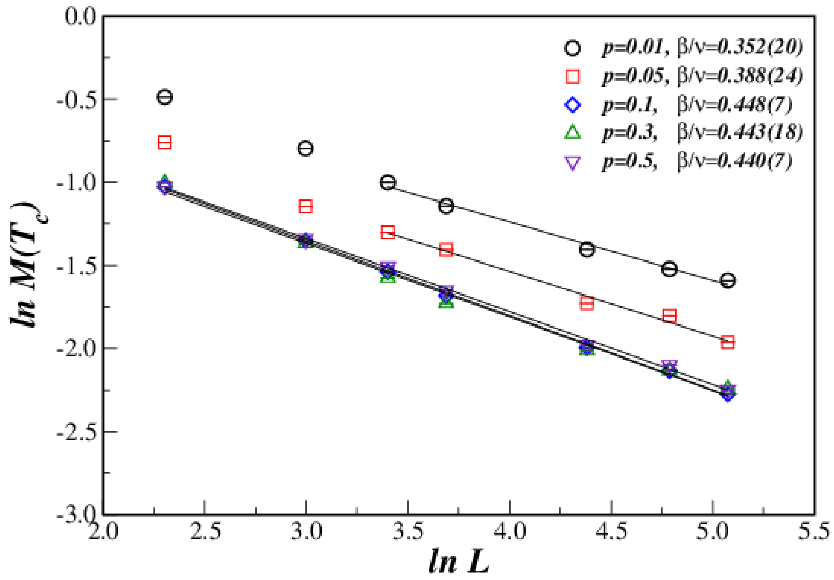

FIG. 3. (color online) ln [Tc(L) - Tc] as a function of L for various values of p. The solid lines are the best linear fits.

In Figure 1, we show the behavior of the magnetizationversus temperature for different lattice sizes and rewiringprobability p = 0:3. From here on, we set J and k_B tounity. One can see a typical behavior of a second-orderphase transition. The critical temperature was estimatedby calculating the fourth-order Binder cumulant givenby eq. (8). It is known from the literature that thesequantities are independent of the system size and shouldintercept at the critical temperature [18]. In Figure 2 thefourth-order Binder cumulant is shown as a function ofT for several lattice sizes for the rewiring probability p =0:3. Taking the largest lattices we have Tc = 3:018(5).To calculate U_4 we note that it varies little at Tc, so wehave U_4^*= 0:293(4). One can see that $U_4^*$is di_erent fromthe universal value $U_4^* \sim 0.61$for the Ising model on theregular d = 2 lattice. By following this same procedure,one can get the corresponding results for other values ofp.

The correlation length exponent $1/\nu$can be estimated from $T_c(L) = T_c + bL^{-1/\nu}$, where Tc(L) is the pseudocritical temperature for the lattice size L, Tc is the critical temperature in the thermodynamic limit, and b is a non-universal constant. In Figure 3 it is shown a plot of ln [Tc(L) - Tc] as a function of lnL for various values of p. One can clearly see that the exponent is, within the errors, independent of p, in agreement with universality ideas. The actual values of $1/\nu$are shown in Table I. In order to go further in the present analysis we have also computed the modulus of the magnetization at the inflection point and the magnetic susceptibility at Tc.

The Log-log plot of these quantities as a function of L are presented in Figures 4 and 5, respectively. A linear fit of these data gives β/ν from the magnetization and γ/ν from the susceptibility. In addition, we plotted in Figure 6 the logarithm of the maximum value of the susceptibility χ_{max} as a function of lnL for several values of p. One can also see that the exponents β/ν and γ/ν are also independent of p, as expected. They are different from $\beta/\nu = 0.125$ and $\gamma/\nu = 1.75$obtained for a regular d = 2 lattice, but obey hyper-scaling relation (within the error bars)

$$2\frac{\beta}{\nu} + \frac{\gamma}{\nu} = d\,, \tag{13}$$

where d = 2. The numerical values of the ratio β/ν and γ/ν are also shown in Table I.

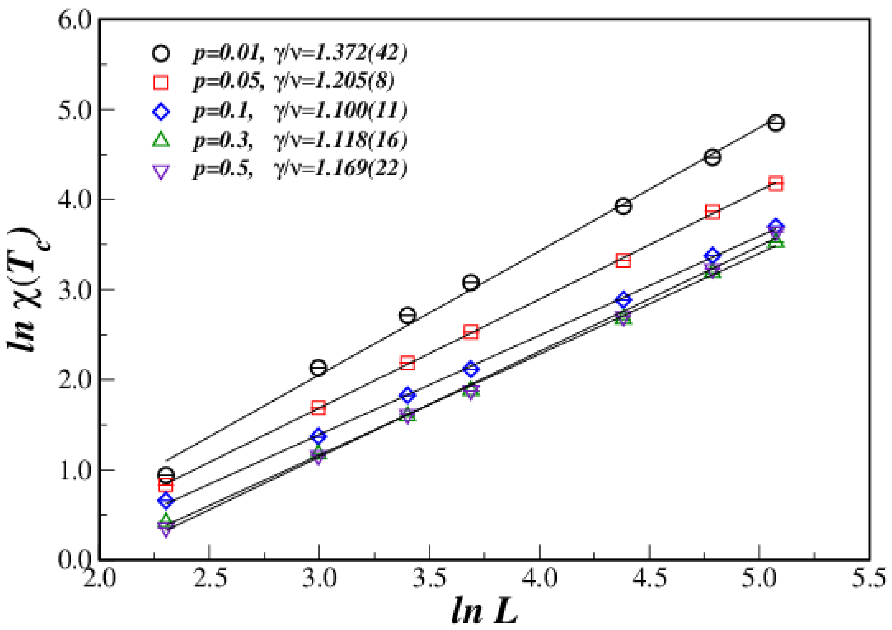

FIG. 4. (color online) Log-log plot of the modulus of themagnetization at the inflection point (Tc) as a function of L.

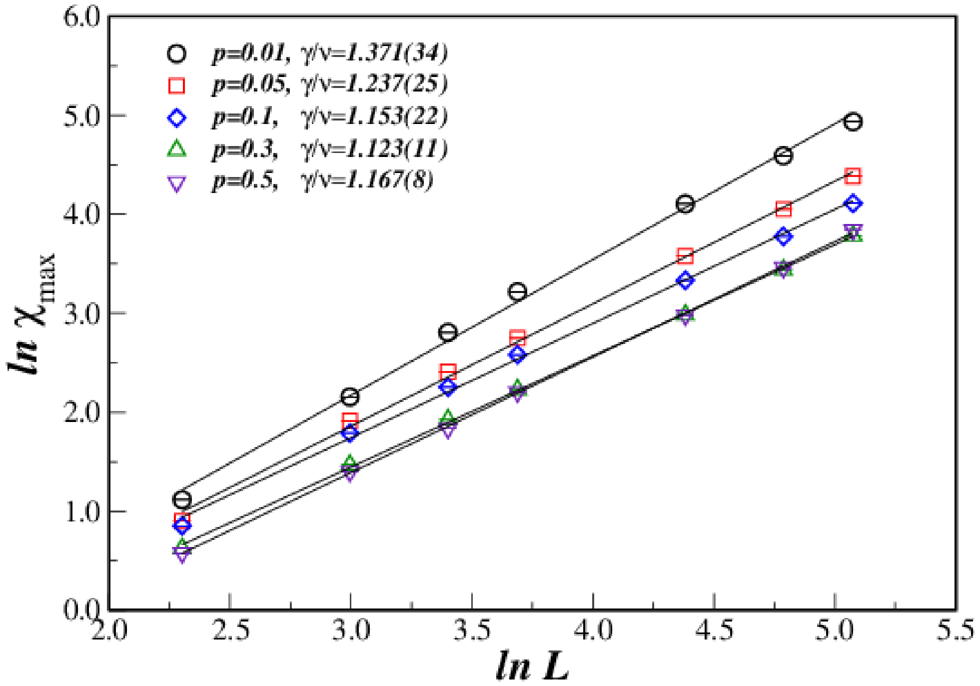

FIG. 5. (color online) Log-log plot of the susceptibilityχat Tc as a function of the logarithm of L.

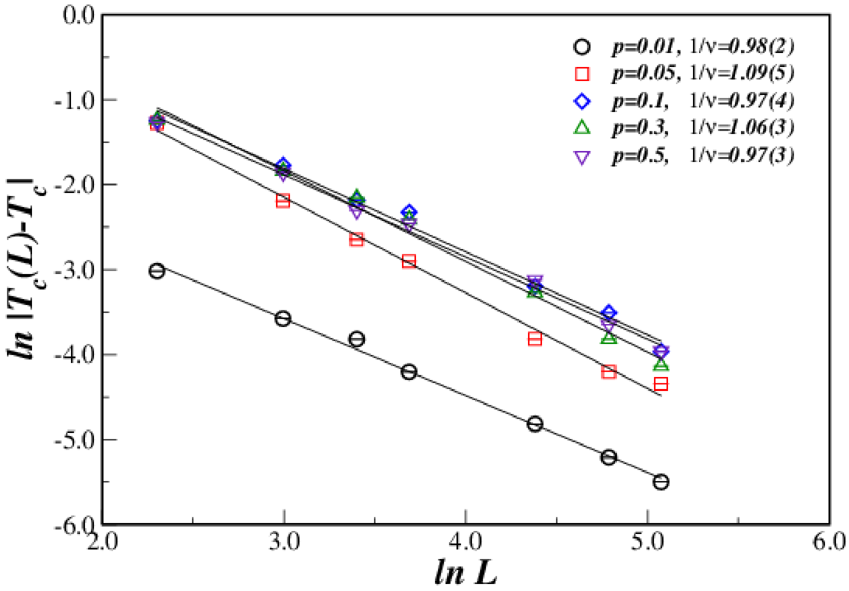

FIG. 6. (color online) Log-log plot of the susceptibility maximaχ_{max} as a function of the logarithm of L.

In Figure 7, we display the data collapse for the magnetization, susceptibility and the Binder cumulant for p = 0:01 (a, b and c) 0.1 (d, e, and f). In these cases, we see that the estimates of the critical exponent ratios β/ν and γ/ν are in good agreement for all lattice sizes. The same qualitative results are obtained for other values of p.

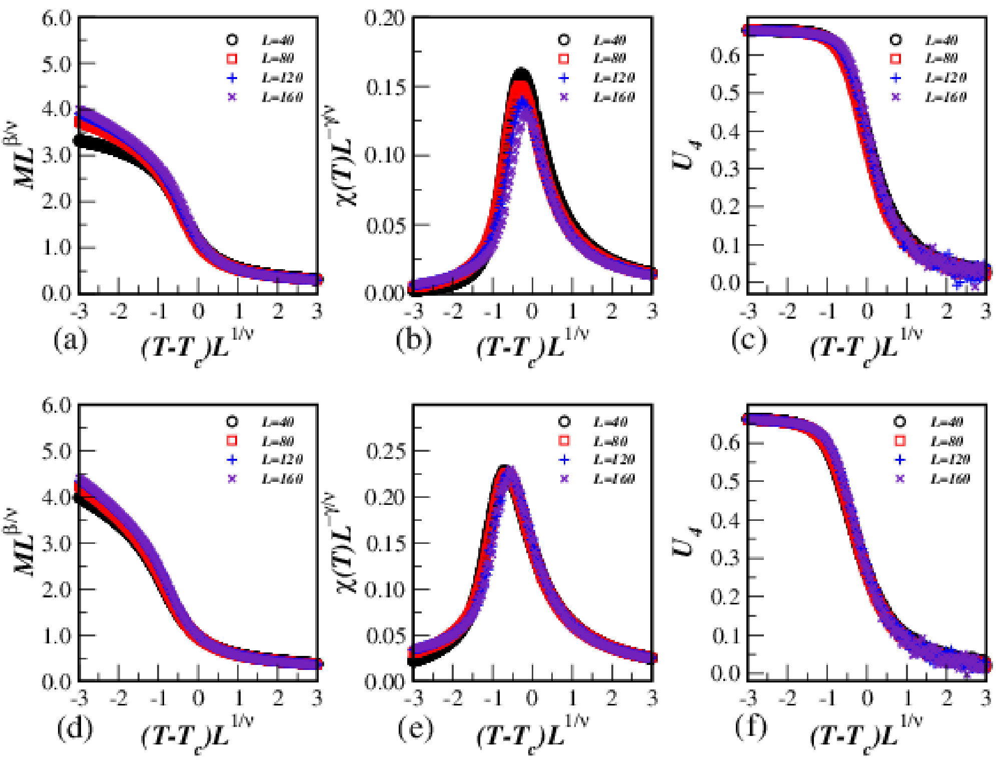

FIG. 7. (color online) Data collapse of magnetization, susceptibility and Binder cumulant for various values of L and for p = 0:01 (a, b and c) and 0:1 (d, e and f).

IV. Conclusion

Through numerical simulations, we have studied thespin-1/2 Ising model on DSWN with non-local interactions. We have calculated the critical temperature and the critical exponents ratios β/v, γ/v and $1/v$ for several values of the rewiring probability p. In summary, from the above results, there is a strong indication that the spin-1/2 Ising model on a non-local DSWN is in a different universality class than the model on a regular two-dimensional lattice. The exponents' ratio here esteemed are independent of p for p = 0.1; 0.3; and 0.5 and dependent on p for p = 0:01 and 0:05 whichare less than p = 1 and for both cases are different fromthe Ising model on regular d = 2 lattices. One possible explanation for this change in the universality can be ascribed to the influence of directed non local interactions that occur with the presence of p directed bonds [13].

However, we observe that the obtained exponents' ratio are independent of the facts of the interactions being local or non-local. We observe also that our results agree with the Harris criterion for DSWN.

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