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On the application of Noether's theorem to the potential energy Lagrangian due to the oxide voltage in a MOS device

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Abstract: *This paper demonstrates the application of the Noether's theorem I to the potential energy Lagrangian due to the oxide voltage in a MOS device. The Lagrange's equations for the force functions have been developed in terms of the change in kinetic or potential energy or the work done. The force functions minimize the potential and kinetic energy Lagrangian functions by expressing them as divergences.*
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I. Introduction

The four forces of nature are: gravitation, electromagnetism, weak nuclear force and the strong nuclear force with the gravitation being the first force to be discovered in the 17^{th} century by Sir Isaac Newton. The unification of these forces initiated and worked on by Einstein for many years when only gravitation and electromagnetic forces were known, is not yet successful. It is now known that this is possible at very high energies of about 10^{28} eV with the present day observations at 10^{13} eV, as presented by the 2004 Physics Nobel laureate David Jonathan Gross (along with Hugh David Politzer and Frank Anthony Wilczek) in one of his 2013 videos titled "A Century of Quantum Mechanics". String theory is one such theory that is attempting to combine gravitation along with other forces.

The author here is particularly focussed on the electromagnetic force due to the electrons which was discovered in the 18^{th} century. The author has determined the effective mass of light hole in the thermal silicon dioxide as 0.58m at 300K [1-3]. This value is now experimentally confirmed by a Japanese research group [4]. Furthermore, the author derived the four equations for the oxide voltages across the Metal-Oxide-Semiconductor (MOS) device having charges, in the magnitude form [2-3, 5]. These equations can be configured in one equation as;

$$V_{ox} = V_{app} – V_{fb} \quad (1)$$

where, V_{fb} is the flatband voltage across the MOS device and V_{app} is the applied voltage. The four equations with magnitudes result from considering electron tunnelling from the cathode having a negative applied voltage and hole tunnelling from the anode having a positive applied voltage, and V_{fb} being positive or negative for negative or positive charges in the MOS device. The immediate observation was that the oxide voltage at the cathode for electron tunnelling is negative of the oxide voltage at the anode for hole tunnelling, showing C-symmetry due to simultaneous conjugation of charge at the cathode or anode and in the oxide due to V_{fb}. The equations for the oxide voltage were determined by applying Gauss's Law and the Gauss's Law can be derived from the Coulomb's Law or vice versa. The rotational, translational and C-symmetry in the potential or Electric field already exists in these Laws and therefore in the oxide voltage.

In this article, the author shows that Noether's theorem I is applicable to the physical system of the MOS device. Noether's theorem states that: If an Integral I is invariant with respect to a group G_p, then a linearly independent combination of the Lagrange expressions become divergences---, and from this, conversely, invariance of I with respect to a group G will follow [6]. Here, the force functions minimize the potential and kinetic energy Lagrangian functions by expressing them as divergences.

II. Theory

First, it is shown that charge is a conserved quantity. The Coulomb's Law can be derived from the Gauss's Law. Starting with the equation of the total flux from an enclosed charge q_1, in a Gaussian surface dS;

$$\Phi_E = \oiint_S E.dS = \frac{q_1}{\varepsilon_0}. \quad (2)$$

$$(E)x(4\pi r^2) = \frac{q_1}{\varepsilon_0}. \qquad (3)$$

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{F}{q_2} \qquad (4)$$

$$Coulomb's\ \underset{\rightarrow}{\ }Force,\ \underset{\rightarrow}{\ }F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad (5)$$

Now, the work done is the potential energy or the work done in bringing a positive charge q_2 from infinity to the point r distance away from the enclosed charge q_1. The charge is not a function of distance and the distance can be interchanged with time as the unit based on the fact that a distance is the time it takes light to travel, where speed of light is constant. So, the charge is not a function of time, as well. Therefore, change in charge with time is equal to zero. In other words, charge is a conserved physical quantity in the above two Laws. Although, the Coulomb's Law is valid for point charges, the principle of superposition is applicable to the forces due to many charges and thus makes the Coulomb's Law compatible with the Gauss's Law, where there is an enclosed total charge and not just a point charge. Before discussing further, it is to be mentioned that "Lagrangian" is the operator that gives the difference between the potential energy and the kinetic energy, unlike the "Hamiltonian" which is the total energy operator, that is, the sum of kinetic and potential energy as for example in the Schrodinger's equation of Quantum mechanics.

A . Lagrange's Equation for the change in kinetic energy of a moving particle of mass m

First, the Lagrange's equation for the kinetic energy has been derived as an example [7] for the development of the equation. For the work done as the change in kinetic energy,

$$L = \int mv\,dv \qquad (6)$$

$$L = (1/2)mv^2. \qquad (7)$$

$$\frac{d}{dt}(mv) = F \qquad (8)$$

$$\frac{\partial L}{\partial v} = mv \qquad (9)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) = F \qquad (10)$$

Also, $\frac{\partial L}{\partial x} = \frac{1}{2}m\left(\frac{2x}{t^2}\right) = \frac{mx}{t^2} = \frac{mv}{t} = ma = F$ (11)

From (10) and (11), we get,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0 \qquad (12)$$

Equation (12) is the Lagrange's equation which expresses Newton's second Law of motion in terms of the kinetic energy as the Lagrangian function. If the second term of equation (12) equals zero, that is, L is invariant with x and therefore having translational symmetry, then the first term is also zero. That is $(d/dt)(mv) = 0$. It shows that momentum does not vary with time and so momentum is a conserved quantity.

B. Lagrange's equation for the change in potential energy of the test charge q located at a distance r when brought from infinity, from an enclosed charge q.

$$L = -\int_r^\infty F.dr \qquad (13)$$

Here F s the Coulomb's Force and r is the distance of test charge from the point charge enclosed in the Gaussian surface

L = Coulomb's Force x distance= Potential energy or the work done to bring the point charge from infinity to distance r.

L=qE x r, (14)

where, E is the Electric Field due to the enclosed charge.

L = q x V, (15)

where, V is the potential as a function of r.

$$L = \frac{Kq^2}{r^2} x \; r = \frac{Kq^2}{r} \quad (16)$$

$$\frac{\partial L}{\partial r} = -\frac{Kq^2}{r^2} = -F. \quad (17)$$

Here, the Force function F minimizes the potential energy function or the work done function. In particular, for r=∞, the Force function will be zero and the potential energy function will be zero or minimum.

Also, $\frac{\partial L}{\partial q} = \frac{2Kq}{r}$ (18)

$$\frac{1}{2}\frac{q}{r}\frac{\partial L}{\partial q} = \frac{1}{2}\frac{q}{r}\frac{2Kq}{r} = \frac{Kq^2}{r^2} = F \quad (19)$$

Now, from equations (17) and (19), we get,

$$\frac{\partial L}{\partial r} + \frac{1}{2}\frac{q}{r}\frac{\partial L}{\partial q} = 0 \quad (20)$$

Equation (20) is the Lagrange's equation which expresses the Coulomb's force in terms of the potential energy or the work done on test charge q at a distance r from another charge q. Here, the force function is minimizing the potential energy function. Going one step further, differentiating equation (15) gives;

$$\frac{dL}{dr} = q\frac{dV}{dr} + V\frac{dq}{dr}. \quad (21)$$

Keeping the speed of light in mind, dq/dr can be treated as dq/dt as discussed earlier. This is zero as the charge does not change with distance and therefore with time. Charge is therefore a conserved quantity. Also,

$$\frac{\partial L}{\partial r} = \frac{dL}{dr} \quad (22)$$

So, equation (21) becomes;

$$\frac{dL}{dr} = q\frac{dV}{dr}. \quad (23)$$

Also, $V = \frac{Kq}{r}$ (24)

$$\frac{dV}{dr} = -\frac{Kq}{r^2} \quad (25)$$

$$q\frac{dV}{dr} = -\frac{Kq^2}{r^2} = -F. \quad (26)$$

So, equation (20) now becomes;

$$q\frac{dV}{dr} + \frac{1}{2}\frac{q}{r}\frac{\partial L}{\partial q} = 0. \quad (27)$$

Dividing by q throughout gives;

$$\frac{dV}{dr} + \frac{1}{2r}\frac{\partial L}{\partial q} = 0. \quad (28)$$

Equation (28) is another version of the Lagrange's equation (20).

III. Results and Discussion

It can be observed in the above descriptions, that the force functions are written in terms of the energy functions. The force functions minimize the kinetic or potential energy functions. The fact, that the potential energy function is continuously differentiable means that at some point in the differentiated function, a minimum or maximum of the function will exist. Consider the Lagrange's equation for the potential energy in equation (13), (15), (20) and (28). The Lagrangian L contains the Force and potential functions, which are functions of two independent variables, q and r. These functions are subjected to transformations such as: dV/dr and $\partial L/\partial q$. V(r) is continuous and continuously differentiable symmetry of the Lagrangian. As can be seen, the linearly independent combination of the Lagrangian can be written as divergences in equation (20) and (28) which leads to the invariance of the Integral in equation (13) within a group, according to the Noether's theorem [6]. Expressing the Lagrangian as a divergence informs us about the law of conservation of charge as in equation (21) [6]. The application of the Noether's theorem to the MOS device is that wherever there is a continuous and continuously differentiable symmetry in the Lagrangian, there is a conservation law applicable and conversely, where there is a conservation Law, there is a continuous symmetry transformation. In this case, conservation of charge has been shown to exist for the symmetry function of potential or oxide voltage in the MOS device. Notice that equation (28) becomes the Lagrange's equation for the Electric field function instead of Force function in terms of the potential energy or work done. The Lagrange's equation (12) for the change in kinetic energy of a moving mass shows the conservation of momentum.

IV. Conclusion

The application of Noether's theorem I to the potential energy Lagrangian due to the oxide voltage in the MOS device has been demonstrated and the Lagrange's equations have been developed. It is shown that for the kinetic energy having translational symmetry, that is invariance with x, momentum is a conserved physical quantity. For the potential energy of a charged particle with transformed functions of potential and field having rotational, translational and C-symmetries, charge is a conserved physical quantity.

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