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Newton's generalized form of second law gives F =ma

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Abstract
Isaac Newton never wrote equation F =ma, it was clearly derived by Euler in 1775 (E479 http://eulerarchive.maa.org/). Also, Newton ignored acceleration throughout his scientific career. It must be noted that acceleration was explained, defined and demonstrated by Galileo in 1638 (four years before birth of Newton) in his book Dialogue Concerning Two New Sciences at pages 133-134 and 146. Galileo defined uniform velocity in the same book at page 128 and applied it in Law of Inertia at page 195 in section The Motion of projectile.
Descartes in book Principles of Philosophy (1644) and Huygens in his book Horologium (1673) used uniform velocity in defining their laws. Huygens also applied gravity in 1673 i.e. 13 years before Newton. Newton also defined first law of motion in the Principia (1686,1713,1726) in terms of unform velocity. Galileo, Descartes and Huygens did not use acceleration at all, as uniform velocity is used in law of inertia. Likewise, Newton ignored acceleration completely, even it was present in literature during his lifetime. So, it is distant point that Newton gave F=ma. The geometrical methods were the earliest method to interpret scientific phenomena. Now there are three main points for understanding of second law. Firstly, genuine equation based on second law of motion F =kdV (it is obtained like $F = Gm_1m_2/r^2$ or $F \propto m_1m_2$, $F \propto 1/r^2$ or $F \propto dV$) . But F = kdV is neglected by scientists completely . Secondly scientists related F =ma with second law. Then they tried to obtain, F =ma from second law, but it is not obtained from it. Thus, scientists made arbitrary assumptions that motion is momentum (mV), in fact motion is velocity. It does not serve the purpose. Then scientists assumed that change in motion (momentum) is equal to rate of change of momentum. As both sides of equation have different units, dimensions and magnitudes so these arbitrary assumptions are completely inconsistent. Thirdly to obtain F =ma from second law of motion is to change definition of the law so that acceleration appears in it. However, Newton has completely neglected acceleration. But now acceleration is arbitrarily brought in second law. The definitions of first and third laws of motion are quoted in the text books and standard references in the same way as given in the Principia.
W W Rouse Ball has changed definition of second law of motion slightly i.e. used phrase change in momentum per unit time. Disagreeing with this I. Bernard Cohen given 4 equivalent equation forms of second law of motion. F =ma is inseparable part of physics, so it can be it can be obtained by changing the definition of the law, "The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed."
Thus, we get F =ma from modified form of law, not from original form.

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I. Early Physics

Gradually ancient minds tried to resolve various mysteries observed around him. There was no short cut, all processes took their own time. First feeling in human mind may be fear of some power which was beyond human's physical and mental control …. thus, religion was developed in different ways in different regions. But eventually should converge at same point, so it must be cohesive force and not divisive. Earlier all branches of knowledge were fused together. The explosion of knowledge may be compared with 'big bang' which took place gradually and slowly. In ancient days man started expressing his insights on the sand or earth with diagrams, thus geometry was first mode of expressions. The science came into being, something we understand and also make others understand repeatedly. The converse is superstition, we cannot understand, cannot make others to understand. Initially all branches had one name, the natural philosophy. *Natural philosophy* was the philosophical study of nature and the physical universe that was dominant before the development of modern science. The various scientists extracted various branches such as mathematics, physics, chemistry, biology etc. emerged from natural philosophy. In physics Newton, Galileo and Aristotle were the pioneers, and numerous others who brought physics to current status.

Earlier Aristotle (385-323BC) stated that force is required for movement of body. The table stops as soon force (may be push or pull) ceases to act on it. It is clearly observed even now due to presence of various resistive forces. The concept of *inertia* was alien to the physics of Aristotle. Aristotle, and his peripatetic followers held that a body was only maintained in motion by the action of a continuous external force. Aristotle implied that rest is natural tendency of body, it is disturbed when external force acts on body; and justified in above example. This doctrine was contested between admirers and critics for centuries.

1.1 Galileo's Dialogue and Newton's Principia are the pioneering books.

Before Sir Isaac Newton's masterpiece *Mathematical Principles of Natural Philosophy* [1,2] , Italian Galileo Galilei did pioneering work in process of initialization of physics. Italian has conducted some experiments with simple equipment about motion of bodies in first decade of 17th century (may be in1604), may be regarded as first genuine physicist. Galileo improved Aristotelian scientific views. Galileo defined uniform motion, acceleration and accelerated motion, he also applied uniform motion in formulating law of inertia (used in refined form as Newton's First Law of Motion). Galileo [3] presented these experiments in his book *Discorsie dimostrazioni matematiche intorno a due nuove scienze* (*Dialogues Concerning Two New Sciences)* in 1638. It is translated by Henry Crew and Alfonso de Salvio. The book was written and published when Galileo Galilei was under house arrest and state of ill health. Galileo has become completely blind in 1636. This book was written in form of dialogues between three men e.g. Simplicio, Sagredo, and Salviati . The significance of this book can be realized form the fact that Galileo explained in it uniform velocity at page 128, acceleration at pages 133-134, 146, law of inertia at page 195 in section of The Motion Of Projectiles. Galileo's this book deserves mention in the beginning due to reason the definitions of uniform velocity, acceleration and law of inertia are still used in the same form as given by Galileo. So much so Newton's First Law of Motion is more refined form of Law of Inertia given by Galileo.

The other book which has formed basis of physics and science is Newton's *Philosophiæ Naturalis Principia Mathematica* (Mathematical *Principles of Natural Philosophy* [1,2]) popularly known as the Principia. Newton initiated physics separating it from natural philosophy i.e. practically from zero state (that was genuine beginning of physics as subject). So he may be called originator of physics. *The Principia* was first published in Latin in 1686 and translated to English by Andrew Motte in 1729.

In this book Newton had given new definitions, axioms or laws of motion and basics of law of gravitation. Between these two books (the Dialogue,1638 and the Principia,1686) French man Rene Descartes published Principles of Philosophy[4] 1644 , Christian Huygens, *Horologium oscillatorium sive de motu pendularium* [5] ,1673). All the interpretations are given in form of theorems, propositions etc. The concepts were justified geometrically and philosophically; not with mathematical equations.

Likewise, Newton [1,2] did not give any mathematical equation in *Mathematical Principles of Natural Philosophy,* 1686 and explained phenomena geometrically and diagrammatically (simple, extremely speculative and complicated) without mathematical equations. Thus, geometry was the earliest method to interpret the perceptions.

1.2 Peculiarities of the *Mathematical Principles of Natural Philosophy (*Principia).

Galileo has defined acceleration but did not apply acceleration in explaining motion of bodies as he applied uniform velocity in formulation of law of inertia. Galileo also defined uniform motion and applied the same in enunciation of Law of Inertia (1638). Descartes (1644) used the Law of Inertia in formulation of his second law of motion, and Huygens (1673) applied the same in formulation of his first hypothesis that bodies move with equal velocity in resistance free systems.

Galileo (1638), Descartes (1644), Huygens (1673) did not use acceleration in explanation of motion of bodies. Also, Newton did not use acceleration purposely. Newton's First Law of Motion is just other form of Law of Inertia. Newton improvised in Second Law of Motion by associating force with motion of bodies or changed kinematical system to dynamical system.

Thus, Newton did not use acceleration in the Principia at all, even when he had opportunity to do so in the third and final edition of the Principia in 1727. Also, Newton did not write F =ma (Force = mass x acceleration), in his scientific career, as he did not write equation for acceleration. Galileo defined acceleration (dV/dt) in 1638 in the book Dialogue at page 133-134 and 146. Thus, acceleration was completely ignored by Newton as given 4 years before his death. In the discussion the scientific literature right from 1604 to 2011 is not only reviewed but critically analyzed impartially.

The unanimous conclusion is that Newton neither defined acceleration nor wrote F =ma in throughout scientific career. It implies Newton did not write equation F =ma for Second Law of Motion (hence it should not be credited to Newton), it was done by following scientists with inconsistencies; this is main point of discussion here. The inconsistent interpretation had been given by following scientists not by Newton. So Newton can never be held responsible for inconsistent explanation. The reason is that Newton left after defining laws and did not give any equation (like F =ma) in any edition of the Principia. The genuine equation based on second law of motion is F =kdV, it is completely neglected.

Jacob Hermann [6] may be regard as having directly given equation G = MdV/dT where G indicates the weight or variable force of gravity in his book *Phoronomia* at page 57 in 1716. Cohen [11] at page 113 apparently related this equation with Newton's second of motion as F =mdV/dt. Euler has given many equations relating to force, mass and acceleration in 1736, 1749, 1752, and 1765. Finally, Euler derived mathematical equation $F = md^2x/dt^2$, F =ma in 1775 which was published in article, *Novi Commentarii academiae scientiarum Petropolitanae* in1776. The equation F =ma was discovered by Hermann and Euler independently at different times. It is confirmed in section (4.6) that there are no clear scientific evidences when F =ma was associated with Newton's second law of motion. The historical reviews of physics and mathematics are required purposely.

Newton initiated physics separating it from natural philosophy i.e. physics may be regarded as at zero state at that time (that was genuine beginning of physics as subject). Newton defined new definitions, laws of motion etc. Newton has categorized motion as absolute motion (Motus absolutus) and relative motion (relativus motus). Newton expressed both absolute and relative motion in terms of velocity. There are evidences that before Newton motion was regarded as velocity.

Newton used Galileo's Law of Inertia in refined form in First Law of Motion. Newton did not explain acceleration at all, as it was easier to explain uniform motion. Newton improvised existing laws by associating force with motion of bodies i.e. put laws in dynamical form. The importance of acceleration was fully realized when differential and integral calculus was developed after death of Newton.

At that time (before and during Newton's time) the phenomena were explained with help of propositions, theorems and geometrical diagrams only. At that time mathematical equations were neither prevalent nor requisite. In fact, initially man started explaining his perception's with help of diagrams only, on the sand or earth. At Newton's time the mathematical equations were not perceived as we have now. Like his predecessor Newton used geometrical diagrams for describing his perceptions.

The first differential equations of motion [7] for systems having more than two mass-bearing points were published in 1743 by John Bernoulli and by D'Alembert. So, Newton originated physics (without mathematical equations, even did not give F =ma, $F =GM_1M_2/r^2$) which was developed later on with combined efforts of many scientists and process is on. Thus, he may be called originator of physics but not developer.

I Bernard Cohen in 1999 in his book *Isaac Newton The Mathematical Principles of Natural Philosophy* had tried to create a void that works of both Hermann and Euler are insignificant and only Newton's equation F =ma (which Newton never speculated or derived) is all in all. However, truth is that Newton neither mentioned about acceleration (as given by Galileo in the Dialogues at pages 133-134, 146) nor wrote F =ma. The equation F =ma was derived for first time by Euler in 1775. Also Cohen has tried to justify Hermann's equation implies Newton's second law of motion.

The foundations of this book/monograph, *Newton's generalized form*
Of second law gives F =ma were laid down during The Euler Society Conference, 2014 held in Texas, USA. The author attended this conference going from India (Shimla) to Texas (Austin, Dallas) covering more than 14,000 kms via Chicago [8]. This conference the best and only opportunity to answer the long-standing queries about origin of F=ma. Thus, scientific quest lead this adventure sponsored by wife Anjana Sharma and spent from own purse. The participants were most mathematicians, agreed that Euler has given F=ma, they showed references which proved very helpful. After interactions with scientists all over the world, it was concluded that all information must be put together in single volume. Finally, various experts in history of physics/mathematics showed a way a valuable critical report on the manuscripts and sent some obscure papers The Covid-19 period and lock down era (March 2020) was the most suitable for starting this book /monograph as smaller articles did not serve the purpose. The aim was to put all information about F=ma in one platform which was otherwise scattered.

1.3 First Translation of the *Philosophiæ Naturalis Principia Mathematica*

Newton has written *Philosophiæ Naturalis Principia Mathematica* in 1686. Newton was encouraged to write Principia by his friend and sponsor of book Edmund Halley. Andrew Motte (1696-1734), brother of one of Great Britain's most famous publishers, Benjamin Motte who encouraged Andrew to translate the 1726 edition of the Principia in English. Thus, Benjamin Motte published English translation in 1729. However, on the other hand Galileo faced hardships in publishing his book *Dialogues Concerning Two New Sciences*. After failing publishing book in France, Germany and Poland. Finally the book was published by Lodewijk Elzevir who was working in Leiden, South Holland.

The second standard translation in this regard is American edition titled *The Mathematical Principles of Natural Philosophy* (1846) edited by N W Chittenden [9] and was published in 1846. This edition it is nothing but Andrew Motte's translation, contains Book I, Book II and Book III of the Principia along with

Newton's book *The System of the World (De Mundi Systemate*) published in 1728 (a year after Newton's death).

The other English editions of the Principia are taken from Andrew Motte's edition. Florian Cajori [10], Swiss-American historian of mathematics published in 1934 the Principia as *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World* (Berkeley: University of California Press). The other or following translations or elaborations have emerged from above books so that contents may be given to reader in simple words.

Also recently I. Bernard Cohen (Victor S. Thomas Professor of the history of science at Harvard University, USA), published in 1999, The Principia , Authoritative Translation [11]. This book has first part A Guide to Newton's Principia (p.3-399) which has 10^{th} chapter as How to Read the Principia (293-368). The second part is Mathematical Principles of Natural Philosophy (p. 400-966), it is not translation but simple interpretation of the Principia from existing translations.

1.4 Jacob Hermann (1716), Leonhard Euler (1736, 1749, 1752 ,1765 & 1775) and F =ma

Newton has written the famous masterpiece the Principia in 1686. As Newton did not give any mathematical equation for second law of motion (due to conceptual limitations at that time). Newton and his predecessors (Galileo, Descartes and Huygens) explained phenomena philosophically and geometrically. It is also independently confirmed by various scientists [11, 12] in renowned publications by American Institute of Physics and University of California Press. These quotations are also found valid when the concerned scientific literature is critically reviewed right from days of Galileo's experiments in first decade of 17^{th} century (1604) till twenty first century.

Thus, definition of the Second Law of Motion should not be misinterpreted to give pre-supposed results. The equation i.e. F =ma was associated with Second Law of Motion after death of Newton inconsistently by following or succeeding scientists. In this regard section (4.6) is very significant. The genuine equation-based definition of Newton's second law of motion is F =kdV, which is not completely neglected by scientists. Thus, following scientists must be held responsible for inconsistencies, not Newton (who has just given definition of second law of motion) and did not any equation. Newton also explained acceleration (as previously derived by Galileo) throughout his life.

(a) Jacob Hermann has given directly (without derivation) equation

G =MdV:dT or G =MdV/ dT

where G weight or variable force of gravity, M is mass, dV change in speed and time dT. This equation appears in his book *Phoronomia* at page 57.

(a) Cohen [11] at page 113 has quoted that

"Newton actually never made a formal statement of the second law of motion in the algorithm of fluxions or the calculus. The first person to do so seems to have Jacob Hermann (1716) in which he writes G =MdV/dT. Thus, according to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann.

Then force G (weight or force of gravity) may be replaced by F (also weight is equated with force of gravity to calculate acceleration due to gravity, g). Thus, we get equation as

$$F = mdV/dt \qquad (1)$$

In this direct quotation Hermann did not use Newton's second law of motion. Here dV/dt is acceleration explained by Galileo in his book *Dialogue Concerning Two New Sciences* at pages 133-134 and 146.

In third and last edition of the Principia in 1726 Newton did not acknowledge eq. (1) as equation for second law of motion. Thus, Newton did not any equation of force, the first edition of the Principia was published in 1686 and last in 1726 i.e., 41 years later.

(b) Euler [13] in his paper titled *Nova methodus motum corportum rigidorum degerminand* , completed the construction of general equations of dynamics by formulating a system of six equations determining the motion of any body, which (except for an additional coefficient) he wrote in the following way at page 222-23.

$P=\int dM \frac{d^2x}{dt^2}$, $Q=\int dM \frac{d^2y}{dt^2}$, $R= \int dM \frac{d^2z}{dt^2}$

Or in general, $F = md^2x/dt^2 = ma$ (2)

Euler's paper has Enestrom number E479 and available online http://eulerarchive.maa.org/ without any membership.

It must be noted Hermann and Euler both were Swiss born at Basel were distant relative to each other. Acceleration was found useful term when differential and integral calculus was developed, then F =ma was associated with Newton's second law of motion by the succeeding scientist but inconsistently. Thus, this association happened after death of Newton so he was ignorant of it. The following scientists are responsible for inconsistencies not Newton. The genuine equation based on Second Law of Motion is F =kdV which was neither mentioned by previous scientists nor mentioned now.

Euler, the discoverer of $F = d^2x/dt^2 = ma$ had become blind of right eye at age of 35 i.e. 1738. And in 1766 i.e. at age of 63 became completely blind of left eye. Euler worked in Switzerland, Russia and Germany at different times. Newton's scientific stature was exceptionally high compared to both these scientists in all respects. Both were very small compared to Newton. Hermann's quoted equation for G *(signifies weight or gravity applied to a variable mass M), it* was just quoted as a line in one paragraph. Euler's equation was derived in a research paper in 1775. Earlier in 1736,1749,1752 and 1765 gave various equations relating to mass, force and accelerations. But Newton did not give any equation in the Principia, not speak of F =ma. Euler has derived eq. (2).

While going through the relevant existing literature it is concluded that Newton neither gave F=ma nor discussed acceleration at all (Galileo has explained acceleration in 1604 and published in 1638). In this regarded various quotations and comments in existing literature are also available. Scientists study them in discrete way. But here an attempt has been made to compile all on the basis of critical analysis and put all relevant information in single book/monograph.

1.5 Existing quotations

(i) Cohen [11] has written at page 117 of that

"Newton did not give equations to his laws."

Galileo may be regarded first genuine physicist as he defined uniform velocity, acceleration and law of inertia. Galileo's Law of Inertia (1638) was used Rene Descartes (1644), Huygens (1673) and Newton (1686) in form of propositions, theorems etc. i.e. by geometrical methods without mathematical equations. Newton did not give F=ma and $F = Gm_1m_2/r^2$ in the Principia.

So, at that time it was neither conceptually feasible nor mandatory to write mathematical equations so Newton did not write mathematical equations for physical laws. The geometrical methods are the oldest methods to understand the phenomena and ancient mathematician including Newton used in the description.

(ii) Also, an article published in American journal of Physics [12] (2011) at page 1015 states that – *"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."*

It is justified that acceleration was defined by Galileo page no. 133-134, 146 of the book *Dialogue Concerning Two New Sciences*. Acceleration is change in velocity divided by corresponding change in time. Earlier Galileo has defined uniform velocity at page 128, and explained it as law of inertia in section The *Motion of Projectiles* at page 195 as

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

Thus Galileo (1638), Rene Descartes (1644), Huygens (1673) and Newton (1686,1713,1726) all explained motion in terms of uniform motion, not in terms of acceleration. Also, Newton did not explain acceleration in

Newton's second law of motion. Newton has simply written alteration or change in motion is proportional to impressed force.

It implies that Newton did not write F =ma for second law of motion.

(iii) Cohen [11] at page 113 has correctly written that

"Newton never actually made a formal statement of the second law in the algorithm of fluxions or the calculus."

It is true Newton did not write any equation for second law of motion. Newton's book The *Methods of Fluxions and Infinite Series* was published in 1736 after 9 years death of Newton. It is believed that Newton had completed this book in 1671 and published 65 years after completion. In this book also Newton did not mention dV/dt as acceleration.

Jacob Hermann seems to have given eq. (1) but Newton did not acknowledge it as equation for second law of motion even in third and final edition of *the Principia* in 1726. Thus, neither Newton acknowledged F =mdV/dt = m x acceleration (acceleration was clearly defined and explained by Galileo in 1638) nor acceleration in the third edition of the Principia. Newton has added and removed contents in other two editions of the Principia. But Newton did neither change the definition section nor axioms or laws of motion in the second and third sections.

(iv) V V Raman has published in an ace pedagogical or academic journal *The Physics Teacher* [14] in March 1972 issue at page 137…

"Although this remark was made over a decade ago, we still find textbooks in which F =ma is called Newton's formula, and which make absolutely no mention of Euler's in this context. "

Truesdell's findings [7] published in journal *Archive for History of Exact Sciences* in 1960 that Euler gave F=ma. Thus, Raman has pointed out that Euler's name should also be mentioned with F =ma. Euler has derived the relations between force, mass and acceleration for years i.e., 1736, 1749,1752 and 1765. In 1775 Euler derived equation $F=md^2x/dt^2$ without using Newton's second law of motion. However, V V Raman [14] has not mentioned contribution of Hermann in this regard. So scientific veils are revealed gradually.

The answer for V V Raman's query is that it is true that F =ma was never given by Newton and why the original discoverer (Euler) is not associated with it. It was repeated and highlighted by V V Raman [14] in *The Physics Teacher* published by *American Institute of Physics* after 12 years. Even about 50 years after publication in *American Institute of Physics* still F =ma is associated with Newton not with Euler. It is again stressed Euler has derived F =ma in 1775 and Newton never wrote. The equation F =ma follows from modified form of Newton's second law not from original form as given in the Principia.

The reason is that this issue was not fully discussed taking all historical and conceptual aspects in account with sole aim of clarification at one place. The conceptual and historical issues are to be discussed at length. This honest and impartial attempt is being made here for first time to reveal hidden truths. So we should teach correct concepts to our coming generations.

Also, Cohen [11] has written that in Chapter 10, pages 293 -370 as 'How to Read the Principia'. There is no mention of above concepts in it.

1.6 **Triple Point**

There are mainly three points. Why equation F =kdV (follows from definition of Newton's second law of motion) is neglected? Why the definition of second law of motion changed in the textbooks or standard references? Why Euler's and Hermann's names are not associated with F=ma ?

(a) Why the genuine form of second law of motion, F = kdV is neglected ?

The definition of Newton's second law of motion [1,2] is

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

The genuine equation based on Newton's second law of motion is

Change in motion or (dV) $\propto$ impressed force (F)

F =kdV

Its derivation is similar to law of gravitation , $F \propto m_1m_2$ and $F \propto 1/r^2$ or $F = G\, m_1m_2/r^2$, $F \propto dV$

The derivation of F =kdV is just like derivation of equation for law of gravitation. Newton did not give above equation in the Principia these were written afterwards by following scientists. The equation for law of gravitation is quoted in every textbook or reference book, but equation F =kdV is not quoted anywhere. Again Newton did not quote $F = G\, m_1m_2/r^2$ in *the Principia* but it is found in in text books and reference books. Then scientists related equation

F =ma

with Newton's second law motion (as F =ma) which was never given by Newton. Newton had neglected acceleration throughout his life. Thus, law must be properly studied as initial level. These equations have not been given by other scientists after death of Newton. The following scientists are responsible for inconsistency, fault or mistake not Newton. As Newton did not give any equation for second law of motion; then ignoring F=kdV, scientists interpreted it in arbitrary way. It is not justified, the equation for law of gravitation is given correctly.

(b) Why the definition of Newton's second law of motion as given in the Principia is changed in textbooks or standard references?

The definition is changed to justify that F = kdV does not follow from Newton's second law of motion, but F =ma follows. The definition of Newton's second law of motion is quoted in different ways than given in the Principia by Newton. The definitions are changed in form of acceleration, which is never quoted by Newton in the Principia.

1. The Encyclopaedia Britannica [15] states the second law of motion as

"The net unbalanced force producing a change of motion is equal to the product of mass and the acceleration of particle."

2. "The net (unbalanced) force acting on material body is directly and linearly proportional to, and in same direction as, its acceleration." [16]

3 'when the resultant force is not zero the body moves with accelerated motion, and the acceleration, with a given force, depends on property of the body known as its mass.' [17]

Thus in standard references the definition of second law of motion is changed so that

(i) genuine equation F =kdV does not follow from it.

(ii) the equation F =ma may follow from it.

Whereas Newton's first and third laws of motion are quoted in the same references as given in the Principia. Why only Newton's second law of motion is changed? Why genuine equation F =kdV which has origin like law of gravitation is neglected. The reason is that scientists wanted to get the equation F =ma for Newton's second law of motion.

(c) Another significant aspect in this regard is that why Euler's and Hermann's name are not quoted along with F =ma?

Euler had given various equations regarding force, mass and acceleration in 1736,1749,1752, 1765. In 1765 he had derived F =ma [18]. According to Cohen it also appears that Hermann had directly given equation F=mdV/dt at page 59 in his book *Phoronomia* in 1716.

As F =ma is inseparable part of physics so it can also be obtained if Newton's second law of motion as given in the Principia is modified or generalized.

1.7 Some genuine questions and generalization of second law of motion

Newton has been never given F =ma. Newton ignored acceleration throughout his scientific career. The genuine equation based on second law is F =kdV, which is neglected by scientists. F = ma has been derived by Euler in 1775 (E479 http://eulerarchive.maa.org/).

Scientists hurriedly associated F =ma with Newton's second law. When scientists tried to derive F =ma from second law then two assumptions are made; as genuine equation from second is F =kdV. The motion (basically velocity) is regarded as momentum (mV); as it does not solve the purpose then 'change in motion' is regarded as 'rate of change of momentum'. It is clear from eq. (5). It is again not consistent as discussed in section (2.1).

Now simple questions are …

(i) Why original definition of Newton's second law of motion as given in the Principia; is not quoted in the textbooks or standard literature?

(ii) Why original forms of Newton's first and third law as given in the Principia; are quoted in the textbooks and standard references?

(iii) Why genuine mathematical equation F=kdV is not even quoted in literature?

(iv)Why $F = Gm_1m_2/r^2$ is retained in literature and F =kdV (for second law) is neglected when both follow from 'method of proportionality" (**$F \propto m_1m_2$, $F \propto 1/r^2$** or $F = Gm_1m_2/r^2$ and **$F \propto dV$** or F =kdV) ?

(v) Why F =ma is associated with Newton's second law of motion when Newton had not derived it? Even Newton did not quote acceleration throughout his scientific career. So how F =ma is written?
(vi) Why name of Euler is not associated with F =ma when Euler had derived it in 1775?

(vii)Why mv-mu = d/dt (mv-mu) is regarded as true? It against all laws of every branch of science? Is it logical to obtain F =ma from definition of second law of motion flouting all laws of science.

It is not justified as both Left Hand Side and Right Hand side have different units (kgm/s & kgm/s^2) , dimensions (MLT^{-1} & MLT^{-2}) and magnitudes. It is discussed in section (2.1).

The answer to these questions lead to modification and generalization of Newton's second law of motion as (better to change definition of law rather than making series of inconsistent assumptions to obtain F =ma)

"*The rate of change of quantity of motion* (Quantitas motus) *with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Motive impressed force = rate of change of 'momentum with time'.
= rate of change 'quantity of motion or Quantitas motus **with** time.'

F = d(mv –mu)/dt

$F = dp/dt$ or $F = ma = mdV/dt = md^2x/dt^2$

Thus generalized form of second law of motion gives equation F =ma without any inconsistency.

2.0 Typical and critical status of Newton's second law of motion in the Principia.

In brief all relevant historical and conceptual facts since Galileo's experiments (1604) have been quoted in this book/monograph. All facts can be simultaneously understood. Thus, there is discussion from days of natural philosophy to modern physics relating to Newton's second law of motion.

2.01 Newton's First Law

"*Everybody perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon".*

Newton's first law of motion is improvisation of Galileo's law of inertia given at page 195, (Third Day) of the book, Dialogues Concerning Two New Sciences [3], published in 1638, Rene Descartes second law of motion published in *Principles of Philosophy* (1644) and Christiaan Huygens [5] first hypothesis published in book *Horologium oscillatorium sive de motu pendularium* in 1673. These subjects are discussed by various scientists at different stages.

2.01 Newton's second law

"*The alteration of motion is ever **proportional to** the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

In Latin

Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimatur.

Newton did not give any equation for second law of motion.

(i) Contribution of Jacob Hermann : Jacob Hermann [7] in 1716, has given equation directly without derivation in his book *Phoronomia* at page 59. Cohen [11] has commented about this at page 113

"Newton never actually made a formal statement of the second law in algorithm of fluxions or the calculus. The first person to do so seems to have been Jacob Herman in his Phoronomia (1716), in which he writes

G = MdV : dT

where he says G signifies weight or gravity applied to a variable mass M. ".

This equation has been written directly by Hermann.

Cohen has related the equation with Newton's second law of motion, so G (weight or applied gravity) can be regarded as force (impressed force F, say). So if G (weight or gravity) is replace by force F (impressed force, say) then we get equation for force F as

$G = MdV/dt = F$ (1)

Here dV/dt is ratio of change in velocity and time. Galileo had defined acceleration at pages 133-134, 146 in the book *Dialogue Concerning Two News Sciences* published in 1638 in eq. (26). So, this equation is independent of Newton's second law of motion.

Thus according to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann. Thus eq. (1) follows from Hermann's direct interpretation as given in his book *Phoronomia* at page 57 published in 1716.

(ii) Also, Swiss Leonhard Euler (a distant relative of Jacob Hermann) derived various equations between force, mass and acceleration in 1736, 1749,1752, 1765. Also Swiss Leonhard Euler (a distant relative of Jacob Hermann) a distant relative of Hermann) in 1775, derived equation [18] in research paper in *Novi Commentarii academiae scientiarum Petropolitanae* at page 222-223

$F = md^2x/dt^2 = ma$ (2)

This issue to be logically justified due to two main reasons.

Firstly, Newton did not write F =ma in any of three editions of the Principia (1686,1713,1726).

Secondly, Newton neglected or ignored acceleration for 85 years of his life. Acceleration (change in velocity /change in time) was given by Galileo in his book *Dialogue Concerning Two New Science* in 1638 at pages 133-134 , 146.

The prevalent form of mathematical equation for Newton's second law is

$F = \text{mass x acceleration} = ma = mdV/dt = md^2x/dt^2$ (3)

Some prudent authors quote above definition of Newton's second law of motion in different ways (directly gives dependence of acceleration) so that eq.(3) may be obtained. However, it must be carefully noted that Newton neither quote acceleration nor gave any equation for second law of motion. Apparently, it is not justified that to quote equation of force for second law arbitrarily if Newton did not give any equation.

2.1 How definition of Newton's second law arbitrarily leads to F =ma?

F =ma is prevalent form of second law of motion. To obtain F =ma from second law of motion; scientists assume motion is momentum, mV (quantity of motion). However, it is discussed below that motion is velocity not momentum (mV). Thus derivation of F =ma from definition of Newton's second Law of motion is understood under two assumptions.

(i) Purposely the scientists assume

Motion = mV (4)

According to definition,

Impressed force $\propto$ change in momentum

But the equation F =ma is not obtained from above proportionality.

(ii) To obtain F =ma from definition of second law of motion, scientists arbitrarily assumed that 'change in motion' (mv-mu) is equal to 'rate of change of momentum with time', [d/dt(mv-mu)].

Alteration or change in momentum = rate of change of momentum with time

mv-mu = d/dt (mv-mu) (5)

m : mass of body , u initial velocity , v final velocity

2.11 Contrast in comparison

It is justified as both Left Hand Side and Right Hand side have different units (kgm/s & kgm/s^2) , dimensions (MLT^{-1} & MLT^{-2}) and magnitudes. These basic issues of physics /science cannot be ignored just to obtain pre-supposed result F =ma from definition of second law of motion. The logics are supreme in science.

Left Hand Side, eq.(5)	Right Hand Side , eq.(5)
Units : kgm/s	Units : kgm/s^2
Dimensions : MLT^{-1}	Dimensions : MLT^{-2}
Magnitude : mv-mu	Magnitude : d/dt (mv-mu)

Thus equality of equation (5) is not justified , hence the equation is arbitrary. But this arbitrary equation is used by scientists to obtain, F =ma.

According to Newton's second law,

F (impressed force) $\propto$ mv-mu

As mv-mu is regarded as ratio of rate of change of momentum .

or F (impressed force) $\propto$ d/dt (mv-mu)

or F (impressed force) $\propto$ d/dt (mv-mu)

F = K md/dt (v-u) = K ma (6)

Thus eq.(5) is based on inconsistent and arbitrary assumptions. Thus equation F =Kma is based on inconsistent assumption.

The value of K is regarded as unity in eq. (6) for defining unit force.

F =ma (1)

From the critical analysis we find that

(i) Newton did not write F =ma

(ii) Newton did not mention acceleration in his 85 years long time. Acceleration was discovered and explained by Galileo in 1638 i.e. 4 years before birth of Newton. So practically acceleration was ignored by Newton but acceleration was present in the existing literature in Newton's time. Newton did not discuss acceleration even

from the literature.
(a) The genuine equation based on second law of motion is F =kdV i.e., eq. (11), this equation is not discussed at all. As F =ma was hurriedly associated with Newton's second law of motion so scientists tried to derive it from the definition of the second law (change in motion $\propto$ impressed force). This association was done by scientists after death of Newton, so Newton cannot be held responsible for any inconsistency regarding this. Thus, some assumptions were made by following scientists.
(b) Firstly, scientists assumed that motion is nothing but momentum. Thus we get
Impressed force $\propto$ change in momentum (mv-mu)
Impressed force = k (mv-mu)
But it does not lead to F =ma
(c) Then scientists assumed (so that F=ma may be obtained from definition of second law of motion) that change in motion is equal to rate of change of momentum as in eq.(5) i.e.
mv-mu = d/dt (mv-mu)
But this assumption is inconsistent as justified above.
(d) Then scientists tried to change the definition of second law of motion as given in the Principia. The definition was changed in terms of acceleration. It is clear from section (7.1). It must be noted that Newton has ignored acceleration throughout his life. The definitions of first and third laws are quoted in the existing literature as given by as given in the Principia.

2.2 First objection on Newton's second law by Walter William Rouse Ball

Then Cambridge mathematician W W Rouse Ball did not apparently agree with eq.(5) i.e. 'change in momentum is equal to rate of change of momentum'.
Rouse Ball meant that 'change in momentum' is not equal to 'rate of change of momentum with time' . Thus both sides cannot be equated , hence
mv-mu $\neq$ d/dt (mv-mu) (5ieq)
It implies that F $\neq$ ma.
Rouse Ball [19] In his book '**An Essay on Newton's Principia'** published in 1893 at page 77 quoted that Newton's second law of motion in the form
*The change in momentum **[per unit of time]** is always proportional to moving force impressed and takes place in direction in which force is impressed.*

So, Ball altered definition of Newton's second law of motion by introducing phrase 'per unit of time'. To derive F=ma from definition of Newton's second law, 'change in motion' is regarded as equal to 'rate of change of momentum'. It is arbitrary. However Rouse Ball divided change in momentum with total time.

2.3 I Bernard Cohen

I Bernard Cohen [11], American Historian of Physics (First PhD of America , in of history of Science from University of Havard in 1947) has quoted in opening paragraph at page 111 in section (5.3) title, 'The Second Law: Force and change in motion', in A Guide to Newton's Principia.

Newton's second law, as stated in the Principia, sets forth a proportionality between a impressed 'force' and resulting "change in motion", by which Newton means change in quantity of motion or change in momentum.

Cohen wrote at page 111

"Since it is not the more familiar version of the second law, in which a force produces an acceleration or a change in momentum in a given (or unit) time , some writers have seen a need to introduce a correction to Newton's statement of the law."

But Cohen objected to Rouse Ball's altered definition and stated that Ball had not understood the Newton's second law of motion properly. Cohen objected to Ball's perception at page 111 in the following the statement,
"It apparently never occurred to him to try find out what Newton meant rather than to introduce "per unit of time."
Cohen's above statement meant that Rouse Ball did not understand the second law properly while dividing by the time (so obtained 'per unit time'.)

Cohen[11] has given solution to the problem by giving four equivalent forms or equations (equal in value, amount, function, meaning, etc.) of Newton's second law of motion. Earlier we had just one equation for second law of motion as F $=md^2x/dt^2$ =ma. Cohen did not justify the need and advantage of four equations for second law of motion. But Cohen also divided by time 'dt' (constant) and Rouse Ball had introduced phrase 'per unit of time'. This issue is discussed in section (11.2). Now one wonders how division by time as done by Rouse Ball is incorrect and division by Cohen (arbitrarily) is correct**?** When Rouse Ball divide with total time, then equation F $=md^2x/dt^2$ is obtained when Cohen arbitrarily divides with dt, then four equations (F =kdV, F = k_1d(mV), F = KdV/dt, F = k_2 d(mV)/dt) for second law of motion is obtained. How it is logical to get four equations with different dimensions instead of one equation (F $=md^2x/dt^2$)?

2.4 Author's modified form of second law of motion

If the all facts are considered impartially then it may not be prudent to regard eq. (5) scientifically correct. Thus the definition of Newton's second law of **motion [20]** is extended or modified by author as

"*The rate of change of momentum with time **is equal to** the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Mathematically,

Impressed force = rate of change of momentum

F = d/dt (mv-mu) = m d/dt (v-u) = ma (1)

This solitary issue of correctness or incorrectness of derivation from definition of Newton's second law of motion as F =ma is completely discussed in various sections here. This issue is further substantiated by the fact Newton did not write equation F =ma and he neglected or ignored acceleration for 85 years of his life. Acceleration was given by Galileo in 1638 i.e. 4 years before birth of Newton. So practically acceleration was ignored by Newton; as it was present in the existing literature. So, it should have been discussed regarding motion of bodies. Newton has refined the law of inertia as first law of motion. The law of inertia was given in the Galileo's Dialogue (p.195) and acceleration is also given in the same book (p.133-134, 145).

2.5 Newton's third law

"*To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts".*

Newton published the first edition of the Principia in 1686, at that time there was no tradition or precedence for writing equations. Newton did not write even the simplest mathematical equation for the third law

Action = **-** Reaction (7)

Thus definition of the law and equation are applicable universally i.e. it is equally applicable in all cases. There is no constraint on the applicability of the law i.e., eq. (7). However, in some typical cases the relevant effects need to be taken in account for critical and complete understanding of the law.

Effect of shape of bodies or artifacts (spherical, semi-spherical, umbrella shaped, triangular, square, cone, long pipe, flat, irregular etc.) in Newton's Third Law of Motion.

For ideal considerations the effects of some characteristics can be minimised or vanished completely but different *shapes* cannot be ignored; this issue is debated in international conferences [21,22] in theoretical discussions. Thus, bodies of rubber or plastic (precisely and precisely having same compositions, characteristics etc.) but of different shapes (spherical, semi-spherical, umbrella shaped, triangular, square, cone, long pipe, flat, irregular etc.) can be considered [23] if bodies have different shapes. For example, we may consider bodies of precisely -2 same plastic but of different shapes e.g. (spherical, semi-spherical, umbrella shaped, triangular, square, cone, long pipe, flat, irregular etc.).

If the composition is precisely -precisely same then various factors become uniform, and only *shape* is the significant factor [24]. Thus, shape of body or artifact is adjustable. Then the magnitudes of action and reaction can be considered when bodies or artifacts of different shapes collide with floor (suitable composition) and rebound. According to third law action and reaction must be same for bodies of all shapes, as eq.(7) implies action and reaction are universally same irrespective of any constraint.

The eq.(7) does not contain any factor which may account for *shape* of body, thus action and reaction has to be precisely same. There is no other third factor in eq. (7) other than action and reaction. The quantitative results of such observations verifying eq. (7), are not available in the existing literature. So both experiments and critical discussion are noble. In case in experiments involving bodies of precisely -2 same composition, but of different shapes can be considered in experiments. If results are different then effect of shape in third law of motion will be confirmed i.e. we need say that reaction is proportional to action. Then additional coefficient [24] in eq. (8) will account for shape of body and other relevant factors (if experimentally confirmed). The generalized form of third law of motion theoretically or speculative way is

Action =Q Reaction (8)

where Q is coefficient of proportionality and its value will depend on experiments. Newton's third law is established law over 335 years. Apparently, the measurements of k and Q (constant or coefficient) must be same scientists measure the gravitational constant **G [25].** All these results due to critical analysis of Newton's Principia.

It is added that the most sophisticated experiments are being conducted to test third law of motion. These are based on speculations of British Engineer Roger Shawyer in 2000 that it is possible to launch a rocket with help of microwaves. The instrument is called EM Drive (A radio frequency (RF) resonant cavity thruster) or it is known as Impossible Drive. In simple words rockets can fly without exhaust (backward gases, smoke, fire, sparks etc.); thus there can be reaction without reaction. It would mean failure of Newton's Third law of motion. After facing huge resistance for the proposition, NASA' s results were published in paper **[26]** *Measurement of Impulsive Thrust from a Closed Radio-Frequency Cavity in Vacuum* in Journal Propulsion and Power (July -August 2017).

"Thrust data from forward, reverse, and null suggested that the system was consistently performing with a thrust-to-power ratio of 1.2 ±0.1 mN/kW."

Thus Roger Shawyer's speculation was justified but it is subjected to many sources of errors. The possible sources of errors are mentioned in the paper by authors. Now many scientists all over the world especially German scientists are at verge of completely experiments removing all possible sources of errors, but the generalized form is capable of explaining such results with help of coefficient Q. In case all the sources of errors are removed only then results would be acceptable and scientists are proceeding in this direction.

The Defense Advanced Research Projects Agency (DARPA) is a research and development agency of the United States Department of Defense responsible for the development of emerging technologies for use by the military. DARPA has invested over a million US Dollars on this project. However, the experiments suggested by author to check shape dependence of third law of motion. Both the utmost sophisticated experiments and shape dependence of third law of motion lead to same conclusions finally.

2.6 First and third laws are written in reference book as given in the Principia Why second law of motion is written in other way than given by Newton?

First and third laws of motion are quoted in the form in the existing literature (text or reference books) as given by Newton in the Principia. For third law of motion eq. (7) is used. But there is no such specific equation for first law of motion. But the original form of Newton's second law of motion as given in the Principia is not quoted in textbooks or reference books. Or it must be explained why Newton's second law is altered? Further along with altered form of Newton's second law by prudent authors, the original definition as given in the Principia is not quoted. Both the altered and original forms of second law of motion must be quoted logically simultaneously.

Then it must be justified conceptually and mathematically that altered form (given by authors in reference or text books) is same as that of Newton's original form. Or it must be quoted why Newton's law is altered? If it not so then altered form would be author's own law not Newton's. Thus, unethically name of law (definition is changed by authors) should not be given to Newton which he has never given. Thus reader may understand both i.e. forms of law given by authors and Newton in the Principia. Thus, it is noble discussion.

The best and the most ethical way to get rid of this issue is that prudent authors should quote Newton's second law of motion in text or reference books as given in the Principia; like Newton's first and third laws of motion. Strictly speaking Newton's second law of motion gives equation F =kdV (it is obtained in the way $F = m_1m_2/r^2$ is obtained i.e., $F \propto m_1m_2$, $F \propto 1/r^2$ and $F \propto dV$) but prevalent form is F =ma. As Newton's second law of motion given by Newton in the Principia, does not lead to F=ma (prevalent form of the law globally); so, in next measure the definition of the law is altered by authors in the literature.

If prudent authors quoted original form of the Principia, then it leads to F =kdV not F =ma. Thus there would be confusion among the students. So prudent authors quote altered form second law of motion (on behalf of Newton, not modified form of Newton). However, there is no such case with first and third law of motion, hence so it is quoted in the same way as given in *the Principia.*

Further Newton's first law of motion is qualitative, and no mathematical equation is given to it by preceding scientists i.e., Galileo (1638), Descartes (1644), Huygens (1673), Newton (1686,1713,1726) and other scientists. Thus the first law of motion is being quoted in original form (refined form of law of inertia).

Law of Gravitation: Newton gave the law in form of propositions (I-X), in Book III of the **Principia [9]**
"The force of attraction between two bodies (particles) is directly proportional to product of their masses and inversely proportional to square of distance between their centres."

Newton did not give this definition in the Principia, Newton explained the law in various propositions as mentioned above. Then result of various propositions is combined as above after Newton's death (this issue needs different and separate discussion), then law of gravitation is obtained. Also, Newton did not give any mathematical equation for law of gravitation in the Principia. The equation is self-explanatory from definition.

$F \alpha m_1m_2 \quad F \alpha 1/r^2 \quad \text{or} \quad F \alpha m_1m_2/r^2$

$$F = G\, m_1m_2/r^2 \tag{9}$$

where terms have usual meanings. m_1 mass of one body, m_2 mass of second body and r is distance between their centres. Newton did not give this equation for the law in the Principia. The constant of proportionality G depends on experimental factors and determined experimentally **[25].**

$$G = 6.6743 \times 10^{-11}\ m^3kg^{-1}\ s^{-2} \tag{10}$$

If value of G is like constant K (both constants of proportionality) in Newton's second law of motion i.e. unity then all data pertaining to heavenly bodies will vary. Thus, it would be interesting to measure value of K like G experimentally under different conditions. But it is regarded as unity. Both are examples in Newtonian Physics.

There should be consistent method for measurement of constants.

2.7 Newton did not give any mathematical equations in the Principia
There was no precedence of writing equations in Newton's time or before

In early days of physics laws were interpreted qualitatively and philosophically. We cannot expect everything (theoretical concepts and experimental data) at the inception, beginning or origin of physics (ancient scientists have started from the state of cipher to develop modern day sophisticated science). The geometrical methods were used for interpretation of the law not mathematical (as such methods were not discovered yet) in the earliest days. Thus, in early days of physics there was no tradition of changing proportionality to equation. Newton has expressed his perceptions in form of proportionality not equation. Thus, science or physics developed gradually.

At that time there was no tradition of writing mathematical equations, even in related works of Galileo [3] in 1638, Descartes [4] in 1644, Huygens [5] in 1673 indicate this. Newton's first law of motion is simply conceptual and philosophical statement. Newton did not justify quantitively the third law of motion. Neither Galileo (1638), Descartes (1644) Huygens (1673) and Newton (1686,1713,1727) nor modern scientists calculated the uniform velocity (Galileo mentioned *uniform and perpetual motion).* Should there be relation between uniform perpetual velocity, mass and applied force or not for proper quantitative understanding of the law. So, Newton's first law of motion, is as qualitative as Galileo's inertia.

With help of mathematical equations, the quantitative predictions are made thus law is specifically studied. However equations for second law of motion (F =kdV or prevalent F =ma) , third law of motion (action =-reaction, negative sign indicates direction as force is vector quantity), law of gravitation ($F = GmM/r^2$), centripetal force ($F_c = mv^2/r$) for speed of sound in media , Newton's law of cooling etc. were given after Newton's death by following or succeeding scientists. Further scientists will keep critically analyzing the laws in view of theoretical and experimental findings.

2.8 Genuine equation of force based on second law of motion.

The definition of Newton's second law of motion establishes proportionality between force and alteration or change in motion (velocity). It is the simplest deduction even for school level students (from Principia's definition of the law). It can be changed to mathematical form as in law of gravitation as eq. (9), as only proportionality is involved. Here we regard motion as velocity, this aspect is elaborated more clearly in sections (3.8).

Scientists have written equation for law of gravitation as given in eq. (9), depending upon simple proportionality method. The scientists did not interpret the same in proportionality form

$F \propto$ change (alteration) in motion

(This proportionality is similar to law of gravitation i.e., $F \propto m_1 m_2$, $F \propto 1/r^2$)

It is explained in the following discussion that motion is just other name of velocity or motion is represented mathematically represented by velocity. In early days of physics as mathematical equations were not prevalent so velocity was represented in motion.

F = k change in motion or F =kdv

$$F = kdV \qquad (11)$$

2.81 Complete derivation of eq.(11) from definition and explanation of second law of motion.

For complete derivation or understanding let us discussed how Newton explained or demonstrated the law , see Principia at page 20

"If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and progressively" .

In original Latin,

"Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, ***sive simul & semel,*** *sive* ***gradatim & successive*** *impressa suerit."*

"sive simul & semel" means or at once

' sive gradatim & successive " means or gradual and progressively

Newton has stated that higher the impressed force (acting at once or gradual and progressively), higher would be change in velocity. If impressed force F changes velocity by V/2, then force 2F will change velocity by V and triple force will increase velocity by 3/2V. Newton has repeatedly ignored word acceleration throughout his scientific career. Newton did not relate force with acceleration.

F (impressed force *at once, or gradually and progressively)* $\propto$ alternation or change in motion (velocity)

F (impressed force *at once, or gradually and progressively)* = kdV (11)

F (impressed force simul & semel, sive gradatim & successive) = kdV

It is simply written as F. In the definition F is impressed force.

$$F = kdV \qquad (11)$$

Thus eq.(11) implies that force may act on body at once or gradually and progressively i.e. force may act for short or long time. Thus, Newton's law is applicable whether force acts for smaller or longer time. Newton did not give two definitions of the law i.e. when force acts at once or gradually and progressively. In both cases definition is the same.

The definition is equally applicable whether force acts *at once, or gradually and progressively i.e.* simul & semel or gradatim & successive. Thus, obviously equation must be the same. However theoretical law can be speculated in any way.

Thus for all cases (force may act for short or for longer time) the definition of the law is the same i.e. impressed force is proportional to change in motion or velocity, and hence mathematical equation is given by eq. (11). In the simplest way it can be written as

$$F = kdV \quad (11)$$

2.9 If motion is regarded as mV

In case we regard or speculate motion as mV (it is explained in the following discussion that motion is mathematically represented by velocity) even then eq.(11) is obtained.

F (impressed force *at once, or gradually and progressively)* $\propto$ alternation or change in motion or m(v-u)
m is constant in in classical physics thus the proportionality hold good.

F (impressed force *at once, or gradually and progressively)* $\propto$ (v-u)

In determination of unit force , the mass of body is considered unity

F (impressed force *at once, or gradually and progressively)* = kdV (11)

F (impressed force simul & semel, sive gradatim & successive) = kdV

$$F = kdV \quad (11)$$

2.91 Further categorization of impressed force by I B Cohen

Further Cohen [11] has categorized impressed force as impulsive force (time dependent) and continually acting force (time independent) and written four equivalent forms (equal in value, amount, meaning, importance etc.**)** of second law of motion. Cohen [11] at page 116 gave four equivalent forms or equations for second law of motion

$F \propto dV$ or $F = kdV$ (11) $F = k_1\, d(mV)$ (12)

$F \propto dV/dt$ or $F = K_1 dV/dt$ (13)

$F = k_2 d(mV)/dt$ (14)

This aspect is discussed in section (11.0).

2.92 **Prevalent equation for Newton's second law motion**

The prevalent equation for Newton's second law of motion is given by

$$F = Kma = Km\, dV/dt \quad (6)$$

where K is constant of proportionality. The value of K is regarded as unity to define unit of force newton or dyne. If using same argument value of G is regarded as unity, then heavenly parameters will vary. Should there be same rules for estimating constants of proportionality or the constants may be interpreted arbitrarily for different laws? The scientific logic implies consistent methods for measurement of constant of proportionality. The eq.(13) is obtained by applying arbitrary conditions.

Nature of constant of proportionality K

All the constants of proportionality must have consistent and logical methods of determination. There should not be different methods for determination of constants in different laws. In universal law of gravitation, G is universal constant and its value is experimentally determined as in eq.(10) equal to $6.6743 \times 10^{-11}\, m^3 kg^{-1} s^{-2}$.

The value of constants or coefficients of proportionality must be calculated experimentally as in other various laws e.g. force due to Coulomb law, force due to viscosity, force of friction, gravitational force, coefficient of thermal conductivity, Hubble's constant, Resistivity in Ohm's law etc. etc. This interpretation must be equally same for various values of k_i's. i.e., eq. (6), eq. (11-14). The values of k, k_1, k_2 and K must be different when LHS is same. Likewise, the dimensions will also vary.

The value of K is determined in eq. (6) by simple assumption. If mass is kept constant (say, 1kg) for standardization, then value of unit of force is determined (newton) then mass and acceleration both are assumed to be unity. The nature of resistive forces of the system is not mentioned, thus ideal system is considered. Newton's first law of motion is just other form of law of inertia. If acceleration 2 m/s^2 is produced in body of ½ kg even then force would be unity (1newton).

For same mass of body, the nature of surface, shape of body, etc. can be different. For standard conditions the shape of body can be chosen square or round etc., the point of impact should be in the middle; thus, various standard conditions may be chosen. However, now the force is determined in terms of acceleration and mass only. The unit of **dyne [27]** was defined in 1874 and that of newton in **1948 [28],** whereas the Principia was published in 1686.

2.10 Galileo's inertia, Descartes' law and Huygens Hypothesis preceded Newton's laws

(i) Galileo's interpretation of Law of Inertia (1638)

Galileo has given the law of inertia (generally known) in his book *Dialogue Concerning Two New Sciences* [3] in 1638 at page 195 in section The Motion of Projectiles.

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

It implies body moves with perpetually uniform velocity in straight line in a system without resistive forces. So, in this case acceleration of body is zero. Galileo has clearly indicated system without resistive forces i.e. ideal system. Thus, under such conditions Galileo meant uniform motion is natural state of motion. Galileo did not mention anything about mass and external force of body. So it is true under all condition. Thus, Galileo's law of inertia is qualitative statement. Currently it is obtained from second law of motion when impressed force is zero or does not act on body.

Whereas according to Aristotle (322-384 BC) mover is required for movement of body; as table stops moving on the floor as external force ceases to act. It was debated for centuries. Galileo distinguished between systems which are friction less or devoid of resistive forces and other systems which possess resistive forces. Aristotle has given example of the system when resistive forces are present. This topic is further discussed in section (1.4) along with impressed force.

It must be noted that Galileo has explained acceleration in *Dialogues Concerning Two New Sciences* [3] in 1638 at pages 133-134, 146. Thus, acceleration was defined 4 years before birth of Newton. But Newton did not use or mention acceleration throughout his life of 85 years. So, Newton ignored already existing acceleration.

Galileo has explained the law of inertia in terms of uniform motion (however he had discovered acceleration); as it was the simplest to explain the doctrines in terms of uniform motion. Galileo did not use acceleration in motion of bodies as Galileo put forth law of inertia using uniform velocity. The uniform velocity is explained at page 128 and law of inertia at page 195 of the same book i.e. Dialogue Concerning Two New Sciences.

(ii) **Descartes' second law of motion** (1644)

Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. (Principles Part II, article 39).

Descartes [4] has given his three laws of motion in his book **Principles of Philosophy** (1644). It implies body tends to continue moving in straight line. Descartes indirectly implies that body moves with uniform velocity. Descartes did not mention about variable velocity or Descartes did not use acceleration.

(iii) **Descartes' third law of motion**

When a moving body collides with another, if its power of continuing in a straight line is less than the resistance of the other body, it is deflected so that, while the quantity of motion is retained, the direction is altered; but if its power of continuing is greater than the resistance of the other body, it carries that body along with it, and loses a quantity of motion equal to that which it imparts to the other body. (Principles Part II, article 40). Apparently Newton's third law of motion (1686) is more refined and compact form of third law of motion given by Descartes (1644). Thus, Descartes has given his third law of motion 42 years before Newton.

Like Galileo, Descartes did not discuss motion in terms of acceleration, as law of inertia was explained in terms of uniform velocity.

(iv) **Huygens' hypothesis (1673)**

Christiaan Huygens [5] published book *Horologium oscillatorium sive de motu pendularium* in 1673. It contains three hypotheses in second part at page 21.

Hypothesis I

If there is no gravity, and the air offers no resistance to the motion of bodies, then any one of these bodies admits of a single motion to be continued with an equal velocity along a straight line.

It implies if body is set in motion it remains in straight line with uniform velocity (equal velocity at every point) provided resistive forces are absent from the system.

Thus the simple conclusion of all these three deductions by Galileo, Descartes and Huygens is that body tends to move in straight line with equal or uniform velocity. Like Galileo and Descartes; Huygens did not discuss acceleration in terms of motion as Galileo has described law of inertia in terms of uniform velocity. Galileo himself has described acceleration 4 years before birth of Newton but Newton did not quote the same.

(iv) Newton's Principia (1686)

Descartes (1644) has given three laws of motion and Christian Huygens (1673) has given three hypotheses. Likewise, Newton (1686) gave three laws of motion (as described above) in section (2.0).

Like Galileo Descartes, Huygens and Newton did not discuss acceleration in terms of motion. Galileo has expressed law of inertia in terms of uniform velocity. Likewise, Newton did not describe the motion in terms of acceleration. Galileo has given acceleration in 1638, but Newton did not mention it throughout his life. Now acceleration is life line of mechanics.

As all three Galileo, Descartes and Huygens used law of inertia in form of uniform motion, Newton did the same. Newton's first law of motion is other form of law of inertia. Newton changed the kinematical system (given by Galileo, Descartes, Huygens etc.) to dynamical form. Newton changed the system to dynamical form i.e. discussed the motion in terms of force in second law of motion.

Newton did not give or write equation F =ma in the Principia. Acceleration was found very useful physical quantity, when differential and integral calculus was developed. Then acceleration was associated with second law of motion F =ma.

Now scientists associated acceleration as F =ma after second law of motion after death of Newton inconsistently. So it is not mistake or fault of Sir Issac Newton but of the following scientists. Newton neither gave F=ma nor mentioned acceleration throughout his scientific career. This is scientific reality, F =ma is associated with Newton's second law of motion after death of Newton. Newton did not change the definition of second law of motion (along with first and third law) in all three editions of the Principia (1686,1713,1726) i.e. continuously 40 years even in 1727 (when the legend breathed his last). But it is explained in section (7.1) that scientists have changed the definition of Newton's second law of motion on their own and called it definition of Newton's second law of motion. But the definitions of first and third laws of motion are kept same as in the Principia.

2.11 **When F =ma was credited to Newton's second law of motion**?

Thus according to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann. Thus eq. (1) follows from Hermann's direct interpretation as given in his book *Phoronomia* at page 57 published in 1716. Jacob Hermann had given equation in his book *Phoronomia* at page in 1716 which Cohen interpreted F =dV/dt. Also, Euler derived F $=md^2x/dt^2$ =ma in 1775. Newton published third and final edition of the Principia in 1726 but did not mention or acknowledge F =mdV/dt as equation for the second law of motion in the Principia. Newton died in 1727 , thus there is no possibility that Newton ever wrote F =ma . Further Newton did never wrote acceleration in life time . It is also justified by various quotations by scientists as

I Bernard Cohen [11] has mentioned at pages 116-117

"Newton did not write any equation for his laws."

Also an article published in American journal of Physics [12] (2011) by Bruce Pourciau at page 1015 states that –

"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."

I Bernard Cohen [11] has correctly mentioned at page 113 that

*"Newton never actually made a formal statement of the second law in the algorithm of fluxions or the calculus.***"**

Here the oldest pedagogical book available for discussion is published in 1871. It is titled '**First Three Sections of Newton's Principia'[28]** designated as Cambridge School and College Text Books published from **London.** The authors of book are John H Evans and P T Main. It quotes definition of Newton's second law of motion as given in the Principia, but F =ma has not been quoted at all even in 1871. Acceleration was discovered by Galileo and published in 1638 in his book *Dialogues Concerning Two New Science* at page 133-134, 146. And equation F $=md^2x/dt^2$ was derived by Euler in 1775 in paper E479 **[13]** in paper *Novi Commentarii academiae scientiarum Petropolitanae* 20, 1776, pp. 208-238 (http://eulerarchive.maa.org/).

So, F =m d^2x/dt^2 was not quoted as equation for second law of motion in the book **First Three Sections of Newton's Principia,** 1871. Otherwise it (equation for second law as F =ma) would have been mentioned in textbooks for students (meant for Cambridge School and College textbooks). The other references can be searched in literature. Apparently books available in libraries of Universities of Cambridge and Oxford, and other renowned institutions would be helpful in this regard. The acceleration 'a' is defined by eq.(27) as given by Galileo in the Dialogues in 1638. For long time its extensive discussions and applications were not considered. But it was readily recognized when applications of differential and integral calculus were developed. Then F =ma was related with Newton's second law of motion. The learning of science is continuous process. We should not expect everything was prevalent at inception of physics.

2.12 Updated form of Newton's second law of motion (2020)

Considering the origin and development of Newton's second law of motion; the law has been updated. This aspect is discussed in section (14.0). Table I shows works of various scientists.

Table I: Contributions of Galileo, Descartes, Huygens and Newton in acceleration and uniform velocity.

Sr. No	Scientist	Book	Year of Publication	Acceleration a = change in velocity	Uniform velocity or

				/change in time	Law of inertia
1	Galileo	Dialogue Concerning Two New Sciences.	1638	Discovered but did not use it.	Discovered and used it
2	Descartes	Principles of Philosophy	1644	Neither discovered nor used it	Did not discover but used it
3	Huygens	Horologium oscillatorium sive de motu pendular	1673	Neither discovered nor used it	Did not discover but used it
4	Newton	Mathematical Principles of Philosophy.	1686	Neither discovered nor used it	Did not discover but used it

Note: Galileo has discovered both acceleration and uniform motion in 1638. However, he had conducted experiments in 1604. Galileo applied uniform motion in Law of Inertia, but Galileo did not apply acceleration or discussed accelerated motion. The reason is that it was the simplest case when law is expressed in terms of uniform motion. In 1644 Descartes used law of inertia in his second law of motion in same sense. In 1673 Huygens used laws of inertia as first hypothesis. Then in 1686 Newton used law of inertia (persistence of body in uniform motion) in first law of motion in more compact and refined way. Newton expressed first law of motion, impressed force and innate force in terms of uniform velocity.

It must be noted that none of four i.e. Galileo, Descartes, Huygens and Newton applied acceleration in describing motion as uniform velocity is used in law of inertia. However, Galileo had discovered acceleration in 1638. Newton related force with uniform motion. In second law of motion Newton related force with change in velocity ($F \propto$ change in velocity). Afterwards scientists related force with acceleration i.e. F =ma is mathematical equation for Newton's second law of motion. It is explained in sections (2.1). However genuine equation for second law of motion is F =kdV.

3.0 Some important Definitions in Newton's Principia

(i) **Definition I Quantity of matter (Quantitas Materiae)**

The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.

Quantitas Materiae est mensura ejusdem orta ex illius Densitate & Magnitudine conjunctim

Thus air of a double density, in a double space, is quadruple in quantity; in a triple space, sextuple in quantity. It is this quantity that I mean hereafter everywhere under the name of body or mass. It is clearly stated by Newton at page 1 of the Principia.

"It is this quantity that I mean hereafter everywhere under the name of body or mass"

Thus Newton clearly stated that he would regard 'quantity of matter' as body or mass.

(ii) Definition II Quantity of motion (Quantitas motus)

Definition II Quantity of motion (Quantitas motus) page1 of the Principia

'*The Quantity of Motion is the measure of the same, arising from the velocity and quantity of matter conjunctively.*'

Quantitas motus est mensura ejusdem orta ex Velocitate et quantitate Materiæ conjunctim.

The 'Quantity of Matter' is defined in DEF.I of *the Principia*. It is very important term in mathematical form of Newton's second law of motion.

It must be clearly noted that Newton did not write for 'quantity of motion', like for 'quantity of matter' as *"It is this quantity (quantity of matter or* quantitas motus) *that I mean hereafter everywhere under the name of motion (motus)"*

This is required to be kept in mind in further explanation about meaning of quantity of motion.

Further Newton has explained motion in Scholium of the same section at page 9 that motion is already explained by various scientists so it is known quantity (not quantity of motion, as it is new definition). Further Newton categorized motion in terms of

(i) absolute motion

(ii) relative motion

Newton defined both separately obviously under different conditions, and both are velocities (when expressed in mathematical equations).

Thus Quantity (amount or quantum or magnitude) of motion is the product of mass and velocity, explained in the *Principia*. Andrew Motte translated quantitas motus as 'quantity of motion' not motion-quantity or quantity-motion (magnitude-motion). Newton did not give any equation. It can be written as

Quantity of Motion (Quantitas motus) = Quantity of Matter × velocity = mV (15)

= 10kg x 40 m/s = 400 kg m/s

So quantity of motion is mv , quantity of velocity or motion is nothing but velocity.

For more clarity eq.(15) can be written as

Quantity or Quantitas (amount or magnitude quantum) of Motion = Quantity of Matter × velocity =mV (15)

Further now we understand
Momentum = mV (16)
Thus quantity of motion is amount or magnitude or quantum of motion ; and motion is movement or velocity when expressed in terms of mathematical form or equations.

Scholium at page 9 of the Principia

Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse. I do not define time, space, place and motion, as being well known to all. Only I must observe, that the vulgar conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which, it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.

In Latin

Hactenus voces minus notas, quo in sensu in sequentibus accipiendæ sunt, explicare visum est. Nam tempus, spatium, locum et motum ut omnibus notissima non definio. Dicam tamen quod vulgus quantitates hasce non aliter quam ex relatione ad sensibilia concipit. Et inde oriuntur præjudicia quædam, quibus tollendis convenit easdem in absolutas & relativas, veras & apparentes, Mathematicas et vulgares distingui.

Thus Newton gave reason for not defining motion that it is already known to all. Newton further categorized, space, place, time and motion in different ways. Newton categorized motion as Absolute and Relative.

(iii) **Absolute space**

Absolute Space in its own nature, without regard to anything external, remains always similar and immovable. Place is part of space where body is placed (just like a point).

Absolute motion: Absolute motion is the translation of a body from one absolute place into another.

In physics, the body is said to be in motion if it changes its position from one point to other.

Translation $means\ movement$ (motus)

Absolute space $means$ point

When motion or movement or translation are expressed in mathematical form or equation then it is velocity.

When body moves from one point to other then it possesses velocity, 20m/s.

Or in simplest words body possesses velocity if it changes its position from one point to other.

Thus motion is nothing but velocity (both have similar definitions in Newton's Principia) as clarified by Newton; as there is no difference between motion and velocity, when expressed mathematically.

In earlier days of natural philosophy (the study of nature and the physical universe that was dominant before the development of modern science), the mathematical equations were not discussed so movement or motion or velocity were the same.

It is clear that when motion is expressed in form of mathematical equation, then it is velocity. When units and dimensions [30,31] in 1822 were defined, then velocity was used as physical quantity (units: m/s, dimensions M^0LT^{-1}). When Jennings defined velocity in eq. (17) then word motion was not used. Earlier motion was used for velocity but velocity is used in mathematical equations.

Thus motion and velocity are the same.

So motion is term for velocity in natural philosophy (the study of nature and the physical universe that was dominant before the development of modern science.). Newton has initiated physics from natural philosophy. When mathematical interpretation for motion was studied then it becomes velocity. The velocity is denoted by 'v' but motion does not have any symbol. The symbol 'm' is denoted for length (meter), not for motion.

The velocity is defined by Jennings as

V =S/t

When units and dimensions [30,31] in 1822 were defined, then velocity was used as physical quantity (units : m/s, dimensions M^0LT^{-1}). Thus, refinements in scientific terminology continue and velocity is measured with rest to reference point or frame of reference.

Now in more refined way we define velocity with respect to a reference point or frame of reference but definition and mathematical equation remains the same. To measure velocity displacement (vector quantity), the magnitude equal to distance and time has to be simultaneously measured.

The **velocity** of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. The velocity is defined with respect to a known point as distance and time both are to be measured.

Practically it means change in position of body w.r.t. to some point or movement of body from one point to other. The motion has not been assigned any symbol, as distance is denoted by S, acceleration by 'a', force by 'F' etc. 'm' does not stand for motion but for meter (unit of distance).

Relative motion: It is the translation from one relative place into another.

Newton explained relative motion or motion in terms of velocity e.g. motion of the ship, it is explained in section (2.13).

Definitions of momentum and velocity were given by Jennings

J Jennings [32] in 1721, in his book *Miscellanea* in Definition I at page 67 has specifically used word Momentum,P (without mentioning quantity of motion). Jennings defined momentum P and velocity V. mathematically. For first time momentum is written as product of mass and velocity,

$$\text{Momentum } P = mV \qquad (16)$$

Similarly Jennings in the Miscellanea in Definition II at page 67 has given mathematical equation for velocity as distance travelled by body divided by time, The velocity is specifically written as

$$V = S/t \qquad (17)$$

Galileo's interpretation at page no.128 of the motion also mean the same thing. Thus, motion is nothing but velocity or motion is old name for velocity. The eq. (16) states momentum is product of mass and velocity.

(iv) **Definition III Innate Force.**

The vis insita, *or innate force of matter, is a power of resisting, by which everybody, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*

The *vis insita*, or innate force of matter, is a power of resisting by which everybody, tries to preserve its present state i.e., state of rest or of uniform motion. This property is also called vis *inertia*, or force of inactivity.

(v) Definition IV of Impressed Force

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

Here Newton meant moving uniformly forward (travels equal distances in equal intervals of time) as moving with uniform velocity.

Thus if body is at rest then impressed force acts on it then it sets body in moving uniformly forward in a right line. (the same is used in definition III in case of the vis vista or innate force of matter). In Newton's First law of Motion, "body *perseveres its state of uniform motion" or state of uniform velocity.* It simply implies that body moves with uniform velocity in straight line.

Practically now phrase 'moving uniform forward in straight line' or uniform motion are expressed as uniform velocity, V.

Table II. Highlights of various characteristics of impressed force, innate force, first law of motion and second law of motion.

Sr. No	Statement /law by Newton	Phrase	Meaning	Mathematical expression	Acceleration
1	Law of Inertia	motion which is uniform and perpetual	Uniform velocity	Velocity V	Acceleration not mentioned
2	Descartes	tends to continue moving	Uniform velocity	Velocity V	Acceleration not mentioned
3	Huygens	continued with an equal velocity	Uniform velocity	Velocity V	Acceleration not mentioned
4	Impressed force	Moving uniformly forward	Uniform velocity	Velocity V	Acceleration not mentioned
5	The vis insita or innate force of matter	Moving uniformly forward	Uniform velocity	Velocity V	Acceleration not mentioned
6	First Law of motion	Uniform motion	Uniform velocity	Velocity V	Acceleration not mentioned
7	Second Law of motion	Change in motion	Change in Velocity	Velocity V	Acceleration not mentioned

Note: The general term for movement was regarded as motion in natural philosophy in some cases. Now motion is regarded as velocity in quantitative explanation using mathematical equations. In natural philosophy scientific doctrines (say in physical sciences) may be philosophical but now there are mathematical equations.

3.1 In quantitative mathematical equations velocity is used not motion (movement).

Now all the phenomena are expressed quantitively i.e. theoretically predictions are confirmed experimentally. The motion is used as velocity (V). We normally say that bus is moving but quantitatively it is understood in terms of velocity e.g. but is moving with velocity 40 km/hr from New York towards London. Velocity is denoted by 'u' or 'v' normally but motion does not have such symbol in physics or science.

At that time (in case of natural philosophy) people were used to say that cart is in motion (terminology for movement at that time); but now we say that a bus is moving with velocity 40km/hr from London to Cambridge. In 1716 John Jennings defined velocity as in eq. (17), then mathematically term velocity was used. Further discovery of vectors and scalars made this interpretation more specific i.e. either speed or velocity was used.

The various kinematical and dynamical equations are expressed in terms of velocity not motion. The unit of velocity is m/s and dimensional formula M^0LT^{-1}; but there is no such notation (as units and dimensions) for motion.

(i) Velocity = Distance or displacement /time (17)

But it is not written as

Motion = Distance /Time (18)

So equation is not represented in terms of motion

(ii) v =u + at (19)

Final velocity = Initial velocity + acceleration x time

But there is no equation

Final motion = Initial motion + acceleration x time (20)

Thus motion is expressed in terms of velocity. There are many such equations.

(iii) The power of an agent is defined as

P = force x velocity = F.V

But it is not defined as

P = force x motion (21)

(iv) We say that escape velocity of body

Escape velocity (V_e) = $\sqrt{2gR}$ (22)

where g is acceleration due gravity and R is radius of the earth. The value of escape velocity is 11.2 km/s. We do not say value of escape motion is 11.2 km/s e.g.

Escape motion = $\sqrt{2gR}$ (23)

Thus in terms of equations , velocity is used not motion. So practically motion (movement) is just velocity, just other name is motion when physics was initiated.

3.2 Quantity of Motion (Quantitas motus), or momentum (MV) and Motion are not synonymous Scholium (explanation) after Definitions section at pages 9-10

Newton has described motion of ship in terms of velocity in the Principia [2] at page 11 in terms of velocity in the following way :

" As if that part of the earth, where the ship is, was truly moved toward the east, with a velocity of 10010 parts; while the ship itself, with a fresh gale, and full sails, is carried towards the west, with a velocity expressed by 10 of those parts; but a sailor walks in the ship towards the east, with 1 part of the said velocity; then the sailor will be moved truly in immovable space towards the east, with a velocity of 10001 parts, and relatively on the earth towards the west, with a velocity of 9 of those parts".

Thus Newton expressed motion in terms of velocity.

Thus, to assess motion of sailor; he is moving towards the east with velocity 1 part; its net velocity towards west w.r.t ship equal to 9 parts i.e., 10-1 =9 parts). The net velocity of sailor w.r.t. earth moving towards east (direction of velocity of earth, 10010), is 10001 parts.

Thus Newton has used the velocity as above; not momentum (mV) to express motion. The magnitude of quantities of motion or momentum, mV (if motion is quantity of motion) of man, ship and that of earth would be entirely different. Newton expressed motion in terms of velocity. So, velocities are considered. But if momenta are to be considered then product of masses of bodies and velocities are needed. The masses of the earth and ship are exceptionally high compared to mass of sailor. So their momentum would be entirely different, Newton has used only velocities to express relative motion.

3.3 Quantitative explanation for velocity given by Newton.

Newton's second law implies that when impressed force is proportional to change in motion or velocity. Newton did not give any equation for this, thus it may be regarded as qualitative explanation. Newton has also given quantitative explanation in Principia's Book II , Proposition XXIV , Theorem XIX [12] as given below;

" For the velocity, which a given force can generate in a given matter in a given time, is

as the force and the time directly, and the matter inversely.
The greater the force or the time is, or the less the matter, the greater velocity will be generated. This is manifest from the second law of motion."
(a) In Newton's first law of motion,
(i) the velocity is independent on time.
(ii) the velocity is only hampered by resistive forces of the system.
(b) In Newton's second law of motion
(i) 'change in motion' or 'change in velocity' is proportional to impressed force.
(c) But in Newton's Book II, Proposition XXIV (which he called Newton's second law of motion),
(i) the velocity is proportional to time t
It implies force may act on body (force may be constant) its velocity must increase with time.
In first law of motion velocity is independent of time.
This issue is explained in the section (9.1).

3.4 Motion is represented by velocity prior to the Principia.

Galileo had expressed motion and acceleration in terms of velocity. Motion is movement which is expressed in terms of velocity.

(i) **Galileo's uniform velocity in book Dialogues *Concerning Two New Sciences* (1638) at page 128.**

By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal.

Galileo has explained it with help of three axioms and six theorems.
Uniform motion is characterized by fact that equal distances are travelled in equal intervals of time
Uniform motion = Distance travelled /corresponding time (24)
Let body travels distance 4 meters in every 2 seconds then from Jennings's equation
Uniform motion = 4m/2s = 2m/s (24)
Also we can write equation for velocity is given by eq.(17)
Uniform velocity = 4m/2s = 2m/s (24)
(ii) Descartes in 1644 also discussed, stating bodies, 'always tends to continue moving' i.e. moving with uniform velocity.
(iii) Huygens in 1673 has directly expressed uniform motion in terms of equal velocity as described in section (2.10) as described.
Also Galileo's acceleration is also understood in terms of velocity.

3.5 Reasons for confusion between quantity of motion and motion in second law of motion.
Is translation is the reason for confusion?

Originally Newton has written the Principia in the Latin as *Philosophiae Naturalis Principia Mathematica* and it was translated by English mathematician Andrew Motte. Perhaps this effort was at the behest of his brother, Benjamin, who was the printer who published the translation. Thus, commercial reasons may be responsible for translation of the masterpiece along with immortal intellectual contents. It is interesting to know that English version was written by Newton as encouraged of Edmund Halley. Halley also spent money on publication of English version. Thus, it is very interesting story of publication of the Principia in the Latin and English. The original author of the Principia (Newton) and translator of the Principia (Andrew Motte) both have encouragements. But Galileo wrote and published books under the adverse conditions.

Definition II Quantity of motion (Quantitas motus)
'*The Quantity of Motion is the measure of the same, arising from the velocity and quantity of matter conjunctively.*'
Quantitas motus est mensura ejusdem orta ex Velocitate et quantitate Materiæ conjunctim.

Just possible if Andrew Motte had translated Quantitas motus (Quantitas means quantity ; motus means motion) as *Quantity-Motion or Motion-Quantity* , then the interpretation would have been different. These terms mean magnitude of motion. The Quantity of motion means amount of motion, but scientists regard it as only motion. Had it been translated by Motte as Quantity-*Motion or Motion-Quantity, then it would have not been understood as motion.* The reason being that quantity-motion and motion-quantity directly imply magnitude of motion.
It would have been clearly given impression as it is magnitude of motion (Quantitas motus) not motion (movement). Here meaning as magnitude has been clearly neglected.

Conclusion: Thus, Newton expressed motion (movement) as velocity; not momentum mV. The term Quantitas motus or 'quantity of motion' and 'motion' are similar so both appear same superficially but conceptually are quite different. It may be possibly due to reason its translation that Quantitas motus is translated as 'quantity of motion' not magnitude-motion or motion-magnitude. The other reason is that earlier scientists hurriedly associated F =ma with Newton's second law of motion then tried to justify that F =ma actually follows from definition of the law.

Quantitas motus and quantity of motion means magnitude of motion; and Newton took it as mV (10kg x40 kgm/s = 400 kg m/s). It is not velocity 40 m/s. Had Motte translated Quantitas motus as Quantity-*Motion or Motion-Quantity,* then it would have been clear that it is 'magnitude of motion', not simply motion.

Quantity of motion (magnitude of motion) = Quantitas motus =Quantity motion = Motion-Quantity =mV = 10kg x 40 m/s = 400 kg m/s

Thus motion is velocity 40m/s; it is not quantity of motion or magnitude of motion 400 kg m/s.

3.6 In Newton's first and second laws motion means velocity

Newton's second law of motion is regarded as central law, because first and third law of motion can be obtained from this. When second law reduces to first law then motion is regarded as velocity. Thus, it removes all doubts that motion is nothing but velocity.

The prevalent form of Newton's second law of motion is

$F = ma = m (v-u) /(t_2-t_1)$

(i) When no force acts on the system i.e. F =0. Now above equation becomes

$v-u = 0$

$u = v$ or Initial velocity = Final velocity (25)

Thus body moves with equal or uniform velocity, provided resistive force in the system is zero. Thus, Newton's first law of motion results in terms of velocity.

Also, we have other equation of force

$F = dp/dt$ (26)

where dp is change in momentum. When F=0,

$0 = dp/dt$ or p =constant or initial momentum = final momentum or mv = mu

$u=v$ or Initial velocity = Final velocity (25)

which is similar result as interpreted earlier. So, Newton just expressed law of motion in terms of velocity.

(ii) When no force acts on the system F =0

Then body does not move, it may remain at rest i.e. u=0 and v also 0; it is obvious from eq.(26) which is nothing but first part of Newton's first law of motion. Thus, Newton's first and second laws of motion are represented in terms of velocity. But they reduce to velocity when quantitatively interpreted.

3.61 The genuine form of Newton's second law of motion i.e. **eq. (11)**

$F= kdV$

If no external force acts on the system

$0 = kdV$

$dV= 0$ or $v-u = 0$ or final velocity = initial velocity (25)

Like this in this case also body moves with constant velocity. Thus, Newton's second law of motion reduces in form of velocity.

Also in this case velocity u =0 i.e. body remains at rest which is first part of Newton's First law of motion. Thus we find Newton's second law of motion reduces to velocity when mathematically expressed. Otherwise statement of the law is in terms of motion. So motion is nothing but velocity when law is mathematically interpreted.

3.62 Cohen expressed motion as velocity

Cohen in 1999 explained that motion is expressed in terms of velocity (V).Thus Cohen [11] has illustrated at page 116 the proportionality ($F \propto dV$ or F= kdV i.e. related impressed force with change in velocity) Cohen called it first equivalent equation of second law of motion. Cohen obtained other three equations starting from F =kdV which assumes motion is velocity (V).

It also implies that Cohen expressed motion in terms of velocity and equation F =kdV. Thus, equation F =kdV is genuine form of second law of motion, other equations were expressed in terms this equation. The remaining three equivalent forms i.e. eqs.(12-14) are based on it. So F =kdV is central term in Cohen's formulation. Here Cohen regarded motion as velocity V.

3.7 Peculiar results if motion is regarded as mV (momentum, quantity of motion or Quantitas motus).

Anyhow if we regard motion as momentum (mV) as postulate and applied in law of motion. It is discussed as below.

At page 19 of the Principia Newton has given

Axioms or laws of motion

Now it would be read as

Axioms or laws of momentum (mV)

The first axiom or law of motion

Now it would be read as

The first axiom or law of momentum (mV). But Newton never called motion as momentum. If Newton has mentioned both terms i.e. quantity of motion and motion, then it does not mean both are same. Newton has defined both differently at different places. Quantity of motion is defined in Definition II at page 1 and motion in Scholium of the definition section at page 9-10.

3.71 The transformation of first law of motion if motion is regarded as momentum,

Thus it would read as

"*Everybody perseveres in its state of rest, or of **uniform momentum (mV)** in a right line, unless it is compelled to change that state by forces impressed thereon".*

Now mass may have different inherent characteristics and shape (sphere, umbrella shaped, cone, square,irregular shape etc.) .

The mass of body is constant in Newtonian Mechanics, so practically the law would be expressed in terms of velocity only. Momentum = mV = constant V, in Newtonian mechanics.

"*Everybody perseveres in its state of rest, or of **uniform velocity (V)** in a right line, unless it is compelled to change that state by forces impressed thereon".*

Thus it is again justified that motion as velocity, the mass is constant in Newtonian Mechanics.

In Definition I Newton has defined 'Quantity of matter' and used wider sense

"The same thing is to be understood of snow, and fine dust or powders, that are condensed by compression or liquefaction; and of all bodies that are by any causes whatever differently condensed."

In Latin [1] Def. I.

Idem intellige de Nive et Pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quæ per causas quascunq; diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo.

Thus Newton's laws hold good for all bodies.

3.72 The second law of motion

Similarly Newton's second law of motion would read as

"*The alteration of momentum is ever **proportional to** the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

change in momentum $\propto$ Impressed force

Impressed force $\propto$ mdV

Impressed force = ZmdV = kdV (k =Zm)

or Impressed force $\propto$ mdV

The mass is constant in Newtonian mechanics. In Newton's Second Law of Motion the proportionality occurs so it holds good in different ways.

Impressed force $\propto$ dV

$$F = kdV \qquad (11)$$

Thus it is again justified that motion is velocity. Thus, it is repeatedly justified that motion is velocity, V. In Newton's second law of motion proportionality occurs which holds good over wider range.

3.73 **Similarities between F =kdV and F =ma**

The genuine equation for second law of motion is F =kdV and prevalent equation F =ma since centuries. F =kdV is not mentioned in the standard references even for seconds. So, it may have been due to some similarities between two; also, there are dissimilarities between two.

(a) Both the equations i.e. F =kdV and F =mdV/dt have impressed force in left hand side; as both are equations of impressed force.

(b) Both equations involve change in velocity.

(c) Newton neither gave F =ma not F =kdV for second law of motion.

3.74 **Conceptual dissimilarities** between **F =ma and F =kdV**

(a) The definition of Newton's second law of motion involve 'change in motion'.

which is initially regarded as 'change in momentum'

Change in motion (velocity) = Change in momentum = mv-mu

But it does not lead to second law of motion.

(b) As initial aim of scientists is to obtain F =ma. So they further assumed change in motion is equal to rate of change momentum.

Change in motion (velocity) = rate of change of momentum (pure assumption)

But in actual reality

Change in motion (velocity) $\neq$ rate of change of momentum = d/dt (mv-mu) (5)

Both the LHS and RHS differ in units, dimensions and magnitudes are different, hence equation does not hold good. This aspect is discussed in section (2.1).

Thus scientists made some arbitrary assumptions to get F =ma from definitions of second law of motion.

(c) F =ma is being used by scientists since centuries but F =kdV not even for seconds.

3.8 Comprehensive discussion: motion is velocity V; not momentum

This question arises due to reason that scientists are ignoring that genuine equation based on second law of motion is F =kdV. Newton had not given any equation, neither F =kdV nor F =ma. Scientists are ignoring F=kdV and adopting F =ma (never given by second law of motion) for second law of motion. When differential and integral calculus was developed then acceleration was found useful. The equation F =ma was derived by Euler in 1775.

It is certain that Newton did not give F =ma. There are no evidences that when F =ma was associated with second law of motion and who were the scientists responsible for doing this. Likewise, Newton did not give $F = Gm_1m_2/r^2$ and it not certain the name of scientists who combined various proportionalities (Propositions 1-X, Book III of the Principia), and wrote eq. (9) i.e. $F= Gm_1m_2 /r^2$. These issues can be resolved after careful and elaborated studies in history of physics and mathematics.

Obviously F =kdV, follows from the second law of motion as $F= Gm_1m_2 /r^2$ from law of gravitation; both equations utilize method of proportionality. Now law of gravitation has number of applications in physics but F =kdV is not even quoted.

Then it was required that equation F =ma (very important equation when differential and integral calculus was developed) should be derivable from the definition of second law of motion. As F=ma has to be derivable from second law, so scientists have to arbitrarily assume motion as momentum (in Definition II Newton defined quantity of motion as mV). Thus, we get mathematical equation F= k(mv-mu). But it does not serve the purpose.

So further scientists have arbitrarily assume

'change in momentum' = 'rate of change of momentum with time' = d/dt (mv-mu) (5)

Or $F = Km(v-u)/(t_2-t_1) = ma \quad (K=1)$ (6)

The dimensions, units and magnitude of both left hand side and right hand side are different; so eq.(5) is completely inconsistent as shown in section (2.1), thus eq.(6) is not justified. So motion means velocity only. Scientists have tried to obtain F =ma from second law of motion which is completely inconsistent. Thus, in the process scientists make arbitrary and inconsistent assumptions, one after the other. It is justified below that motion is simply velocity,

(i) In Definition I of the Principia Newton has defined quantity of matter (Quantitas Materiae). Further Newton wrote that

"It is this quantity that I mean hereafter everywhere under the name of body or mass"

In Definition I at page 1, Newton clearly stated that he would regard 'quantity of matter' (Quantitas Materiae) as body or mass.

In Definition II at page 2 , Newton never wrote in Definition II (few lines down) as he would regard quantity of motion ('Quantitas motus) as motion. However, Newton has defined and explained motion separately with help of examples.

(ii) At page 9, in scholium Newton did not define space, time, place and motion as these are already known. Here Newton further categorized motion in terms of absolute motion and relative motion, both were defined by Newton and mean velocity. It is explained in section (3.1).

(ii_a) Already known terms:

(p) Law of Inertia (1638) given by Galileo in Dialogues p.195

Phrase: 'motion which is uniform and perpetual' …it simply implies the motion of body uniformly perpetual; i.e. body perpetually moves with uniform velocity.

(q) Descartes second law (1644), (Principles Part II, article. 39). It is given by Rene Descartes in book Principles of Philosophy

Phrase :'*tends to continue moving, not in any oblique path but only in a straight line'*…. implies uniform velocity

(r) Christiaan Huygens (1673): Hypothesis I p 21, Book Horologium oscillatorium sive de motu pendularium

Phrase: continued with an equal velocity

Hence Huygens has more clearly implied that body moves with uniform of equal velocity.

Thus before Newton by word motion scientists meant velocity. Then Newton also meant motion as velocity

(s) Isaac Newton (1686,1713,1726) Principia , Definition III The vis insita or innate force of matter

Phrase : Moving uniformly forward in right line, means uniform velocity

(t) Isaac Newton (1686,1713,1726) Principia , Definition IV An Impressed Force

Phrase : Moving uniformly forward in right line , means uniform velocity

(u) Isaac Newton (1686,1713,1726) Principia, First Law of Motion

Phrase : uniform motion in right line, means uniform velocity

(w) Isaac Newton (1686,1713,1726) Principia, Book II , Proposition XXIV , Theorem XIX [12]

Phrase /statement : " For the velocity, which a given force can generate in a given matter in a given time, is as

the force and the time directly, and the matter inversely.
The greater the force or the time is, or the less the matter, the greater velocity will be generated. This is manifest from the second law of motion."
So Newton has expressed second law of motion in terms of velocity. Newton has stated the reason for not defining motion as it is already defined motion because it is well known to all. Thus, we find before Newton Galileo (1638), Descartes (1644) and Huygens (1673) has defined motion as velocity. Also Newton himself either meant motion as velocity or directly implies motion as velocity. The various aspects are highlighted in Table II for quick reference and clarification.
When the motion is expressed in terms of mathematical equations then it means velocity. The mathematical equation for velocity was given by Jennings in 1721 as eq. (17) i.e. V =S/t.
(ii_b) At page 9, in scholium Newton has also categorized motion
as absolute motion and relative motion.
When Newton has explained absolute motion and relative it also implies velocity. It is explained in section (3.0).
Thus Newton meant motion as velocity not momentum (mV). The quantity of motion (Quantitas motus) is momentum mV, as defined in Definition II.
Thus, by motion Newton implied motion as velocity. The motion is expressed in velocity mathematically.
(iii) At page 11 (while explaining motion as velocity) Newton discussed relative motion sailor on moving ship, then motion was regarded as velocity. The reason is that Newton in calculation of relative velocity added and subtracted velocities, not momenta. The velocity of earth is regarded as 10010 *parts* (units of velocity, as we have m/s, now*); towards east*, the ship moves towards west with 10 parts, the sailor walks in ship with velocity 1 part towards east. In Newton's time, units of velocity were not defined (beginning or inception of physics), the units and dimensions **[30,31]** were defined in 1822.

Now while calculating the relative motion, Newton did not consider momentum of earth (mass of earth x velocity of earth), momentum of ship and momentum of sailors. But Newton considered 10010 parts. 10 parts, 1 part as velocities. Newton calculated relative motions equal to 10001 parts or 9 parts or 1 part as velocities of earth, ship and sailor. So, Newton regarded, relative motion (one category of motion) as velocity. So, motion is expressed in terms of velocity not momentum. For calculation of momentum masses of earth, ship and sailor need to be taken in account which are not taken in account. Newton has used velocity as 10010 parts, 10 parts and 1 part as units of velocity, dimensions etc. were defined much later.

(iv) Cohen has given four equivalent forms or equations of second law of motion; the first form or equation is F=kdV. Thus, Cohen has regarded motion as velocity (V) to get equation F =kdV.

(v) Prior to Newton, Galileo has defined uniform motion, acceleration when these are put in mathematical form these are in terms of velocity. Thus, motion is nothing but velocity. It is evident from section (2.15).

(vi) The motion or movement are old terms for velocity. Mathematically velocity was defined in eq.(17) in 1721 in book Miscellanea in Usum Juventutis Academicae. As it is quantitative term involving for moving but Newton did not quote it in third and final edition of the Principia in 1726. Now motion is not a physical quantity as it does not possess symbol, units and dimensions. The velocity is represented by V, but motion is not represented by m, the symbol m represents length (meter). The symbol, units and dimensions for velocity are V, m/s and M^0LT^{-1}.

(vii) In mathematical equation velocity is taken not motion e.g., v = u+at (final velocity = initial velocity +at, not final motion = initial motion +at) P =FV (Power = force x velocity). $V_e = \sqrt{2gR}$ (it is known as escape velocity not escape motion). Thus, mathematically we always use velocity. So motion is always represented by velocity, not by momentum.

(viii) Newton's second law of motion is regarded as central law of motion i.e. it reduces to first law when no external force acts in the system (F=0),
$F = m(v-u)/(t_2 - t_1)$
$0 = m(v-u)/(t_2 - t_1)$
or u =v or initial velocity = final velocity
Thus Newton's first law of motion is expressed in terms of velocity. Now the definition of the can be understood as
"*Everybody perseveres in its state of rest, or of uniform motion (uniform velocity) in a right line, unless it is compelled to change that state by forces impressed thereon".*

(ix) Further, Newton himself considered vertical motion of bodies as velocity i.e., in case of falling bodies in Proposition XLI, General Scholium of the Book III of the Principia and Scholium of Corollary VI at page 31 of the Principia. Thus, Newton has expressed vertical motion in terms of velocity.

We normally say that bus is in motion. Then it may move with velocity 10m/s or 5m/s or different. Thus when bus is motion it may have any velocity. However, when we say that bus is moving with velocity 10m/s, then velocity is definite.

3.9 Perception of uniform motion

The perception of equal or uniform velocity existed before Newton's time.

Galileo (1642), Descartes (1644), Huygens (1673) has explained motion in form of uniform of uniform motion. Newton also stated first law of motion in terms of uniform motion. Newton's second law also reduces to first law of motion, which is in form of uniform of motion. Thus, uniform motion or velocity is very significant term and must be properly understood.

3.91 Galileo

Galileo, who may be regarded as first, greatest experimental and theoretical physicist as he even used domestic and rhetorical instruments to conduct experiments of far-reaching importance. Galileo has given the law of inertia in his book *Dialogues Concerning Two New Sciences* [3] in 1638 at page195 in section The Motion of Projectile. At page 128 Galileo defined uniform motion as with help of 4 axioms and 6 Propositions and 6 theorem .

By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal.

Velocity = distance travelled /time taken = S/t

When velocity is calculated by eq.(17), then motion is not used.

Further Galileo applied it to the motion of bodies in form of Law of Inertia in The Motion of Projectile at page 195

Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits.

At that time people were used to say that cart is in motion (terminology for movement at that time); but now we say that a bus is moving with velocity 40km/hr from London to Cambridge . In 1721, John Jennings defined velocity as in eq. (17), then mathematically term velocity was used. Further discovery of vectors and scalars made interpretation more specific i.e. either speed or velocity was used. It is concluded that motion is velocity. Now velocity is defined with respect to reference point or frame of reference.

3.92 Descartes (1644)

Descartes [11] has too assumed bodies move essentially with constant velocity in straight line in his book Principles of Philosophy in 1644 in Law II (Principles Part II, article. 39). Thus Descartes continued the Galileo's perception.

(ii) **Descartes' second law of motion** (1644)

Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. (Principles Part II, article 39).

Descartes [4] has given his three laws of motion in his book **Principles of Philosophy** (1644). It implies body tends to continue moving in straight line. Descartes indirectly implies that body moves with uniform velocity as he did not mention about variable velocity.

(iii) **Christiaan Huygens Hypothesis I**

Christiaan Huygens [5] published book *Horologium oscillatorium sive de motu pendularium* in 1673. It contains three hypotheses in second part at page 21.

If there is no gravity, and the air offers no resistance to the motion of bodies, then any one of these bodies admits of a single motion to be continued with an ***equal velocity*** *along a straight line.*

It implies if body is set in motion it remains in straight line with uniform velocity (equal velocity). Just like Galileo, Descartes, Huygens and Newton also preferred to express first law of motion using uniform velocity or from law of inertia.

Huygens perception of gravity.

In Huygens first hypothesis (of book *Horologium oscillatorium sive de motu pendularium,* second part page 21) the gravity is significant term. Huygens has perceived or foreseen gravity in 1673 i.e., 13 years before Newton's Principia. Just like air resistance gravity also hinders motion of bodies; thus, gravity is the force which is attractive in nature i.e., due to gravity earth attracts bodies towards itself. Thus, gravity hinders the perpetual horizontal motion of bodies. Similarly, in hypothesis II, Huygens considers falling bodies downwards due gravity. This hypothesis implies that Huygens has foreseen gravity and its attractive applications on bodies before Newton. In Newton's masterpiece 'The Mathematical Principles of Natural Philosophy' Book III, the Law of gravitation has been stated in various propositions (I-X). The equation for law of gravitation F =

Gm_1m_2/r^2 was written after death of Newton.

(vi) **Newton's First Law and Law of Inertia**

"*Everybody perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon".*

The body perseveres in its state of uniform motion in a right line means maintains state of uniform velocity, V .

The mathematical interpretation of the first law of motion also implies motion is nothing but velocity, it is mathematically justified. Thus, Newton's first law also implies that bodies move with uniform velocity (state of uniform motion). When no force acts on the system (F=0), then prevalent form of second law of motion implies

$0 = m(v-u)/t$

or u= v or Initial velocity = Final velocity (25)

So body moves with uniform or equal velocity, when no resistive force acts on moving body. Here it is assumed that body is already moving. So, first law of motion is expressed in terms of equal or uniform velocity. Thus, motion expressed as velocity. The similar results are obtained if eq. (11) i.e. F =kdV is considered. If F=0,

dV = 0 or v-u = 0 or u =v or Initial velocity = Final velocity

Thus in this case also body moves with uniform velocity. Thus eq. (11) also gives similar results as eq. (2). So, first law of motion is expressed in form of equal uniform velocity. Thus, motion is expressed in terms of velocity.

When body moves with uniform velocity then distance travelled is given by

S =Vt (17)

where S is distance travelled in time t when body moves with uniform velocity V.

In addition Newton extended application of motion in circular motion. Galileo had stated that law of inertia in form of uniform velocity i.e. bodies move with uniform velocity in horizontal motion. Further Newton has justified that celestial bodies constantly pursue their revolutions in orbits (move with equal velocity in vacuum, $v = r\omega$: v linear velocity, r is radius and ω is velocity). Newton was a primarily mathematician, so he speculated that bodies also descend with equal or uniform motion in vertical motion also. He did not conduct any experiment regarding this, also did not suggest any experiment for measurement of the same.

4.0 **Origin of acceleration**

Galileo originated acceleration (1638) but Newton ignored it throughout his life

Galileo may be regarded as first genuine experimental physicist who conducted experiments in 1604 and also explained them theoretically. Galileo's conclusions are even now basis of physics even today. The experimental data and its interpretation were published in 1638 when he was under house arrest and his eyesight had deteriorated beyond restoration. In this book he described acceleration which is now regarded as lifeline of mechanics. Galileo in his book *Dialogues Concerning Two New Sciences* has elaborated acceleration at pages 133-134, 146. Galileo defined accelerated motion at page 134. Also, uniform velocity was defined at page 128, and applications of uniform motion as Law of Inertia at page 175 in section The Projectile Motion.

Acceleration was used to derive equation F =ma for Newton's second law of motion after death of Newton. Euler derived equation $F = md^2x/dt^2 = ma$ in 1775. Jacob Herman is believed to have directly quoted equation F =mdV/dt (without mentioning dV/dt as acceleration). Also, Newton never mentioned acceleration (already defined by Galileo in 1638). Newton had opportunity to express F =mdV/dt as equation for second law of motion. But he neither wrote F =mdV/dt as second law of motion nor dV/dt as acceleration.

Also an article published in American journal of **Physics [12]** (2011) at page 1015 Bruce Pourciau states that – *"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."*

It is also justified from discussion in the Principia. Newton ignored acceleration for 85 years of his life.

4.01 **Definition and interpretation**

Accelerated Motion : *A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed.*

Galileo has interpreted at page 133 as,

Thus if any equal intervals of time whatever have elapsed, counting from the time at which the moving body left its position of rest and began to descend, the amount of speed acquired during the first two time-intervals will be double that acquired during the first time-interval alone; so the amount added during three of these time-intervals will be treble; and that in four, quadruple that of the first time interval.

Linear acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) =dV/dt (27)

Let body starts from the rest then after 1s, 2s, 3s and 4s its velocity will become

1m/s , 2m/s , 3m/s , 4m/s respectively then it will fall with uniform acceleration as

Linear acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) = $1m/s^2$ (27)

Thus uniformly accelerated motion is also represented in terms of velocity.

4.02 Galileo's demonstration of acceleration

Galileo has described simple experiments to understand the accelerated motion at page 146 of the Dialogue as :

A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball……..

So Galileo completely described acceleration in the book **Dialogue Concerning Two New Sciences** published in 1638 at pages 133-134 and 146. However, the relevant experiments were conducted by Galileo in 1604.In addition Galileo also explained uniform velocity which he applied in explanation of law of motion.

But Galileo did not apply acceleration in such a way. The worth of acceleration was understood afterwards. The significance and importance of acceleration was fully understood in differential and integral calculus. **The first differential equations [7] of motion for systems having more than two mass-bearing points were published in 1743 by John Bernoulli and by D'Alembert.** Then it was also related to Newton's second law of motion F =ma, mathematically. However this issue needs to be settled finally with research in history of mathematics and physics that when and by whom F =ma has been related with second law of motion. The genuine equation for second law of motion is F=kdV, the genuine discoverer of F =ma is Euler who discussed it in 1875. Hermann is also regarded to have directly given equation F =mdV/dt in 1716. Neither Euler nor Hermann are mentioned as discoverer of F =ma. However, Newton's is known as discoverer of F =ma who never discovered F=ma. It is exceptionally strange.

4.1 Acceleration not mentioned in the Principia

Newton's acceleration-less physics.

This discussion is also related or in continuous with sub-section, 'Origin of acceleration'. It is true that acceleration was initially described by Galileo. But Galileo explained law of inertia in terms of uniform velocity only. Now the critical analysis implies that acceleration was not included in Newton's terminology at all. Newton did not write equation F =ma during his long and prospective research career. Also, acceleration was related with Newton's second law of motion as F =ma inconsistently by succeeding scientists. This aspect is further discussed.

Apparently, it may have been done when acceleration was found exceptionally useful in differential and integral calculus. However, Euler had derived equation $F =md^2x/dt^2 =ma$ in 1775 and Hermann is regarded to have directly given equation F =mdV/dt in 1716. Hermann did not mention dV/dt as acceleration. Also Newton did not mention acceleration. Newton used Galileo's law of inertia in first law of motion and in second law he related force with 'change in velocity' or change in motion. Acceleration is different from 'change in velocity'. Strictly speaking the development of acceleration itself may lead to other separate discussion.

(i) Acceleration was not mentioned in Definitions (I-VIII)

Newton [2] did not mention word acceleration neither in new definitions (I-VIII) i.e. quantity of matter (mass), Quantity of motion (mV), impressed force, the innate force of matter (inertia) , centripetal force, various types of centripetal force nor in already known terms (time, space ,palace and motion). This significant term (acceleration) was not discussed neither in definitions nor in already known quantities such as time, space, place and motion. Thus, acceleration was not in Newton's terminology.

(ii) Acceleration is not mentioned in laws of motion

Newton did not mention acceleration neither in definition of second law of motion nor explanation given in the Principia at page 19. The law is defined as

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Thus, 'alteration or change or difference in motion' is not acceleration.

Final motion (velocity) – Initial motion (velocity) $\neq$ Acceleration (a)

Acceleration is defined in section (4.0) in eq. (27) as given by Galileo. Further definition of impressed force and first law of motion are independent of acceleration.

Acceleration was defined by Galileo in 1638 i.e. 4 years before birth of Newton. Newton did not mention acceleration even 1727 throughout his long scientifically productive life of 85 years. So, Newton ignored acceleration and it was insignificant in his scientific vocabulary.

First Law of motion

"*Everybody perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change*

that state by forces impressed thereon".
Thus law deals with uniform velocity not with rate of change of velocity i.e. acceleration as in eq.(27).

Definition IV of Impressed Force

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.
Even definition of second law of motion is independent of acceleration. The phrase 'moving uniformly body forward' means moving with uniform velocity.

Third and final edition of the Principia

Newton had another opportunity to use or explain acceleration as dV/dt when in 1716. Cohen [11] at page 113 deduced that Herman had directly given equation F = mdV/dt. Herman has directly quoted this equation without derivation. Newton did not acknowledge F =mdV/dt as second law of motion in third and final edition of the Principia in 1726. It was opportunity for Newton to write dV/dt as acceleration (acceleration was actually discovered by Galileo in 1638 i.e. 4 years before birth of Newton).

Newton did not point out dV/dt in equation as acceleration. Galileo has derived acceleration (change in velocity/change in time) in 1638. Newton did not give any equation for second law of motion. The first two editions were published in 1686 and 1713.

Newton added and removed the contents in second and third edition of the Principia when he was President of the Royal Society, London. But Newton did not change three laws of motion and definitions in all three editions of the Principia. Newton completed above book *The Method of Fluxions and Infinite Series* in 1671, but published in 1736 i.e. 9 years after Newton's death. The word fluxions mean derivatives. Newton did not write dV/dt as acceleration in this book also.

Newton published first edition of the Principia in 1686; third and last edition in 1726, a year before death. But acceleration was not discussed at all.

The reason is that it is simpler to explain the laws in terms of uniform velocity. In one way or other preceding scientists, Galileo, Descartes and Huygens etc. explained the law in terms of uniform velocity. They too have neglected accelerated motion or they could not perceive its applications and importance at that time. They used applied form of uniform velocity as in case of Law of Inertia (but in similar way i.e. in applied form acceleration is not used). So, Newton did exactly like his predecessors. Like them Newton neglected terms, 'acceleration' and 'accelerated motion' as he used uniform motion and law of inertia in explanation of Newton First Law of Motion. Newton has ignored acceleration (hence accelerated motion) for 85 years long life (1642-1727). However, Newton associated force with 'change in motion' in second law of motion. Acceleration was associated with Newton's second law of motion F =ma after death of Newton.
I Bernard Cohen [11] has mentioned at pages 116-117
"Newton did not write any equation for his laws."
Also an article published in American journal of Physics [12] (2011) at page 1015 states that –
"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."

Newton believed acceleration-less physics.

4.2 For the sake of simplicity, Newton ignored acceleration.

Newton initiated Physics separating it from natural philosophy (Natural philosophy or philosophy of nature was the philosophical study of nature and the physical universe that was dominant before the development of modern science).

Before Newton Galileo has defined uniform motion at page 128, uniform acceleration at page 133-134, 146 in book Dialogues Concerning Two New Sciences (1638). Galileo explained law of inertia at page 195 in form of uniform motion as,

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

After Galileo, Descartes in the book *Principles of Philosophy* (1644) and Huygens in book *Horologium oscillatorium sive de motu pendularium* (1673) **explained the laws in terms of uniform velocity. Thus Galileo, Descartes and Huygens used uniform motion, not accelerated motion.** Similarly, Newton explained motion in terms of uniform velocity.
Now keeping the precedence of preceding scientists like Galileo, Descartes, Christiaan Huygens and Newton expressed laws in terms of uniform velocity. Newton followed the simplest path like his predecessors and stated first law of motion in terms of uniform velocity which is other form of law of inertia. It was easier to explain motion in terms of uniform motion than accelerated motion.

Euler has given equation $F = md^2x/dt^2 = ma$ in 1775. Thus, it the simplest case to explain motion of bodies in terms of uniform velocity, so Newton and his proceeding scientists (Galileo, Descartes and Huygens) explained motion in terms of uniform velocity. Also at that time there was no established mathematical basis for explaining the phenomena, so interpretation was largely qualitative (no specific prediction of mathematical equations).

4.3 Insignificance of acceleration for Newton

Newton neglected acceleration throughout his life. Acceleration was defined and explained by Galileo in 1638 i.e. 4 years before birth of Newton (1642). Thus at time of Newton acceleration was present in the literature like uniform velocity.

(a) Newton [2] did not mention word acceleration neither in new definitions (I-VIII) i.e. quantity of matter (mass), Quantity of motion (mV), impressed force, the innate force of matter (inertia) , centripetal force, various types of centripetal force nor in already known terms (time, space ,palace and motion). Newton further categorized motion as absolute motion and relative motion, these are velocities not momenta. The absolute motion and relative motion both are expressed in terms of velocity.

(b) Newton did not mention acceleration neither in definition of second law of motion nor explanation given in *the Principia* at page 19.

So acceleration is not mentioned in the definition, it simply involves impressed force, change in motion etc.

Thus, 'alteration or change or difference in motion' is not acceleration.

Final motion (velocity) – Initial motion (velocity) $\neq$ Acceleration (a)

(c) **The Methods of Fluxions and Infinite Series**

It is believed that Newton had completed this book, *The Methods of Fluxions and Infinite Series* in 1671 but published in 1736 i.e. 9 years after Newton's death. In Newton's terminology fluxions means derivatives. In this book Newton did not write acceleration as, a =dV/dt.

The book was published 65 years after its completion. The no reason is given in the literature for the delay in publication. Thus, it is evident that Newton like his predecessors, Galileo, Descartes, Huygens; Newton did not apply in applications of acceleration to motion of bodies. They all explained motion in terms of uniform motion, as in Galileo's law of inertia.

So Newton presumed that acceleration did not exist at all during his life time. Or it was not useful term at that time.

So acceleration was not used by Newton in his conceptual terminology for whole life of 85 years.

After death of Newton, especially in Euler's era, acceleration was found very useful physical quantity in differential and integral calculus. The equation for second law of motion is inconsistently derived as F =ma i.e. related with force and acceleration. Also Euler had derived equation $F = md^2x/dt^2 = ma$ in 1775 and Jacob Hermann gave directly F =m dV/dt in 1716.

The following scientists like Descartes (second law of motion) , Huygens (first hypothesis) , also expressed their laws in form of uniform velocity.

Just like Galileo, Descartes and Huygens, Newton also expressed first law of motion, impressed force, innate force in terms of uniform velocity. However, Newton stated second law of motion in dynamical form i.e. related force with change in velocity in proportionality form. But he did not write $F \propto dV$ or F =kdV. So Newton neglected or ignored acceleration throughout his life, as it existed in literature.

Also, Newton did not write F =ma. So acceleration was completely insignificant and unknown for Newton. Newton did not include acceleration in his terminology and in the Principia where he put forth laws of motion.

4.4 Acceleration remained veiled in Newton's time.

It must be noted that Galileo defined basic terms such as uniform velocity, acceleration and law of inertia. But Newton just took law of inertia and defined and changed it in form of first law of motion; he did not use acceleration at all. Newton was primarily mathematician, who dealt physical phenomena with methods of geometry. The reason is that analytical methods had not been discovered in Newton's time or there was no trend of equations. Cohen [11] has stated at pages 116-117 that Newton did not give any equation. Newton initiated physics as subject separating from natural philosophy i.e. from initial or naïve or zero state. But Newton used only law of inertia as given by Galileo. Newton completely neglected acceleration.

There were no mathematical equations at that time. In Newton's time it was just beginning of physics. Earlier Italian Galileo Galilei has defined uniform motion, acceleration and law of inertia conducting experiments and published in book *Dialogues Concerning Two New Sciences* in 1638. At that time there were no mathematical equations or there was no requirement of mathematical equations. The laws were explained with statements largely qualitatively and philosophically.

Acceleration was defined and explained by Galileo in the Dialogues. Newton ignored this throughout his life of 85 years, as acceleration already existed in the literature. The importance of acceleration was realized by scientists when differential and integral calculus was developed. Then they associated acceleration with Newton's second law of motion as F =ma. But this equation is not derivable from second law of motion and second law of motion gives equation F =kdV.

His predecessors have explained laws philosophically and in terms of uniform velocity. There were no significant experimental data about mechanics at that time. Newton did not perceive experiments like Galileo. So Newton put forth some new terms (Definitions) and gave axioms or laws of motion for description of phenomena. Acceleration was not included in these.

At that time no mathematical methods were discovered except geometrical methods. These were used at initial stages of development of physics /science. The other analytical techniques were developed afterwards, we should not assume these were present as such in Newton's time. Scientists discovered various methods in due course of time and applied to Newton's axioms or laws. In differential and integral methods, acceleration is the most suitable term but not used by Newton. But Newton did not cite dV/dt as acceleration, even in his book **The Methods of Fluxions and infinite series [33]** or third and final edition of the Principia.

The origin and development of acceleration is not abrupt process but it continuously developed with direct interactions of various or numerous scientists.

(a) Earlier Aristotle (385-323BC) stated that force is required for movement. The table stops as soon force (may be push or pull) ceases to act on it. It is clearly observed even now due to presence of various resistive forces. The concept of *inertia* was alien to the physics of Aristotle. Aristotle, and his peripatetic followers held that a body was only maintained in motion by the action of a continuous external force. Aristotle implied that rest is natural tendency of body, it is disturbed as long as external force acts on body; and justified in above example. This doctrine was contested between admirers and critics for centuries.

(b) Jean Buridan's Impetus theory (1295-1360) was propounded, as an arrow moves when left from the bow. So it moves even when no direct force acts on it, the movement is regarded due to impetus of force. The similar theory was proposed by Johannas Philoponus (550) but it was not given due consideration as Aristotelian doctrines prevailed.

(c) Galileo emerged as first experimental and theoretical physicist genuinely, he took observations with simple rhetorical or domestic equipment. He took observations even without clocks or watches. For measurement of time he used water falling from the pot in naked eye experiments. Galileo has given the law of inertia in his book *Dialogues Concerning Two New Sciences* [3] in 1638 at page 195 regarding projectile motion.

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

This implies that body, once set in motion keeps on moving with uniform velocity. For this we need an ideal system devoid of resistive forces. Thus, Galileo put forth uniform motion is the natural tendency of body, and it is hampered by resistive forces of the system of body and medium.

(d) Then Descartes (1644) put forth three laws of motion, the first two laws are directly based on Galileo's law of inertia. The second law of motion implies that body moves with uniform velocity in straight line i.e. does not change its direction by its own. Thus, it also implies that body keeps on moving with uniform velocity, as change in velocity is not mentioned.

(e) Christiaan Huygens (1673) put forth three hypotheses in his book. The first hypothesis implies that in absence of resistive forces the body moves with an equal velocity in straight line.

(f) Thus like Descartes and Huygens, Newton also put forth three laws of motion. Newton's first laws is improvised form of Galileo's law of inertia, Descartes first law of motion. Huygens first hypothesis, that bodies keep on moving with equal or uniform velocity in straight line. Newton put forth the law of motion in more precise and poetic form, but general elaborations are already given by Galileo, Descartes, and Huygens.

Newton gave second law of motion in the dynamical form i.e., related force with velocity, this is addition to previous laws. According to Newton's second law of motion impressed force is proportional to change in motion (velocity). But Newton neither wrote the law in form of proportionality ($F \propto dV$) nor as equation F =kdV. The force was written as F =ma after death of Newton; however, Newton had not originally given this equation.

Thus, in general it is very important to define a region or domain or area of applicability of various laws. Newton did not use acceleration from existing literature? Newton did not write F =ma. F =ma was derived by Euler in 1775. The definition of Newton's second law of motion is quoted in literature. It is true that Newton defined second law of motion in the Principia but neither gave F =ma nor F=kdV.

4.5 After death of Newton

It is justified in above discussion that due to conceptual limitations and historical prospective of development of science. Newton completely neglected or ignored acceleration. In other words, acceleration was not in terminology of Newton. Galileo has proposed uniform velocity, acceleration and law of inertia. Out of these

Newton only took law of inertia and wrote First Law of Motion completely neglected acceleration.

At time of Newton the laws were initially expressed in terms of uniform motion (the simplest case). Before Newton the preceding scientists Galileo (1638), Descartes (1644), Huygens (1673) etc. explained motion in terms of uniform motion (applied to law of inertia). Newton did the same as by his predecessors. Galileo defined uniform velocity and used it in interpretation of inertia of motion. Descartes defined second law essentially in form of uniform motion and Huygens first hypothesis is in terms of equal velocity.

Newton also defined is first law of motion in terms of uniform motion only as per in existing precedence. In Newton's time acceleration was not discussed but it existed in literature. After death of Newton in applications of differential and integral calculus, acceleration was found useful then following scientists related F =ma with Newton's second law of motion. Euler derived F $=md^2x/dt^2$ =ma in 1775 and Hermann had given directly in F =mdV/dt. Both have given laws without using Newton's second law of motion.

Afterwards scientists found acceleration as significant term especially when laws were expressed in differential or integral forms as defined acceleration. Thus, force is given by F =ma. Or Euler's equation may have been directly related with Newton's second law of motion. When they tried to justify the same (connection of Newton's second law with definition acceleration i.e. F =ma), then they have to make inconsistent and arbitrary assumptions. Thus, genuine equation of Newton's second law of motion eq. (11) i.e., F =kdV. The equation F =ma must be credited to Jacob Hermann and Leonhard Euler.

The significant issue left for further discussion is that who related F =ma with Newton's second law of motion? When it is done? Why it is done? Also why genuine equation (F =kdV) based on second law of motion is neglected? Was ever it was considered? This equation (F =kdV) was derived by method of proportionality like that of F $=Gm_1m_2/r^2$. The definitions of second law of motion are given in textbooks or standard references which are not given by Newton. It is neither scientifically consistent not logical. This issue needs to be discussed.

It is basic nature of human to speculate about existing laws and new possibilities which has been reason for development of science since beginning.

4.6 Some significant questions.

When equation F =ma was associated with Newton's second law of motion and by whom?

It is evident that Newton did not mention about acceleration throughout his scientific career. Also, Newton did not write F =ma. Even in research [7] and pedagogical **[14]** journals it is discussed that Euler has given F =ma and should be associated with Euler's name. Thus, it is confirmed that Newton did not write F =ma, this deduction is obvious from various aspects of current discussion. In the existing textbooks and references, Newton is regarded as sole discoverer of F =ma.

But there are also some evidences in the existing literature that F=ma should not be credited to Newton; but these are not given due consideration and credit of discovery of F =ma is given to Newton only.

Consider the followings:

I Bernard **Cohen [11]** has mentioned at pages 116-117

"Newton did not write any equation for his laws."

I Bernard **Cohen [11]** has correctly mentioned at page 113 that

"Newton never actually made a formal statement of the second law in the algorithm of fluxions or the calculus."

Also an article published in American journal of Physics [12] (2011) by **Bruce Pourciau** at page 1015 states that –

"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."

V V Raman has published in an ace pedagogical or academic journal *The Physics Teacher* [8]in March 1972 issue at page 137…

"Although this remark was made over a decade ago we still find textbooks in which F =ma is called Newton's formula, and which make absolutely no mention of Euler's in this context. "

Who related F =ma with Newton's second law of motion? When it was related with Newton's second law of motion? These are unanswered questions.

Why genuine equation F =kdV based on Newton's second law of motion is neglected?

Why equation

(mv-mu) = /dt (mv-mu) is regarded as correct?

It is discussed in section (2.1), that equation is not justified.

However it is obvious that equation F =ma (F =mdV/dt) was initially directly given by Jacob Hermann in 1716 in his book *Phonoromia*. Euler has derived the same equation as F = md^2x/dt^2 =ma in 1775. Earlier in 1736,1749,1752 and 1765 Euler has given different equations relating to force, mass and acceleration. Now it is regarded as Newton's second law of motion. So above questions appear to be irrelevant. But it is required to be answered.

It really required patient and impartial understanding of history of science, and opens new avenues historical research. The basic laws must be clearly studied from all angles.

4.61 '**First Three Sections of Newton's Principia' published in 1871'**

Here the oldest pedagogical book available for discussion is published in 1871. It is titled '**First Three Sections of Newton's Principia'[13]** designated as Cambridge School and College Text Books published from **London.** The authors of book are John H Evans and P T Main. It quotes definition of Newton's second law of motion as given in the Principia, but F = mdV/dt $=md^2x/dt^2$ ma has not been quoted at all even in 1871 i.e. 229 years after publication of the first edition of the Principia (1686). Acceleration was discovered by Galileo and published in 1638 in his book *Dialogues Concerning Two New Science* at page 133-134, 146. And equation $F =md^2x/dt^2$ was derived by Euler in 1875 in paper **E479 [18]** in paper *Novi Commentarii academiae scientiarum Petropolitanae* 20, 1776, pp. 208-238 (http://eulerarchive.maa.org/).

So, $F =m\, d^2x/dt^2 = ma$ was not quoted as equation for second law of motion in the book **First Three Sections of Newton's Principia,** 1871. Otherwise it (equation for second law as F =ma) would have been mentioned in textbooks for students (meant for Cambridge School and College textbooks). The other references can be searched in literature. Apparently books available in libraries of Universities of Cambridge and Oxford, and other renowned institutions would be helpful in this regard. The acceleration 'a' is defined by eq.(27) as given by Galileo in the Dialogues in 1638. For long time its extensive discussions and applications were not considered. But it was readily recognized when applications of differential and integral calculus were considered. The learning of science is continuous process. All this discussion is also supported by facts from the existing literature the various quotations are cited above.

The significant issue left for further discussion are:
Who related F =ma with Newton's second law of motion? When it is done? What are the reasons given for relating F=ma with Newton's second law of motion? Also, why genuine equation (F =kdV) based on second law of motion is neglected? Was ever it was considered? This equation (F =kdV) was derived by method of proportionality like that of $F =Gm_1m_2/r^2$. Why equation mv-mu = d/dt (mv-mu) is regarded as valid? It is justified in section (2.1), the units, dimensions and magnitudes in LHS and RHS are different?

5.0 All bodies descend (fall) with equal velocity or zero acceleration: Newton

Further in this regard Newton has given statements which are not justified with the experimental observations. Newton clearly stated in the Principia bodies fall with equal velocity in vacuum. The reason is that Newton ignored acceleration throughout his research career. Newton's First Law of Motion, impressed force, innate force are based on essence of law of inertia. In addition to this Newton has given such statements which neglect accelerated motion and interprets phenomena in terms of uniform velocity when bodies fall in vacuum.

5.1 **Scholium of Corollary VI at page 31 of the Principia**

" When a body is falling, the uniform force of its gravity acting equally, impresses, in equal particles of time, equal forces upon that body, and therefore generates equal velocities."

The velocity of moving body is considered with respect to reference point A or frame of reference. **It implies that in** case of a falling body (w.r.t reference frame) force of gravity generates equal velocity or due to gravity the bodies fall with equal velocity. At every point body has velocities but of equal magnitude.

5.2 **Proposition XLI, General Scholium of the Book III of the Principia**.

"*Bodies projected in our air suffer no resistance but from air. Withdraw the air, as done in Mr. Boyle's vacuum, and the resistance ceases; for in this void a bit of fine down and piece of solid gold descend with **equal velocity.**"*

Let the body falls from reference point A with respect to which distance and time to be measured as required in equation S =Vt as body descend with equal velocity. But the velocities are always in classical region not relativistic region when measured with respect to reference point A. It is equally true for bodies moving upward and downward.

Now velocity in defined as
The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. The distance of same body is different from different points, so reference point is needed for quantitative measurements.
Here Newton discussed both types of bodies projected upwards and descending downwards. Newton has stated bodies projected in our air suffer …….

5.3 Bodies projected in air or projected upwards.

Let body is placed at some reference point A from that body is projected upwards then its times and distances are measured from reference point. Newton perceived that if bodies are projected upwards by external agency suffer resistance only due to air. However, bodies are also attracted by the earth due to gravitational force. Thus, external force is applied on the body and gravity compete with each other. If bodies are considered in vacuum, then resistance due to air is eliminated whether bodies are projected upward or descend. When bodies descend in vacuum then fall due to gravity only.

Let bodies are projected upward (in vacuum suffer no resistance from air) from reference point A with same external force, then lightest body is observed to rise to greater height than heavy body. In this case weight of body acts downward and external force in upward direction. We can throw upward lighter body and heavier body from suitable reference point with same force. The lighter body moves quickly and travels higher distance compared to heavier body. Both the bodies reach at respective highest points the velocities of bodies become zero. We can measure the difference in times both the bodies (higher and heavier) reach back at the reference point A.

The upward velocity of body at different points (hence acceleration) can be calculated while ascending. These should not only be theoretically assumed, as now scientists and technocrats have developed precise experimental techniques for measurement of velocity at various points. The quantitative conclusions drawn experimentally lead to concrete results.

When body projected upward then its acceleration is theoretically regarded as -g , -9.8m/s^2(while falls down its acceleration is regarded as +g). The body moves against the gravitational pull. Should in all cases when body move upwards the acceleration due to gravity be regarded as -9.8m/s^2 or it should be experimentally determined in vacuum. Now we have enough sensitive experimental techniques for measurements.

Definitely upwards deceleration must be experimentally determined for various bodies thrown upward with different forces. Theoretically it is regarded as – g for all ascending bodies, as +g for all falling bodies. But scientists have exceptionally sensitive instruments now (compared to 19th century) all observations can be taken experimentally and compared with theoretical perceptions.

Newton has given principle of launching of satellite in the Book *The System of the world* published one year after death of Newton in 1728. But the rockets are pushed upward with exceptionally higher external force. The escape velocity of any body is regarded as constant 11.2 km/s. When uniform velocity is body is more than 11.2 km/s (escape velocity of body from the Earth's gravitational pull) then it escapes earth's gravitational pull. The escape velocity is given by

$V_e = \sqrt{2gR}$

If upward velocity of body is uniform, then change in velocity and hence acceleration must be zero. But body moves upward with against force of gravity, so external force acts on it (both forces are competitive in nature. When body falls downwards then it only falls due to gravity and no external force is needed as it falls naturally. This is conceptual difference between upward and downward motion. In case of descending bodies, the body is attracted by the Earth only.

5.31 Descending or falling bodies

Newton stated that falling bodies (with respect to reference point A /frame of reference) due to gravity at different times fall with equal or uniform velocity in vacuum. Also descending bodies may be the lightest i.e., a bit of fine down (a part of lightest feather of young bird) or heavy body (the solid gold, the heaviest element discovered at that time) fall with equal or uniform velocity with respect to point A (from where it is dropped). The bodies may fall down with velocity 0.4m/s, 4m/s or less or more. Newton did not say anything about magnitude of constant velocity.

In vertical or horizontal motion Newton considered equal or uniform velocities during movement. In general scholium of Proposition XLII Problem II in Book III of the Principia . Newton also assumes that bodies also move in vacuum in circular orbits with uniform or equal velocity.

"And the parity of reason must take place in the celestial spaces above the earth's atmosphere; in which spaces, where there is no air to resist their motions, all bodies will move with the greatest freedom; and the planets and comets will constantly pursue their revolutions in orbits given in kind and position, according to the laws above explained."

So Newton assumed that in horizontal, vertical and orbital velocities of bodies are uniform or constant. Thus equal or constant velocity is main deduction from Newton's second law of motion.

Newton had never mentioned acceleration throughout his life as given by Galileo in 1638 i.e. 4 years before birth of Newton.

5.4 **Theoretical analysis**

Newton has stated that body descends (pass from reference point A at higher place to lower place) with equal velocity (no increment in velocity occurs) in the interval. Thus, Newton maintained that bodies descend in vacuum with equal or constant velocity (or zero acceleration) i.e., no increment in velocity of body occurs in the interval irrespective of time. Galileo has discussed both accelerated and uniform motion in the Dialogue. Galileo has given law of inertia in terms of uniform motion. Like Galileo, Descartes and Huygens also gave the law and hypothesis in terms of uniform velocity. Newton chose to discuss uniform motion only. Newton neglected accelerated motion.

If graph is drawn (time on x-axis, velocity on y-axis); then for body (say wooden ball) falling with equal velocity (0.4m/s, say) then it would be straight line parallel to x-axis (S=Vt).Newton did neither give magnitude of equal velocity nor any method to determine it; thus he made purely qualitative statement. So, velocity of body remains same irrespective of time when it descends in vacuum.

If bodies descend in medium then upthrust (as given by Archimedes principle) is taken in account and relevant other factors like shape of bodies. But upthrust in vacuum is zero. It must be noted that Archimedes (288-212 BC) may have given his principle in 250 BC but mathematical equations became feasible in 1888 when standard value of acceleration due to gravity g is determined equal to 9.80665 m/s². The quantitative equation for Archimedes Principle became feasible after 2138 years of its enunciation.

Upthrust = Volume x density of liquid x 9.80665 m/s²

The body displaces volume of liquid from vessel equal to its own volume.

Newton has neglected acceleration as defined by Galileo at page 133-134, 146; and preferred to choose law of inertia (bodies move with uniform or equal velocity given at page 195) in terms of uniform velocity.

5.5 Quantitative Explanation

Newton neither suggested value of equal velocity of descending body nor gave any method to estimate it. Thus he qualitatively mentioned about falling body with equal velocity. So Newton explained phenomena qualitatively without giving any mathematical equations. Here Newton has stated (in corollary VI page 31 and **Proposition XLI, General Scholium of the Book III**) that bodies descend with equal velocity.

Thus Newton stated all bodies (with respect to reference point A/frame of reference) i.e. bit of fine down (soft feather of young bird) or lighter body and a piece of solid gold (heavier body); descend downwards with equal or constant velocity. Thus this velocity may be 0.4m/s, 4m/s , 44m/s , less or more , but velocities remain in classical region not in relativistic limits. The distance and time are measured with respect to reference point A as required in equation S =Vt.

When a cork falls from the rest from a reference point A then its velocity becomes 9.8 m/s after 1s; 19.6 m/s after 2s, 29.4 m/s after 3s , 39.2m/s after 4s etc. So, in first second (0-1s) velocity of body increases from 0 to 9.8 m/s and in next, second (1s-2s) the velocity varies from 9.8 to 19.6m/s. We can experimentally judge increase in velocity in 0-0.5 seconds or in 3.0-3.4s experimentally. There are enough sensitive equipment for verification. Like this velocity of bodies consistently vary in the interval in vacuum. Likewise, velocity of bodies projected upward can be measured continuously at various instants. In the current literature acceleration is regarded as -g.

Thus, velocity is variable, not equal in the interval. So, Newton's deduction that all bodies descend with equal velocity (velocity may be 0.4m/s, 4m/s, 44m/s, less or more) is not justified. Newton's deduction is qualitative as he had never given exact value of velocity nor any method to determine the same. Now Newton has written that body falls or descends with equal or uniform velocity, then distance travelled is given by equation,

S =Vt

5.6 **Measurement of standard gravity in 1888**.

The standard acceleration due to gravity was measured in 1888 equal to 9.8099 m/s^2 , The International Bureau of Weights and Measures (BIPM) was established in **1875.** Thus standardization of various physical quantities started about 189 years after first edition of Newton's Principia. **The** Third General Conference on Weights and Measures took place in 1901 and adopted this value for acceleration due to gravity as standard.

6.0 How equation F =ma obtained in history of science?

Galileo gave law of inertia at page 195 of the book Dialogue Concerning Two New Sciences published in 1638. The law of inertia simply states body maintains its uniform velocity in straight line if resistive forces are not present in the system. The force is not related with velocity. Galileo defined and explained acceleration but did not express acceleration with motion of bodies as in case of uniform motion and law of inertia.

Afterwards Descartes (1644) and Huygens (1673) expressed their laws in terms of uniform velocity. They too did not relate force with motion of bodies. In 1686 Newton also expressed first law of motion just like law of inertia. Newton stated his second law of motion in terms of force i.e. proposed dynamical system. Newton did not give any equation for second law of motion i.e., relating force with motion in any edition of the Principia (1686,1713,1726). Newton also did not use word acceleration in the Principia.

6.1 The **first appearance of equation F =MdV/dT**, **in Hermann's Phoronomia**

Jacob Hermann [6] in 1716, has given equation directly without derivation in his book *Phoronomia* at page 57. **Cohen [11]** has commented about this at page 113

"Newton never actually made a formal statement of the second law in algorithm of fluxions or the calculus. The first person to do so seems to have been Jacob Herman in his Phoronomia (1716), in which he writes

$$G = MdV : dT$$

where he says G signifies weight or gravity applied to a variable mass M."

dV is change in velocity and dT change in time. Cohen has related the equation with Newton's second law of motion, so G (weight or force) can be regarded as force and dV/dT is acceleration.

$$G = MdV/dT = F \qquad (1)$$

or $F = mdV/dt$

Cohen[11] has mentioned that Hermann has quoted the differential form of second law of motion. According to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann. Thus eq. (1) follows from Hermann's direct interpretation as given in his book *Phoronomia* at page 57 published in 1716.

6.2 Leonhard Euler's various equations relating to mass, acceleration and force.

Euler has derived $F = md^2x/dt^2$ in his paper in 1775. Apparently the work of Swiss theoretical physicist and mathematician Leonhard Euler [1703-1783] about equations relating to impressed force and acceleration is not properly interpreted by Cohen [11]. It is pertinent to mention here, that Mathematical Association of America (MAA) has complied all works (papers, books, scientific documents) of Euler, 866 in number on line at http://eulerarchive.maa.org/. [13]**.**

This section of website is initially developed in 2007-2008 and continuously updated. Each document is given unique Enestrom number (E1 to E866 etc.) for easy access of the documents. All the writings are originally in the Latin language and some of them are also voluntarily translated to English. It is deliberated in The Euler Society Conference 2014 that Euler's contribution [8] which may not be accessible earlier; but now it readily available online at **website [13].** Thus, Euler's significant work may be given due consideration as it relates force, mass and acceleration i.e. F =ma. Earlier Euler's work is either misinterpreted or under estimated.

It is believed by scientific community on the basis of paper of Clifford Truesdell [7] published in 1960 that Euler is the first who published F =ma, in paper titled "Discovery of a new principle of mechanics", published in 1752 (earlier in presented in Berlin Academy 1750). This paper **[13]** has (Enestrom number E177) in the website http://eulerarchive.maa.org/. However this belief is not based on the scientific facts, as Truesdell has neither quoted nor reported Euler's above paper properly as Euler has written in it F =2ma; not F =ma in Truesdell's paper. Truesdell [7] has written the equations

$$F_x = Ma_x,\ F_y, = Ma_y,\ F_z = Ma_z, \qquad (29)$$

where the mass M may be either finite or infinitesimal, as the axioms which " include all the laws of mechanics". Truesdell [9] stated that eq.(16) is quoted by Euler in 1752, but it is not true .

In fact Euler [13] has quoted in his paper E177 at page 196 available at website [13]

$$F_x = 2Ma_x,\ F_y, = 2Ma_y,\ F_z = 2Ma_z, \qquad (30)$$

The RHS of equation cannot be arbitrarily divided by 2. So there is scope for improvement in Truesdell's paper [7]. The factor 2 cannot be omitted willfully as done by Truesdell. F=ma is basic equation in physics so all existing results vary by factor of 2. The division of RHS of any equation by any number is illogical. It must be noted that Euler has derived the set of equations independently, without using Newton's second law of motion. It must be noted that Truesdell has either edited or co-edited six volumes of collected works of Leonhard Euler, in addition to his own original work in mechanics. But there is always scope for betterment or improvement. The same set of equations (30) has also been quoted by Euler in paper Enestrom number E112 at page 103 published in 1747. Euler has given equation $F = md^2x/dt^2$ in 1775 in paper E479. Euler's all papers and books (E15, E112, E117, E289, E479 etc.) are available online at **website [13],** http://eulerarchive.maa.org/

(a) In Euler's book Mechanica Vol I (Enestrom number E15) originally written in the Latin published in 1736 **[34].** This book is translated by Ian Bruce, Chapter II, page 59, Proposition 17 Euler has written

"The force of inertia of any body is proportional to the quantity of matter, upon which it depends."

$$F = k\,M \qquad (31)$$

Thus Euler related mass with force of inertia directly. It is true that force is expressed as $F = Ma = MdV/dt = md^2x/dt^2$ by Euler later on.

(b) Euler **[34]** in his book Mechanica Vol. 1 (Enestrom number E15) at page 64 in chapter II Euler has given equation of force as

$$F = ma/n \quad (32)$$

where n is constant and a is acceleration.
Originally Euler has written at page 64 corollary I

$$dc = npdt/A \quad (33)$$

or $p = Adc/ndt$ or $p = mdc/dt\,n$ or $F = ma/n$ (34)
where dc is change in speed or velocity, p is force, A is mass of body or particle , and n is constant.
(c) In his paper E112, presented in 1747, originally published in Mémoires de l'académie des sciences de Berlin 3, 1749, pp.103, has given equation of force as **[13]**

$$F =2Ma \quad (34)$$

Originally Euler has written **[13]**

$$X = m2ddx/dt^2, \quad Y = m2ddx/dt^2, \quad Z = m2ddx/dt^2 \quad (35)$$

where X, Y, Z are forces along x, y, and Z axes.
or $F = 2ma$ (36)
(d) The same equation has also been quoted in paper E177 at pp. 6 (published in 1752) in English and in paper E177a at page 6 in Latin. Stacy Langton has translated this paper in English **[13].**

I. $2m\, ddx = P\, dt^2$, II. $2M\, ddy = Q\, dt^2$, III. $2M\, ddz = R\, dt^2$ (35)
or $F = 2ma$ (36)

This paper is widely discussed in literature [6] that Euler has given F=ma, however paper states F =2Ma.
(e) In 1765, in his book Theoria motus corporum solidorum seu rigidorum (Theory of the motion of solid or rigid bodies) E289, http://eulerarchive.maa.org/, Chapter 4 , page 135) Euler has given equation of force

$$F = ma/2g \quad (37)$$

g is constant used by Euler, not acceleration due to gravity as commonly known now.
Originally Euler has written that **[13]**

$$ddx = 2gpdt^2/A \quad (38)$$

where p is force, A is mass of body or particle and g is constant .
Thus ,

$$p = 2Addx/dt^2\, g = ma/2g \quad (37)$$

(f) Euler [14] in 1775 in his paper E479 titled *Nova methodus motum corportum rigidorum degerminand* , completed the construction of general equations of dynamics by formulating a system of six equations determining the motion of any body, which (except for an additional coefficient) he wrote in the following way at page 222-23.

$$P=\int dM \frac{d^2x}{dt^2}, Q=\int dM \frac{d^2y}{dt^2}, R= \int dM \frac{d^2z}{dt^2} \quad (38)$$

Or in general, $F = \int dM \frac{d^2s}{dt^2} = ma$ (39)

Thus Euler directly related equations with mass and accelerations ; and also gave equation F =ma.

7.0 Newton's Second axiom or Law of Motion in the Principia in Latin.

Explanation

The original Latin version of Newton's law is [1,14]
Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimatur.
The literal English translation of this would be [2,14]

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

F (impressed *at once, or gradually and progressively)* ∝ alternation or change in velocity
or F (impressed *at once, or gradually and progressively)* = k alternation or change in velocity (11)
In simple notation,

$$F =kdV \quad (11)$$

This derivation is based on proportionality like law of gravitation ($F \propto m_1m_2$, $F \propto 1/r^2$ or $F \propto dV$). The eq.(11) is derived in section (2.8)

7.1 Different definitions of the Principia's second axiom or law of motion in literature.

Different definitions of second law of motion on behalf of Newton : Illogical and inconsistent

Definition and explanation of the second law of motion is in terms of motion (velocity) not acceleration. The definition of second law of motion implies change in motion. In the explanation Newton stated that -

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively.

Thus, both in definition and explanation Newton has used change in motion (velocity); but not acceleration as used by scientists above. So, interpretation of second law of motion in terms of acceleration is not justified. Thus, arbitrary interpretation is just to obtain F =ma from second law of motion. The equation for second axiom or law of motion is given by different authors in different ways in the following three examples:

1. The Encyclopaedia Britannica [15] states the second law of motion as

"The net unbalanced force producing a change of motion is equal to the product of mass and the acceleration of particle."

2. "The net (unbalanced) force acting on material body is directly and linearly proportional to, and in same direction as, its acceleration." Holten [16]

3. 'when the resultant force is not zero the body moves with accelerated motion, and the acceleration, with a given force, depends on property of the body known as its mass.' [17]

The above definitions of second law are not quoted in *the Principia.* The prudent authors quote them as definitions given in *the Principia.* But these are altered or distorted definitions of second law of motion, as not given in *the Principia.* It is both unethical and unscientific to quote Newton's law in altered form, and giving it name of Newton. The original definition of Newton's second law of motion as given in the Principia is given by

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Whereas the definitions of first and third laws of motion are same in textbooks or reference books and in the Principia. Why there is only alteration in second law of motion? The definition and explanation given by Newton is in terms of motion (velocity) but altered definitions are in terms of acceleration. Newton has neglected acceleration throughout his life of 85 years. The reason for such alterations is that equation F =ma may be obtained from second law of motion.

7.2 Mathematical Equations

The learned authors present the definition of Newton's second law of motion in different ways, but their mathematical equations are same i.e., F =ma, not eq. (11) as derived in section (2.8).

The mathematical equation based on Newton's definition can be written as eq.(11).

$$F \propto dV \quad \text{or} \quad F = kdV \qquad (11)$$

(i) The statement for force in Encyclopaedia Britannica implies that (force equals product of mass and acceleration)

$$F = ma \qquad (1)$$

The same equation has been quoted by Resnick [31] in book Physics Part I without mentioning the definition given by Newton in the Principia. Resnick did not quote any definition of second law of motion but only equation as F=ma.

Resnick and Encyclopaedia Britannica quoted the first and third law in the same way as in Newton's Principia.

(ii) Equation of force in view of definition given by Holton (force is directly proportional to acceleration) [16]

$$F \propto a \quad \text{or} \quad F = K_1 a \qquad (40)$$

where K_1 is constant of proportionality. It must be regarded equal to m

$$F = ma \qquad (1)$$

Thus K_1 is not constant but coefficient of proportionality as it varies from one form to other.

(iii) 'when the resultant force is not zero the body moves with accelerated motion, and the acceleration, with a given force, depends on property of the body known as its mass.' [17]

$F \neq 0$, motion is accelerated, force depends on mass

$$F = \text{Constant } m = K_2 m \text{ (when motion is accelerated)} \qquad (41)$$

If the value of K_2 is regarded as unity to define unit of force (dyne or newton). K_2 is dimensionless and unity in magnitude. Thus,

$$F = ma \qquad (1)$$

7.3 Inconsistencies between above definition and equation.

(a) The above definitions and equations are in terms of acceleration. But it is confirmed that Newton has ignored acceleration throughout his scientific career.

(b) Newton has not written equation F =ma for second law of motion but here the equation is quoted purposely. Also, Newton did not interpret equation F =mdV/dt for second law of motion, even when he had opportunity to do so.

Constant of proportionality.

Thus if law is stated arbitrarily then values of constants of proportionalities are also different (K_1 = mass of body, m). The force will remain unity if mass is regarded as ½ kg and acceleration is 2m/s^2. The value of constant of proportionality is experimentally calculated or determined as shown in eqs.(9-10) in section (1.1).

$F = GmM/r^2$ (9)

The value of universal gravitational constant was measured for first time in 1798 by Cavendish and its measurements are continue even now **[34].**

$G = 6.6743 \times 10^{-11}\ m^3kg^{-1}\ s^{-2}$ (10)

But here constant of proportionality is regarded as unity.

7.4 **The first and third laws are taught in the same form as given in the Principia**

Newton's the first and third laws of motion are quoted in the same way as given in the Principia. But the definition of second law of motion is not quoted as given in the Principia. Or altered definition of second law of motion is quoted in the reference books. When learned authors quote the altered form, they do not quote Newton's original second law of motion with them for comparison. Thus, incomplete information is given to the reader. However, first and third laws are quoted in textbooks, as given in the Principia. Newton's second law of motion is quoted in different ways by different authors.

It would be prudent to quote the original form of the law as well (as given by Newton in the Principia). If authors are not quoting their own forms of second law of motion along with Principia's original form, then they are hiding something.

Further authors who give deviated form of Newton's second law of motion in the reference books; may argue that the definitions quoted by them, also follows from Newton's second law of motion. Then the prolific science authors and scholars must justify that how their deviated forms are same as Newton's form of second law of motion. If Newton's original definition of second law and their deviated definitions are the same, then why to quote the deviated definition; then best option would be to quote original definition only. There will not be any issue. How Newton's original form of the second law of motion leads to same mathematical equation as given by their changed from?

If Newton's original form of second law is also quoted in the textbooks then readers would raise questions for deviations. There should be transparency in presentation of scientific laws.

If somebody quoted Einstein rest mass energy is $E = mc^3$, and stressed it is given by Einstein. Then it is absolutely wrong, the reason being that reader knows that Einstein's rest mass energy equation is $E = mc^2$. In fact, $E = mc^3$ is not dimensionally inconsistent. The reader does not know Einstein's rest mass energy equation is $E = mc^2$, then he would believe that $E = mc^3$ equation is given by Einstein. If this perception persists for longer time then it may be difficult to convince the readers that original form of mass energy equivalence is $E=mc^2$, not $E = mc^3$.

8.0 V V Raman's inconsistent Interpretation

The hidden aim for Raman's interpretation is to obtain F =ma from definition of Newton's second law of motion. In fact definition of Newton's second law of motion gives equation F =kdV .The prevalent form of Newton's second law of motion is F =ma There are two ways to obtain F =ma.

(i) Firstly write the definition of second law of motion in altered form (i.e. different way as given in the Principia), it is done by some authors as given in the section (7.1)

(ii) Secondly, write the original definition as given in the Principia but make arbitrary assumptions, V V Raman has done so by writing , motion as momentum.

Motion = mV (4)

and assuming change in motion as rate of change of momentum

mv-mu = d/dt (mv-mu) (5)

It is confirmed in sections (2.1) , these are arbitrary assumptions as units, dimensions and magnitudes of Left Hand Side and Right Hand Side are different. It is highlighted in the section (8.1).

Thus, ultimate aim of authors is to obtain equation from F =ma from the definition of second law of motion; but both the approaches are not scientific. It can be easily understood as discussed below.

Raman's **interpretation [14]** of second law of motion in March 1972 issue of the Physics Teacher at page 136

"*By 'motion' Newton meant 'quantity of motion' which he had defined as the product of mass and velocity, i.e. what he would call momentum. The crucial expression is 'change in momentum'. The usual tendency is to take this to mean 'rate of change of momentum*".

It is discussed in details that motion can be regarded as velocity. The motion cannot be regarded as mV. It is discussed in section (3.8).

It can be understood in two parts:

(i) "*By 'motion' Newton meant 'quantity of motion' which he had defined as the product of mass and velocity,*

i.e. what he would call momentum.
Motion is velocity: It is justified in sections (3.8) that motion is velocity. Newton defined quantity of motion at page 1 in Definition II. Newton defined motion at page 9 in the scholium. Newton further stated motion in terms of absolute motion and relative motion. Newton did not state motion is momentum (mV). When Newton defined absolute and relative motions, these are nothing but velocity. Thus it is explained at sections (3.8) motion is velocity. Thus, V V Raman's deduction is not consistent.
(ii) *The crucial expression is 'change in momentum'. The usual tendency is to take this to mean 'rate of change of momentum*".
Further it is stated that the ***usual tendency*** to regard *'change in momentum'* as '*rate of change of momentum'.*

8.1 Usual Tendency…. Is it scientific term?

change in momentum = rate of change of momentum with time (usual tendency) (5)

There is no law in physics which implies that in 'usual tendency in change in momentum' is regarded 'as rate of change of momentum w.r.t time'.
Usual tendency of change in momentum ≠ Rate of change of momentum with time

$$mv-mu \neq d/dt\,(mv-mu)$$

The scientific laws are interpreted not by usual tendency but specific repeatable scientific logic. It is scientific not political or philosophical interpretation.

In physics we deal with exact and specific interpretations not with tendencies. Raman may have called it 'usual tendency' as all preceding scientists were interpreting it like eq.(5). In physics we have specific arguments.

$$\text{Change in momentum} = mv-mu \qquad (42)$$

It is entirely different from 'rate of change of momentum with time '

$$\text{Rate of change of momentum} = m\,(v-u)/(t_2-t_1) \qquad (43)$$

Raman has written that
'usual tendency of change in motion', is equal to rate of change of momentum.

$$mv-mu = m(v-u)/(t_2-t_1) \qquad (5)$$

But it is not consistent
Eq. (5) is not justified as units, dimensions and magnitude of both Left Hand Side and Right Hand Side are different. It is justified as both Left Hand Side and Right-Hand side have different units (kgm/s & kgm/s^2), dimensions (MLT^{-1} & MLT^{-2}) and magnitudes. These basic issues of physics /science cannot be ignored just to obtain pre-supposed F =ma from definition of second law of motion. The logics are supreme in science, not the results.

Left Hand Side	Right Hand Side
Units m/s	Units m/s^2
Dimensions M^0LT^{-1}	Dimensions MLT^{-2}
Magnitude mv-mu	Magnitude $m(v-u)/(t_2-t_1)$

Hence we find that

$$mv-mu \neq m(v-u)/(t_2-t_1)$$

According to Newton's second law of motion:

Force $\propto$ change in motion

According to V V Raman, $mv-mu = m(v-u)/(t_2-t_1)$

or $F \propto$ rate of change of momentum with time
or F = k rate of change of momentum with time.

$$F = k\,m(v-u)/(t_2-t_1) = kma$$

However practically change in momentum (mv-mu) is not equal to rate of change on momentum with time.
It is not justified.

$$F \neq k\,m(v-u)/(t_2-t_1) \neq kma$$

or $F \neq ma$
Hence F =ma is not obtained in Raman's interpretation, it is the actual interpretation.
If there are inconsistencies in precedence then it must be logically improved scientifically, instead contributing to inconstancies by silent approval. This inconsistency is used in deriving F =mdV/dt or F =ma from second law of motion in existing literature.

8.2 Rate of change of momentum may also vary with distance, as $d(mv-mu)/(x_2-x_1)$

Raman have written –
'*The crucial expression is 'change in momentum'. The usual tendency is to take this to mean 'rate of change of momentum*'.
Thus,

'Change in momentum' means 'rate of change of momentum '

Raman wrote 'rate of change of momentum' as 'rate of change of momentum with time', $m(v-u)/(t_2-t_1)$, it is

usual tendency.
But it can also be 'rate of change of momentum with distance', $m(v-u) /(x_2-x_1)$.

It can also be interpreted as 'rate of change of momentum with distance' or with some other quantity; it is not necessarily rate of change with time. It is not only assumed by Raman, but also by preceding and followings scientists.

Raman has simply written that crucial expression change in momentum is regarded as 'rate of change of momentum with time'.
This expression may also be regarded as 'rate of change of momentum with distance' i.e.

$m\, d(v-u) /(x_2-x_1)$ or mdv/dx

This is equally feasible. Thus, in this case F =ma can never be obtained.

8.3 Reason for arbitrary manipulations.

Newton did not give any equation in the Principia. Newton also ignored acceleration in the Principia. The equation based on definition of Newton's second law of motion is F =kdV (Impressed force $\propto$ change in motion or velocity). This derivation is similar to law of gravitation $F = Gm_1m_2/r^2$ ($F \propto m_1m_2$; $F \propto 1/r^2$). The equation $F = Gm_1m_2/r^2$ is used but equation F =kdV is not even mentioned. When differential and integral calculus was developed then acceleration was found exceptionally useful term and it was associated with second law of motion as F =ma.

Next step was to derive F =ma from definition of Newton's second law of motion. Realistically equation F =kdV
follows from definition of the second law; but the equation F =ma does not follow from it in any way.
Realistically Newton never wrote any equation including F =ma.
(i) In pursuit to derive F =ma from second law of motion; the scientists assumed that motion as momentum (mV). Thus

$F \propto$ change in motion
or $F \propto$ change in momentum or $F = k(mv-mu)$

which is not F =ma. Thus under this assumption F =ma is not obtained.
(ii) Then scientists assumed that change in motion is equal to rate of change of motion i.e.

Change in motion (momentum) = rate of change of momentum = $d/dt\,(mv-mu)$ (5)

$F \propto d/dt\,(mv-mu)$ or $F = k\, d/dt\,(mv-mu) = kma = ma\ (k=1)$

Thus scientists assume eq.(5) which is completely inconsistent as discussed in section (8.1). Thus, it has been arbitrarily done to obtained F =ma from definition of second law of motion. The scientific logic is the most important in science.
(iii) Then scientists also altered definition of the law on behalf of Newton as shown in section (7.1). So, all these inconsistencies have been done to obtain F =ma, which can be easily and logically obtained by generalizing the definition of second law as in section (14.0). The following scientists are responsible for inconsistencies not Newton.
Thus, scientists have just one motive to derive F =ma, irrespective of scientific logic. However, the most logical step is that to modify or generalize Newton's second law of motion is to obtain F =ma; in this case we need to have inconsistent steps. It is evident from section (14.0).

. **9.0 Phrase 'in a given matter' and 'in a given time'**
'In a given matter' is mass m , 'In a given time' is time t

(i) Let us consider explanation in Principia's Book II , Proposition XXIV , Theorem XIX [12] ,
" For the velocity, which a given force can generate in a given matter in a given time, is as the force and the time directly, and the matter inversely.
The greater the force or the time is, or the less the matter, the greater velocity will be generated. This is manifest from the second law of motion."

This proposition can be can easily understood as it is in proportionality form as in case of law of gravitation ($F = Gm_1m_2/r^2$ ($F \propto m_1m_2$; $F \propto 1/r^2$).

It implies higher the impressed force acts on body, higher is the velocity attained by body ($V \propto F$).
If the force acts on body for longer time more would be velocity ($V \propto t$).
Further the force acts on the heavier body, then it moves with smaller velocity ($V \propto 1/m$).
So Newton has taken meaning of phrases 'in a given matter' and 'in a given time' as appears in the text. The second sentence justifies it.

Here meanings of phrases 'in a given matter' and 'in a given time' are similar and very important to understand. Newton has himself defined the meaning of phrases 'in a given matter' and 'in a given time'; and in view of it wrote second sentence. It must be noted that meaning of both are identical. The second sentence offers the clear mathematical form of first sentence or both sentences are self-explanatory.

9.01 The phrase in a given matter and its meaning as clarified by Newton.

The phrases 'in a given matter' (in particular value of mass, say 1kg) and 'in a given time' (in a particularly stated time or in a specifically quoted time/instant) have simple meanings. Newton did not mention 'in a given time interval', so he meant in a specific time only. Here word 'a' means one or specifically quoted value in both the cases.

'In a given matter' ≡ in a given matter e.g. body of mass 1kg (say) or for body of mass m.

'In a given matter' ≠ d/dm

It is obvious that for the phrase 'in a given matter ' cannot be written as d/dm, there is no logic behind this. No other meaning can be given to the above phrase. For the phrase 'in a given matter' Newton simply mean in 'a given mass 'm'. It is evident from the second sentence.

So we should not arbitrarily introduce the same, as it suits our explanation or willfully projected results. So it is not prudent to give own and arbitrary interpretation to the terms to get desired results.

The meaning of phrase 'in a given matter 'is in no way implies 'ratio of change in quantity to change in corresponding mass' i.e.

$(Q_2-Q_1) / (m_2-m_1)$ or rate of change of quantity with matter (dQ/dm).

9.02 Phrase 'in a given time' and its meaning as clarified by Newton

Both the phrases 'in a given matter' and 'in a given time' are same and have identical meanings. Thus,

'In a given time' ≡ in a given time or instant e.g. 11 hours, 23 minutes and 46 seconds or at time t.

'In a given time ' ≠ d/dt

It is obvious that for the phrase a given time cannot be written as d/dt , there is no logic behind this.

So phrase 'in a given time' is time at a given instant e.g. 11 hours, 23 minutes and 46 seconds or at time t, not rate of change with time i.e. d/dt . For the phrase 'in a given matter' Newton meant in the given mass 'm'. It is clear from second sentence. The greater the force or time; greater velocity will be generated. Newton has never written in the Principia the **phrase [12]** 'rate of change' and acceleration in second law of motion; thus d/dt is not feasible.

The meaning of phrase 'in a given time' in no way implies that 'ratio of change in quantity to change in corresponding time' i.e.

$(Q_2-Q_1) /(t_2-t_1)$ or rate of change of quantity with time (dQ/dt) .

This interpretation is similar to phrase 'in a given matter'.

So, Newton did not write 'rate of change' in second law of motion, thus d /dt is not feasible.

Newton simply justified it in the second sentence that in a given matter mean (in given mass, 1kg say) or in body of mass m. Further phrase 'in a given time' mean 'at given instant' e.g., 11 hours, 23 minutes and 40s or at time t. Here Newton did not write 'in a given matter' is d/dm and 'in a given time' it is d/dt. It is obvious from second sentence, written by Newton. We should not interpret phrases for our own convenience.

9.1 Mathematical equation based on statement_Proposition XXIV Book II

Obviously, Newton stated in second sentence (Book II, Proposition XXIV) that velocity generated is directly dependent on applied force, time and inversely proportional to mass. Again, this interpretation is like law of gravitation, the force increases directly with product of masses of bodies; and inversely proportional to r^2 as summed up in eq. (9). Both the laws (second law of motion and law of gravitation) are given by same scientist in the same book.

Thus mathematically following proportionality is self-evident:

$V \propto F$; $V \propto t$; and $V \propto 1/m$

$$V = K_3Ft/m \qquad (44)$$

where K_3 is constant of proportionality.

Critical analysis of Eq. (44)

(i) If a body (say , a gunny bag filled with sand and stone having mass 50 kg) is pushed or pulled for 10s, with force F. The magnitude of velocity is given by eq.(44) as

$$V = K_3F10/m = 10(K_3F/m) = 10z \text{ m/s (say)} \qquad (45)$$

(ii) If the same body (say , a gunny bag filled with sand and stone having mass 50 kg) is pulled with same force F for 100s , under similar conditions then eq.(44) becomes

$$V = 100(K_3F/m)= 100z \text{ m/s (say)} \qquad (46)$$

(iii) If the same body (say , a gunny bag filled with sand and stone having mass 50 kg) is pulled with same force F for 500s , under similar conditions then eq.(44) gives value of velocity as

$$V = 500(K_3F/m) = 500z \text{ m/s (say)} \qquad (47)$$

Now following conclusions can be drawn from above equations as

(a) **Prediction** : When body is pulled with force F for 10s, 100s, or 500s, then predicted values of velocity are given by eqs.(45-47). Thus, at different times the values of velocity are predicted as 10z m/s, 100z m/s and 500z m/s. These predictions are not experimentally consistent as discussed below.

Experimental observations: But experimentally in this case the when a body (say , a gunny bag filled with sand and stone having mass 50 kg) is pulled with force F for longer time (10s, 100s, and 1000s) then velocity does not increase to 10V, 100V and 500V. The body moves with constant velocity; but distance travelled by body increases. The predictions of eq. (44) are clear contradictions.

The velocity will be more if body travels distance in shorter time. The velocity of body does not increase if force F is applied for longer time. This is basic difference between Newton's perception and experimental situation. Also, we have observation, with different references i.e. equation S =Vt or V =S/t i.e. if body travels distance S in shorter time; then more would be velocity.

Should we speculate an equation of force relating with velocity, distance and time for practical systems (resistive forces are present in systems). Thus, it is obvious that Newton never meant 'in a given time' as d/dt (rate of change with time). If it is so regarded then it is illogical and arbitrary. It simply implies that 'at a given time' Newton meant at a specific instant or time.

at a given time ≠ d/dt

Thus Newton himself never meant or wrote 'in a given time' d/dt; and 'in a given mass' d/dm. Thus there is always scope in every book.

9.2 Arbitrary meaning of phrase 'in a given time' in centripetal force.

It must be noted that Newton himself used phrase 'in a given mass' as m and 'in a given time' as time t in the section (9.0) in Proposition XXIV, Book II Theorem XIX. Thus, there was specific purpose of above discussion.

Definition V.

A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.

Newton has defined the centripetal force (centre seeking force) in Definition V that it acts towards centre of orbit when bodies (a stone, whirled about a sling or bodies revolved about orbit) etc. moves in circular orbit. So in this case it pertains to circular motion, not linear motion.

The three kinds of centripetal forces, the *absolute quantity of a centripetal force*, accelerative *quantity of a centripetal force, and motive quantity of a centripetal force* we may, for brevity's sake, call by the names of motive, accelerative, and absolute forces. It is like modulus of elasticity which has three types e.g. Young's modulus (Y), bulk modulus (K) and modulus of rigidity (R**).** All the moduli have same units and dimensions. Thus, various types of centripetal forces have same units and dimensions.

Definition VI: *The absolute quantity of a centripetal force is the measure of the same proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about.*

Definition VII: *The accelerative quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time.*

In Latin

Vis centripetæ quantitas acceleratrix est ipsius mensura Velocitati proportionalis, quam dato tempore generat.

Vis centripetæ quantitas acceleratrix est ipsius mensura Velocitati proportionalis, quam dato tempore generat.

Definition VIII: *The motive quantity of a centripetal force, is the measure of the same, proportional to the motion which it generates in a given time.*

Practical meaning of the *accelerative quantity of a centripetal force*: Further in definition VII of the Principia [2] Newton had defined at page 7.

"the *accelerative quantity of a centripetal force"* to the place of the body, as a certain power or energy diffused from the centre to all places around to move the bodies that are in them."

Accelerative quantity coined by Cohen, Newton has given accelerative force.

Here first thing is that Newton never wrote, term 'accelerative quantity', it is coined by Cohen **[11]** at page 103. In fact Newton has written in short form 'accelerative force' for accelerative quantity of centripetal force. Newton has specifically mentioned 'accelerative quantity of centripetal force' in case of centripetal force (denoted by accelerative force).

Cohen's terminology can be misleading; also, it is not mentioned by Newton.

9.21 Acceleration

Galileo has defined acceleration at pages 133-134 and 146 of *the Dialogues* published in 1638. The value of acceleration is given as :

Linear acceleration = change in velocity /change in time (27)

Let body starts from the rest then after 1s, 2s, 3s and 4s its velocity will become 1m/s , 2m/s , 3m/s , 4m/s respectively then it will fall with uniform acceleration as

Linear acceleration = change in velocity $(v_2\text{-}v_1)$ /change in time $(t_2\text{-}t_1)$ = $1m/s^2$ (27)

Galileo has defined acceleration as cited above in case of linear motion.

Cohen [11] has written at page 103
"The 'accelerative quantity' is the measure of gravitational force "proportional to velocity it generates in a given time." In modern terms, this is dV/dt, or the acceleration."
Galileo has defined acceleration for linear motion as in eq.(27). Newton has defined phrase 'accelerative force' for "accelerative quantity of centripetal force" for centripetal force; logically and scientifically it has units and dimensions of force ($kg.m/s^2$, MLT^{-2}) **.** But Cohen has called centripetal force ("accelerative quantity of centripetal force.") as dV/dt (linear acceleration) which is not justified. The centripetal acceleration is v^2/r (m/s^2. M^0LT^{-2}).

9.22 Phrase 'in a given time' means 11 hours, 23 minutes and 46 seconds or at time t.

Now Cohen's above phrase '**'**accelerative quantity' for "accelerative quantity of centripetal force" cannot be regarded as acceleration. Newton has called it in short 'accelerative force'. It can be easily justified. Let us quote **Newton's** Proposition XXIV, Book II. It is already discussed in section (9.0) that phrase
'In a given time' ≡ in a given time or instant e.g. 11 hours, 23 minutes and 46 seconds or at given instant t .
Newton has also used this phrase Proposition XXIV, Book II Theorem XIX as
'In a given matter' ≡ in a given matter e.g., body of mass 1kg (say) or for body of mass m.
'In a given matter' ≠ d/dm
'In a given time' is not 'a rate of change' i.e., d/dt which is never used by Newton in the Principia. Newton has not written acceleration and rate of change in second in law of motion **[12].** Or rather has not defined as acceleration.

The 'velocity which it generates in a given time' ≡ velocity of body at 10 hours, 12 minutes and 23 seconds (say). The 'velocity generated in a given time' is not dV/dt i.e. rate of change of velocity. It is justified from current interpretation and existing literature.
Cohen **[11]** at page 113 has correctly written that
"Newton never actually made a formal statement of the second law in the algorithm of fluxions or the calculus."
Also, an article published in American journal of **Physics [12]** (2011) at page 1015 states that –
"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."
In Newton's terminology fluxions means derivative. So, Newton did not write a =dV/dt. Cohen **[11]** stated that Hermann's equation F =mdV/dt represents differential form of second law of motion. But in the third and final edition of the Principia Newton did not acknowledge F =ma as equation for second law of motion or did not acknowledge acceleration as a =dV/dt.

9.23 **More** m**isinterpretation by Cohen**

Cohen [11] misinterpreted ion Definition VII, the phrase 'velocity generated in given time' as dV/dt (fluxion or derivative). Thus, it is evident that
'velocity generated in a given time' ≠ Change in velocity w.r.t. time ≠ dV/dt or rate of change of velocity.
'velocity generated in a given time' = velocity at 8 hours, 22 minutes and 43 seconds (say).
It is velocity of body at 8 hours, 22 minutes and 43 seconds (say). Newton himself explained the phrase 'velocity generated in a given time' as velocity of body at given instant, in proposition XXIV, Book II. Also, 'in a given mass' it is interpreted as body of mass m.
'In a given matter' ≡ in a given matter e.g. body of mass 1kg (say) or for body of mass m.
So accelerative quantity of centripetal force is not linear acceleration (dV/dt) as deduced by Cohen. The centripetal force acts towards the center.

It is already concluded independently that Newton did not write **[12]** phrase 'a rate of change' and linear acceleration in the Principia. So, we cannot write 'rate of change' and 'acceleration' here. Newton did not write 'acceleration' neither in definition nor explanation.

Acceleration was defined by Galileo in 1638 in book *Dialogues Concerning Two New Sciences* at pages 133-134, 146 i.e. four years before birth of Newton. It is discussed in the following section that Newton has ignored acceleration (did not mention as it existed in his time given by Galileo 4 years before birth of Newton.)
But Newton ignored it throughout his life time.

So it is completely arbitrary and inconsistent to introduce by acceleration by misinterpreting terms, just to obtain F=ma, from definition of second law of motion. Newton never wrote F=ma in his life time. It must be noted the genuine equation based on second law of motion is F =kdV.

9.3 Galileo's insight of uniform acceleration
At pages 7-8 Book I in the Principia.

" Thus the force of the same load-stone is greater at a less distance, and less at a greater: also the force of gravity is greater in valleys, less on tops of exceeding high mountains; and yet less (as shall hereafter be shown), at greater distances from the body of the earth; but at equal distances, it is the same everywhere; because (taking away, or allowing for, the resistance of the air), it equally accelerates all falling bodies, whether heavy or light, great or small."

In Latin

Uti Virtus Magnetis ejusdem major in minori Distantia, minor in majori: vel vis gravitans major in Vallibus, minor in cacuminibus prœaltorum montium (ut experimento pendulorum constat) atq; adhuc minor (ut posthac patebit) in majoribus distantiis a Terra; in œqualibus autem distantiis eadem undiq; propterea quod corpora omnia cadentia (gravia an levia, magna an parva) sublata Aeris resistentia, œqualiter accelerat.

Both the Latin original and English translations have been quoted for clarity. Newton has mentioned that it equally accelerates (*to move faster)* all falling bodies downwards in vacuum. Newton had not used acceleration throughout his life of 85 years, it existed in the literature before birth of Newton. Newton has expressed that bodies fall down with equal or uniform velocity from the state of rest. Newton did not mention that bodies fall with equal acceleration i.e., velocity increases regularly as in eq. (27).

9.31 Newton ignored acceleration throughout his life

(i) Newton [2] did not mention word acceleration neither in new definitions (I-VIII) e.g. quantity of matter (mass), Quantity of motion (mV) impressed force, i.e. the innate force of matter (inertia) , centripetal force, various types of centripetal force nor in already known terms (time, space , palace and motion), absolute space , absolute motion etc. This significant term (acceleration) was not discussed neither in definitions nor known quantities which Newton discussed in the beginning of the Principia pages 1-9.

(ii) Further word acceleration does not occur neither in definition of second law of motion nor in its explanation at page 19 of the Principia.

(iii) Galileo has conducted experiments regarding acceleration in first decade of 17^{th} century i.e. 1604 (say). But the results were published in 1638, in the book *Dialogues Concerning Two New Sciences* at page 133-134, 146.

Galileo explained the law of inertia at page 195 of the Dialogues. Newton neglected elaboration of acceleration at pages 133-134 , 146 i.e.

Linear Acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) (27)

Newton did not quote or apply this definition of acceleration in enunciation of law of motion throughout his life, like his predecessors Galileo, Descartes, Huygens etc.

Newton chose uniformly accelerated motion in Galileo's law of inertia as

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

Galileo has defined and explained acceleration. But it is evident from his above quotation that Galileo explained motion in terms of uniform velocity. Galileo did not express the law in terms of acceleration. Descartes and Huygens also explained the laws and hypotheses in terms of uniform velocity. Newton also followed the path taken by the predecessors and used uniform velocity in understanding of impressed force, innate force and first law of motion.

As discussed above Newton did not defined acceleration neither in known or unknown terms or quantities.

(iv) In 1716, Hermann may be regarded to have given eq. (1) i.e. F =mdV/dt, where dV/dt is acceleration, according to Galileo's definition it is acceleration.

But Newton did not acknowledge F =mdV/dt as equation of force for second law of motion in third and final edition of the Principia in 1726. Also Newton did not acknowledged acceleration as dV/dt.

(v) Newton's book The Methods of Fluxions and infinite series. Fluxions means derivative, but Newton did not acceleration here as dV/dt. This book was published in 1736 i.e. 9 years after death of Newton.

(vi) Further Newton explained that bodies fall with equal velocity or accelerate (move faster) from rest to definite velocity. So, Newton implies bodies move with equal velocity. Newton did neither mention magnitude of equal velocity nor gave any method to determine the same. Newton has ignored acceleration (we mean to say Newton did not apply acceleration in motion of bodies, as Galileo applied uniform motion in case of law of inertia) throughout his life of 85 years , which was defined in 1638 i.e. four years before birth of Newton.

9.4 Scholium of Corollary VI at page 31 of the Principia.

" When a body is falling, the uniform force of its gravity acting equally, impresses, in equal particles of time, equal forces upon that body, and therefore generates equal velocities."

Proposition XLI, General Scholium of the Book III of the Principia.

*"Bodies projected in our air suffer no resistance but from air. Withdraw the air, as done in Mr. Boyle's vacuum, and the resistance ceases; for in this void a bit of fine down and piece of solid gold descend with **equal velocity.**"*

Thus we conclude that in case of falling bodies force of gravity generates equal velocity at all times i.e. due to force of gravity bodies fall down with equal velocity in vacuum at every point.

So, Newton did not mention about acceleration throughout his life, however he specifically mentioned that bodies fall with uniform or constant velocity in vacuum. The dictionary meaning of word 'accelerates' is that to move faster; it can be understood that when body is at the rest (u=0), then force of gravity sets it in motion and accelerates it (from rest to constant velocity). Thus, body moves with uniform or constant velocity from rest (u =0). It is not logical to conclude that the law implies that bodies fall with equal acceleration ($9.8m/s^2$).

Newton did not explain acceleration even when he had opportunity to do so e.g., in 1716 Hermann is regarded to have given equation F =mdV/dt but Newton did not acknowledge it as equation for second law of motion in third and final edition of the Principia in 1727. So it implies that Newton has disregarded acceleration throughout his life. Acceleration was discovered four years before birth of Newton. Newton clearly stated that bodies fall with equal velocity. Thus, statement that bodies accelerate (*to move faster)* should not be regarded as acceleration as Newton has ignored the acceleration throughout his life. Newton clearly stated that bodies when dropped from rest fall with uniform velocity (thus their velocity increases or accelerates from state of rest to uniform or constant velocity). Newton's explanation is qualitative as he did not calculate uniform velocity.

10.0 Rouse Ball's alteration in definition of second law of motion

W W Rouse Ball apparently did not agree with assumption that change in motion is equal to rate of change of momentum i.e.

mv-mu = d/dt (mv-mu) …. Rouse Ball disagreed with this.

The both sides have different units, dimensions and magnitude. It is justified in section (8.1)

Rouse ball **[19]** suggested some change in definition of Newton's second law of motion in book titled An *Essay to Newton's Principia* published in 1893 at page 77.

"*The change in momentum **[per unit of time]** is always proportional to moving force impressed and takes place in direction in which force is impressed."*

Mathematically

Change in momentum /Total time ∝F

or F= k (mv-mu)/50 (48)

Here force is regarded as to act for 50s. Thus, we have not to assume that change in motion is equal to rate of change of momentum.

11.0 Cohen's objection to Ball's explanation and 4 equivalent forms or equations.

Cohen [11] has quoted in opening paragraph at page 111 in section (5.3) title, 'The Second Law: Force and change in motion' in part A Guide to Newton's Principia of the book.

"Newton's second law, as stated in the Principia, sets forth a proportionality between a 'force' and resulting "change in motion", by which Newton means change in quantity of motion or change in momentum.

Since it is not the more familiar version of the second law, in which a force produces an acceleration or a change in momentum in a given (or unit) time, some writers have seen a need to introduce a correction to Newton's statement of the law."

Cohen stated it that Ball had not understood the Newton's second law of motion properly. Cohen objected to Ball's perception at page 111 in the following the statement,

It apparently never occurred to him to try find out what Newton meant rather than to introduce "per unit of time."

In this regard Cohen found eq.(11) correct and thus recognized motion as velocity V

F (impressed force *at once, or gradually and progressively)* ∝ alternation or change in motion (velocity)

F (force impressed *at once, or gradually and progressively)* = kdV (11)

or F = kdV (11)

The various equivalent forms (statements) or equations of second law as given by Cohen at page 116 are:

(1) F = kdV, F∝dV original or central equation. first equivalent form of second law of motion.
It is mathematical form of Principia's second law of motion as given in section (1.2). Thus Cohen accepts motion as velocity (V).

(1a) $F = k_1d(mV)$ Second equivalent form of second law of motion.
Arbitrary multiplication by m to statement (1)

(2) F = KdV/dt Third equivalent form of second law of motion
Arbitrary division by dt to statement (1)
Newton has neglected acceleration

(2a) $F = k_2\ d(mV)/dt$ Fourth equivalent form of second law of motion
Arbitrary multiplication by m to equivalent form three
Already obtained by Hermann and Euler (if k_2=1)

These equations are obtained from eq.(11), by arbitrary division by dt and multiplication by mass m in right hand sides of equations. About impulsive and continually acting force Cohen [11] has simply written at page 116

" that is, in statement (1) the force is impulsive, and whereas in statement (2) force is continually acting."

11.1 Origin of impulsive force and continually acting force

Cohen has clarified the meaning of impulsive or continually acting forces at pages 113-116. Eq.(11) is equally applicable irrespective of the fact force acts at once , gradually and progressively as the definition and explanation of the second law of motion.

Cohen categorized impressed force in form of impulsive force and continually acting force. Cohen assumes that genuine equation for second law of motion is eq. (11) i.e., F =kdV, and motion is velocity. Further Cohen believes that W W Rouse Ball's proposition (introduction of phrase 'per unit of time' in second law) is incorrect. However, Cohen himself divides with time dt.

Initially Cohen confirms validity of eq. (11) correct and motion as velocity, V. Then Cohen's maintains that equation for second law of motion is $F = md^2x/dt^2$ is correct. Further Cohen maintains the there are four equivalent equations for second law of motion ($F = kdV$, $F = k_1d(mV)$, $F = KdV/dt$, $F = k_2\, d(mV)/dt$). Thus Cohen categorizes impressed force as impulsive force and continually acting force.

(i) Impulsive force: Cohen [11] at page 114 has regarded instantaneous impulsive force as thrust given to body, so impulsive force is just a push or pull applied to body. Thus, Cohen has implied impressed force (thrust) as impulsive force when it acts instantaneously.

Cohen represented or categorized this in form of impulsive force and continually acting force; when force acts instantaneously or gradually and progressively. Cohen correctly regarded motion as velocity.

Mathematically Cohen represented impulsive forces as

(1) $F \propto dV$ or $F = kdV$ (11)

(1a) $F = k_1\, d(mV)$ (12)

So the eq.(11) may be regarded as equation for impressed force (as impulsive force), and Cohen regarded this as central equation in formulation of various equivalent forms. Thus, Cohen regarded this as impulsive force. Thus, impulsive force (thrust as quoted by Cohen) is new name given to impressed force when it acts instantaneously. It is time independent form.

Continually acting force: The sequence of infinitesimal impulses (impulsive forces) which act continuously is called continually acting force, as Cohen mentioned at page 115-116. Practically when impressed force or thrust or impulsive force when acts continuously is termed as continually acting force.

We understand that when impressed force acts instantaneously it is impulsive force, when impressed force acts infinitesimal impulses continuously, it is continually acting force. So, both impulsive force and continually acting force are form of impressed force i.e., eq. (11) depending upon time it acts. The force may act instantly or continuously and gradually.

When sequence of infinitesimal impulses is considered then impulsive force becomes continually acting force. The continually acting force is secondary force.

Cohen has written at page 116; Newton has used limiting-process to make a transition from primary impulses to secondary forces that act continually. It means Cohen obtained continually acting force, from impulsive force.

Mathematically, Cohen represented continually acting forces (time dependent) as

(2) $F \propto dV/dt$

Cohen intentionally put

Cohen stared from eq. (11) i.e., F (force impressed *at once, or gradually and progressively)* = kdV

This equation is in terms of velocity (V). But it is arbitrarily changed to in the form of acceleration by dividing with dt. Cohen's ultimate aim has been to change it to force equals mass multiplied by acceleration which was prevalent form of second law of motion since centuries.

Newton has ignored acceleration throughout his career, it is confirmed beyond any doubt.

or $F = K_1dV/dt$ (13)

Now multiplying right hand side of eq.(13) with mass m of body.

(2a) $F = k_2d(mV)/dt$ (14)

In this case constant of proportionality also varies. Cohen means eq.(14) i.e. k_2mdV/dt is nothing but mdV/dt or md^2x/dt^2 (if $k_2 = 1$). Hence, he obtains the desired equation without mentioning status of $F = kdV$, $F = KdV/dt$ and $F = k_1\, d(mV)$; how these equations become equal to $F = md^2x/dt^2$. Cohen obtained these equations from F =kdV which he obtains from the definition of Newton's second law of motion. F =kdV is genuine form of second law of motion. **Cohen [11]** has simply written at page 116

"that is, in statement (1) the force is impulsive, and whereas in statement (2) force is continually acting."

Cohen objected R Ball's introduction of phrase 'per unit of time' (i.e. division

by total time; but himself divided by 'dt'. Is division of 'change in momentum' by total time by Rouse Ball is invalid?
Then how division of 'change in momentum' by 'dt' is valid? The laws and logics are consistently applicable in all cases. Further, Cohen also arbitrarily multiplied right hand side of equations with mass m. Did Cohen try to provide derivation for second law of motion with arbitrary method to prove Ball's perception inconsistent? Cohen did not mention V V Raman's paper in his voluminous book as it does not exist in literature. It is very important paper regarding derivation and understanding of Newton's second law of motion. Further, Cohen completely underestimated Euler's work. Thus, Cohen wanted to prove that F=ma follows from Newton's law but to neglect the established facts is unscientific.

I. Bernard Cohen maintained that Rouse Ball's proposition is inconsistent. Cohen did not explain what is utility of giving four equations very qualitatively ($F = kdV$, $F = k_1d(mV)$, $F = KdV/dt$, $F = k_2 d(mV)/dt$) for second law of motion , when we have one equation prevalent over centuries, $F = md^2x/dt^2$. How equations $F = KdV/dt$ or $F= k_1d(mV)$, are helpful in this regard?
Further, Cohen has mentioned at page 116 that
"Here, once again, we see Newton's use of an intuitive limit-process to make a transition from primary impulses to secondary forces that act continually."
Further,
"Newton's transition from impulsive (instantaneous) to continually acting forces essentially bids us to conceive of these forces as sequence of infinitesimal impulses."

11.11 Meaning of Four equivalent equations.

Cohen has called all equations equivalent (equal in value, amount, function, meaning, etc.)
$kdV= k_1d(mV)$, $kdV= KdV/dt$, $kdV= k_2 d(mV)/dt$.
Hence, $kdV = k_1d(mV)= KdV/dt = k_2 d(mV)/dt = md^2x/dt^2$.

Dimensions

How equations with different dimensions, units and magnitudes can be equal? It is not logical way for proving that Rouse Ball's perception is incorrect and Cohen's method is correct.
Cohen did not state any advantage of manipulating such equivalent forms of second law of motion. Are magnitudes of all equivalent forms are being equal? Thus, the values of k, k_1, k_2 and K must be experimentally measured.

The values of k, k_1, k_2 and K are neither measured nor method is given to determine them by Cohen. The value of constant of proportionality is determined experimentally as value of G in eq.(9) as in eq.(10) equal to 6.6743×10^{-11} m^3kg^{-1} s^{-2} and further refinements are on.

The equation was being used by scientists and students over centuries. He did not interpret the equations further and did not give advantages of his equivalent equations i.e. eq. (11-14) over equation $F = md^2x/dt^2$.

Cohen quoted at page 92 that

"It is, in fact, because the Principia sets forth a dimensionless physics that Newton can make a transition from an impulsive force $F \propto d(mV)$ to $F \propto d(mV)/dt$, basing the shift on dt being a constant. The modern reader will be troubled by this example because the constant of proportionality in the two cases must be of different dimensionality."
The dimensional formulae for various constants as
$k : ML^0T^{-1}$, $k_1 : M^0L^0T^{-1}$
$K : ML^0T^0$ $k_2 : M^0L^0T^0$
These dimensional formulae are different for different constants , k , k_1, K , k_2 .

The concept of units and dimensions [21-22] are developed in 1822. Does it mean any law, phenomena, scientific concepts discovered before 1822 be explained arbitrarily as we wish to do? Such laws cannot disobey the basic norms. For example, one plus one is equal to two (1+1=2), not eleven (11), neither in Newton's time nor now. In fact truth is that Newton's second law of motion (as given in the Principia) gives $F = kdV$, and not $F = ma$.

Can we divide RHS of equation with any number we wish, keeping left hand side unchanged? Both left hand side and right-hand side of equations has to be divided by same term logically. The standards in scientific theories cannot be set arbitrarily to get desired results (F =ma follows from Principia's definition of second law of motion). The perceptions of units and dimensions are equally applicable to new and old theories. There should be no exception.

The old or new theories have to obey units and dimensions for second law of motion. Cohen has written at page 117 that Newton did not give any equation in the Principia (including second law of motion), but he is writing equations arbitrarily it is not justified e.g., eqs. (11-14).

11.2 Arbitrary way of obtaining various equivalent forms.

Assumption of infinitesimal 'particle' of time.

Cohen [11] has written at page 116 that: Newton's transition from impulsive (instantaneous) to continually acting forces essentially bids us to conceive of these forces as sequence of infinitesimal impulses. Purposely Cohen assumed that time consists of **infinitesimal** "particles" of time having magnitude 'dt' which was assumed constant.

As 'dt' is constant so it may be denoted with T_c (T Constant). Further in arbitrary way Cohen has divided right hand side of statement (1), with 'dt' (regarding dt as constant). Both possibilities are discussed, the result must be same in both cases.

Obviously 'dt' implies 'change in time' (gives impression of a part of derivative and variable) which may be misleading, so it is deceptive way of choosing constant. Thus, magnitude of 'dt' or T_c is such that it is infinitesimally small but non-zero (practically time is fraction of second or less). But in mechanics intervals of time are of the order of minutes, hours etc.

Arbitrary division by dt and multiplication by m in the right-hand side is not allowed.

Cohen has divided right hand side of statement (1) or eq. (11) by dt, leaving left hand side unchanged.

(2) $F \propto dV/dt = K_1 dV/dt$ (13) (Cohen's second equivalent statement of SLM).
$Fdt = K_1 dV$

The statement (1) and (2) may look like force if right hand side is multiplied with mass (leaving left hand side unchanged).

Then Cohen multiplied the right hand sides of eqs.(11,13) with mass m leaving left hand side unchanged (exceptionally arbitrary way).

(1a) $F \propto d(mV)$ or $F=k_1 d(mV)$ (12) (Cohen's third equivalent form of SLM)
(2a) $F \propto d(mV)/dt$ or $F = k_2\, d(mV)/dt$ (14) (Cohen's fourth equivalent form of SLM)
or $F\, dt = k_2\, d(mV)$

Thus, Cohen has arbitrarily divided or multiplied the equation to obtain to pre-supposed or desired equation. It is unscientific.

11.3 If T_c is used instead of dt, as both are arbitrary constants.
Writing constant 'dt' is arbitrary and deceptive.

The time dt is infinitesimal and constant but it is written in form of variable (dt), then it is deceptive and arbitrary way of writing. The proper way of writing constant time is T_c. The divisions by both 'dt' or T_c are equally feasible (both have same magnitude but 'dt' appears variable and T_c as constant). Cohen has assumed dt or T_c both are constant.

(2) $F \propto dV/T_c$ or $F = k\, dV/T_c$ (49) (Cohen's third equivalent statement of SLM)
$F\, T_c = kdV$
(2a) $F \propto d(mV)/T_c$ or $F = k_2\, d(mV)/T_c$ (50) (Cohen's fourth equivalent statement of SLM)
$F\, T_c = k_2\, d(mV)$

11.4 Equation , F =kdV can be multiplied with 'dt' or T_c instead of division
The proportionality holds good in both cases whether equation is multiplied or divided by a constant.

(2) $F \propto dt\, dV$ or $F = k_3 dtdV$ or $F= k_3\, T_c\, dV$
or $F = K_3 dS$ (51) (Cohen's third equivalent statement of SLM)
($S = Vt$ or $dS= T_c\, dV$)
$F \propto dt\, d(mV)$ or $F = K_4\, dt\, d(mV)$ or $F =k_4 T_c\, d(mV)$
or $F= K_4 m\, T_c dV = K_4\, mdS$ (52) (Cohen's fourth equivalent statement of SLM)

11.5 Demonstration of eqs. (51,52) i.e. $F = K_3 dS$ and $F= K_4\, mdS$ **is under experimental and realistic conditions**

Results differ from $F=ma = m(v-u)/t_2-t_1)$

The eq.(51) implies that force depends on distance travelled i.e. to push or pull body to longer distance more force is required. In the existing physics force is acceleration dependent, $F =mdV/dt = md^2x/dt^2 =ma$. Practically on a rough road if a gunny bag filled with sand, stones etc., to be dragged to greater distance more force is required to be applied by person.

Practically gunny bag does not move automatically, the bus also does not move longer distances automatically if once set in motion. In this case the constant may be better called coefficient as it varies from one situation to other as would depend on mass, its shape, inclination of path, resistive forces etc. So, it highlights practical situations encountered in everyday life.

The eq. (52) implies that higher the mass of body higher the magnitude of force required to set body in motion. Newton's first law of motion is based on law of inertia (i.e. does not account for resistive forces).

Whereas according to Newton's first law, when body is set in motion, it keeps on moving with uniform velocity (in absence of resistive forces). So body keeps on moving with uniform velocity irrespective of time. It is similar to Galileo's perception of Inertia as given in section (1.2).

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

The eq.(52) implies that higher the mass of body higher force would be needed to set body in motion with velocity V. The body of smaller mass can be set in motion with velocity V, with smaller force. The prevalent laws simply state if body of any mass (m) once set in motion, then keep on moving with speed or velocity (V) But it is meant for frictionless system, however in practical systems resistive forces are present. The eq.(46) implies that more force is required to be applied if body is to be pushed to higher distance. So, force is dependent on distance travelled.

Apparently, Newton's second law of motion is independent of resistive forces; hence other deductions from it. Thus, neither magnitude of mass nor that of velocity comes in picture. The second law also reduces to first law of motion.

Can we speculate an equation of force which takes in account directly force, velocity, time and distance? But it would be different from current perception that force is acceleration dependent i.e. $F=m(v-u)/t_2-t_1$. So critical analysis leads to noble results, as science is dynamic not static.

11.6 Infinitesimal time written as dt or T_c gives different results.

If we write dt as T_c then equivalent forms are not obtained.

$\int dt$ = t but $\int T_c$ is not defined.

Cohen assumed that time consists of **infinitesimal** "particles" of time having magnitude 'dt' which was assumed constant. It is evident from page 116 from his **book [11].** The constant time can also be written as T_c . By writing time as 'dt', it gives conceptual and mathematical impression that it is variable and can be integrated within suitable limits (t_2, t_1 say).

$\int dt$ = t or t_2-t_1 (53)

However the same **infinitesimal** "particles" of time dt is written in other physical form T_c then integral $\int T_c$ is not defined.

as $\int 9$ is not defined.

So Cohen has written constant in arbitrary way which suits the pre-assumption or pre-supposed result F =ma , follows from definition of second law of motion. However scientific logic should always prevail. Cohen's arbitrary operation is not justified scientifically.

12.0 I Bernard Cohen's omission or oversight of some significant facts.

Cohen obtained equation F = kdV as first or central equation for second law of motion. Thus, Cohen assumed that motion is velocity (V).

Cohen devoted 10 chapters, 399 pages to first part i.e. A Guide to Newton's Principia, and should have quoted the above facts from Galileo's Dialogues Concerning Two New Sciences (1638) as these are very significant regarding understanding of mechanics. Cohen has written that in 10th Chapter, pages 293 -370 as 'How to Read the Principia'.

However, it is clear that Cohen has omitted some significant factors involving second law of motion. Also some quotations of Cohen need to be elaborated for proper understanding.

(i) **The developmental facts about linear acceleration should have been quoted:**

Acceleration is lifeline of mechanics, hence of physics and science. Newton never mentioned acceleration throughout his scientific career or rather Newton ignored acceleration as it was present during his life in the literature. Acceleration was given and elaborated by Galileo in 1638 i.e. 4 years before birth of Newton. But Galileo did not did not use acceleration as he used uniform velocity in case of law of inertia at page 195 of his book *Dialogues Concerning Two New Sciences.* The following scientists Descartes, Huygens and Newton also did not use acceleration. Newton did not mention anything about acceleration in the Principia.

Firstly Newton [2] did not mention word acceleration neither in new definitions (I-VIII) i.e. quantity of matter (mass), Quantity of motion (mV), impressed force, the innate force of matter (inertia) , centripetal force, various types of centripetal force nor in already known terms (time, space, palace and motion),. Newton categorized motion as absolute motion and relative motion. This significant term (acceleration) was not discussed neither in definitions nor known quantities which Newton discussed in the beginning of the Principia pages 1-9. Thus, acceleration was not in Newton's terminology.

Secondly Newton did not mention acceleration neither in definition of second law of motion nor explanation given in the Principia at page 19. It is evident from section (2.0) where definition is given, also in section (7.0) where Newton demonstrated the law.

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Thus, 'alteration or change or difference in motion' is not acceleration.

Final motion (velocity) – Initial motion (velocity) $\neq$ Acceleration (a)

Acceleration has been defined by Galileo at page 134 as

A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed.

Linear acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) (27)

(a) Cohen [11] has written at page 117 of that

"Newton did not give equations to his laws."

(b) Also an article published in American journal of Physics **[12]** (2011) at page 1015 states that –

"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."

(c) I Bernard Cohen in 1999 in his book Isaac Newton The Mathematical Principles of Natural Philosophy at page 113 has stated that

"Newton never actually made a formal statement of the second law in the algorithm of fluxions or the calculus."

The author has concluded that Newton did not write any equation for his laws .So Newton's explanation is geometrical, philosophical and qualitative only.

(d) V V Raman has stressed that the name of Euler must be mentioned along with F =ma , as Truesdell in 1960 F =ma has been given by Euler. V V Raman has published in an ace pedagogical or academic journal *The Physics Teacher* **[14]** in March 1972 issue at page 137…

"Although this remark was made over a decade ago we still find textbooks in which F =ma is called Newton's formula, and which make absolutely no mention of Euler's in this context. "

(ii) Exact translation of the Principia has not been done from Latin text : At page 9 of the Principia [2] in Scholium after Definitions section Newton has written that

"Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse. I do not define time, space, place and motion, as being well known to all."

Cohen neither mentioned Newton's above quotation in A *Guide to Newton's Principia* at page 111 and in *Translation* section at page 408. At page 10 of the Principia Newton has expressed motion in terms of absolute motion and relative motion in sections (3.0). Both the absolute motion and relative motion means velocity. Here Newton discusses motion thus differentiates between motion and 'quantity of motion' (mV). The scholium (explanation) cannot be neglected.

Newton has given reason for not defining motion as it is already well known. Newton further categorized motion in terms of absolute motion and relative motion. Thus, Newton expressed motion in terms of velocity.

(iii) Contradictory statements by Newton ignored: Further in this regard Newton has not elaborated statement which is contradictory to experimental observations.

Scholium of Corollary VI at page 31 of the Principia

" When a body is falling, the uniform force of its gravity acting equally, impresses, in equal particles of time, equal forces upon that body, and therefore generates equal velocities."

Proposition XLII (general scholium) of Book III of the Principia reads as .

"Bodies projected in our air suffer no resistance but from air. Withdraw the air, as done in Mr. Boyle's vacuum, and the resistance ceases; for in this void a bit of fine down and piece of solid gold descend with ***equal velocity."***

This issue is discussed with details in sections (5.0). It is experimental observation that bodies fall in vacuum with equal acceleration or variable velocity.

Thus we conclude that in case of falling bodies force of gravity generates equal velocity at all times i.e. due to force of gravity bodies fall down with equal velocity in vacuum.

(vi) Euler's significant work should have been properly quoted: About Euler Cohen stated at page 211 that

'Euler's use of "F =ma" formulation of Newton's second law of motion in this work appears to have been a main influence for its subsequent use.'

Euler has derived his equation (F=ma) independently without using Newton's second law of motion nor mentioning name of Newton at various stages of his work. Newton has not written this equation for his second law of motion. Euler gave many equations relating to mass and acceleration in 1736, 1749,1752 and 1765. Finally, Euler wrote $F = m\,d^2x/dt^2$ in 1775. Euler's equation $F = ma = md^2x/dt^2$ is available in paper *Novi Commentarii academiae scientiarum Petropolitanae* 20, 1776, pp. at page 222-223. Euler work is available at website of Mathematical Association of America http://eulerarchive.maa.org/).

In 1960 Truesdell **[7]** in journal *Archive for History of Exact sciences,* has written that F =ma was given by Euler (not in 1752 paper E177, but in 1775 paper E479). The Euler's contribution is also discussed by V V **Raman [14]** in 1972. It should be noted that Cohen had discussed Newton's second law of motion at pages 111-117. Thus, Euler name should have been quoted along with F =ma (prevalent from Newton's second law of motion).

Raman [14] has written in *The Physics Teacher* [March 1972 , page 137] of American Institute of Physics that

" Although this remark was made over a decade ago we still find textbooks in which F =ma is called Newton's formula , and which make absolutely no mention of Euler in this context".

But Cohen **[11]** neither mentioned Euler's original work completely nor V V Raman's work. As this work has been quoted in research and pedagogical journals so should have been interpreted by Cohen.

But there in neither mention of V V Raman's paper nor Euler's work in Cohen's book. Cohen who have made elaborated attempts to explain historical aspects of the Principia; in 10 chapters (399 pages) in *A Guide to Newton's Principia,* did not mention V V Raman's paper in The Physics Teachers, March 1972. Cohen has just given marginal references in footnote at page 211 that Euler's equation F =ma follow from Newton's second law of motion. But Euler did not give any reference of Newton's work. Newton did not write F =ma. Thus Cohen's book need to be updated in view of existing facts.

13.0 Some Quotations by Cohen

Some interesting quotations of Cohen are discussed below,

(a) **Oldest form of second law is now called as the modern form**.

Cohen **[11]** has quoted at page 104 of his book, the equation F =mdV/dt or F =ma as modern form of Newton's second law.

If it is the modern form then what is old form? How old form is inadequate that modern form is needed? In fact, F = mdV/dt is the oldest form of Newton's law; it was quoted directly by Hermann in the *Phoronomia* in 1716 at page 59. So the oldest form cannot be the modern form if we logically analyze the facts.

Further Cohen did not say about old and new old forms of third law of motion. There is one interesting fact about the Newton's laws. We quote Newton's first and third laws exactly in the same way as given in *the Principia* by the legend. Newton's second law of motion is quoted in the altered form i.e. change in motion is regarded as change in momentum. But original form of Newton's second law of motion is not quoted in the textbooks. Thus it is incomplete information is provided to readers.

(b) It was impossible for Galileo to know about Newton's laws published 44 years after his death.

Further Cohen **[11]** at page 113 has given very immature statement

Galileo certainly did not know about Newton's first law.

Galileo (1564-1642) died in 1642, the year when Newton (1642-1727) was born. Newton gave First law of Motion in 1686 i.e. 44 years after Galileo's death. How could Galileo know about the laws written 44 years after his death?

Galileo did not have any idea of Newton's laws as these were discovered 44 years after death of Galileo. Galileo Galilee died in 1642 and Newton's first law was published in the Principia (1686). The truth is that Newton's first law of motion is nothing but re-statement (in precise form) of Galileo's law of inertia given at page 195.

Thus, there is always scope for updating even in voluminous masterpieces written by celebrated authors. Thus

(c) **Cohen tried to create void by underestimating Euler's work**

Regarding Euler and F =ma Cohen [11] has written (at footnote page 211) that

"Euler's use of F =ma formulation of Newton's Second Law of Motion in this work appears to have been a main influence for its subsequent use. But it was not the first time this formulation of the law had appeared in print. Jacob Herman has presented the second law in terms of differentials in this form in his Phoronomia of 1716."

Cohen presented Euler's equation $F = md^2x/dt^2$ is derived from Newton's second law of motion. But it is not true. Euler has derived various equations involving, mass, acceleration and force in 1736, 1749 ,1752, 1765. Finally, Euler gave equation $F = ma = md^2x/dt^2$ in 1775. Euler did not use

Newton's definition of second law of motion in derivation of F =ma.
Cohen [11] at page 116-117 has written that
'Newton did not write any equation for his laws."

Had Newton given equation F =ma earlier, then it would have been presented in the Principia. However, Newton did not use word acceleration throughout his life, so the perception of F =ma by Newton is impossible. So, there were severe conceptual and mathematical constraints in time of Newton.

Had Euler written F =ma earlier then Euler would have simply re-quoted the same equation as given by Newton; but it only speculation as Newton did not give any equation in the Principia. Only then it could have been concluded that Euler's equation is Newton's equation. But Newton did not write F =ma, but Euler did, without mentioning Newton's second law of motion. Thus, Euler's equation is independent of Newton's second law of motion. Similar is the case of Hermann. Newton even did not write acceleration in any edition of the Principia

Raman [14] has written in The Physics Teacher [March 1972 , page 137] of American Institute of Physics that
" Although this remark was made over a decade ago we still find textbooks in which F =ma is called Newton's formula , and which make absolutely no mention of Euler in this context".

Also Cohen [11] at page 113 stated that Newton did not quote second law of motion in differential form and Hermann has done so in 1716 in the book *Phoronomia*. Hermann has given this equation directly. Again Jacob Hermann wrote eq. (1) without using Newton's second law of motion. Newton did not give any equation to his laws [11].

Thus he did not give F =ma. Newton himself did not write the equation F = mdV/dt as equation for second law of motion in third and final edition of the Principia in 1727. Thus, works of both Euler and Hermann are independent of Newton's second law of motion and must be quoted along with F =ma.

So, Cohen tried to prove that Hermann's equation F = mdV/dt and Euler's equation F =md^2x/dt^2 =ma follow from Newton's second law of motion. But neither Hermann nor Euler used Newton's second law of motion in derivation of equations.

Likewise, Cohen tried to prove that Euler's equation F =ma as given in 1775 is based on Newton's second law of motion. But Euler also gave various equations in 1736,1749,1752,1765 relating to mass, acceleration and force. So, Euler did not use Newton's second law of motion in his continuous work in 1736, 1749, 1752, 1765 when he gave various equations relating to force, mass and acceleration are due to second law of motion.

So, Cohen tried to prove that Euler's work (specifically F =ma) is also due to Newton's second law of motion. Euler's equation F =ma involves acceleration, but Newton did not quote acceleration throughout his scientific career. Also, Newton did not write F =ma. So, Cohen tried to create void that except Newton none has given F =ma but this perception is not based on existing scientific facts.

The truth is that Newton neither mentioned about acceleration nor wrote F =ma in his scientific career. So as discussed by V V Raman in American Journal of Physics, Euler's name must be mentioned with F =ma.

Contribution of Euler : Cohen [11] has written at page 211 that
"Recherches sur le mouvement des corps célestes en général" Memoires de l academie des sciences de Berlin (3) (1747): 93-143; reprinted in Leonhari Euleri Opera Omnia , ser. 2, vol.25 (Zurich , 1960), pp.1-44 Euler's use of the F =ma formulation of Newton's second law of motion in this work appears to have been a main a influence for its subsequent use. But it is not for first time this formulation appeared in print. Jacob Hermann has presented the second law in terms of differentials in this form in his Phoronomia of 1716.

From *Cohen's* misperception is clear that in the above paper (published in 1747) Euler has defined F =2ma, not F =ma . This paper has Eneström **Index E112 (**http://eulerarchive.maa.org/**)** The factor of 2 is very-2 significant in quantitative measurements. It clearly implies that Cohen was not aware of Euler's work in totality. Euler has given F =ma = d^2x/dt^2 in his paper Novi Commentarii academiae scientiarum Petropolitanae published in 1776, however equation F =ma was given 1775.

Further Euler's next statement i.e. '*Euler's use of the F =ma formulation of Newton's second law of motion'* also looks dubious. This conclusion is due to two reasons i.e. firstly Euler did not use Newton's second law of motion and secondly Newton did not F =ma. Thus, it is very important to find out when F =ma was given and who is the scientist responsible for this. The oldest book available **First Three Sections of Newton's Principia** was published in 1871. This book quotes Newton's second law of motion as given by Newton in the Principia but does not quote F =ma. Does it mean that F =ma was associated with afterwards? This book is designated as text book for school and college students.

Thus Cohen wrote that Newton's second law of motion has been presented by Hermann in 1716 and after him by Euler in 1747.

Cohen's statements requires corrections

(a) In the paper *"Recherches sur le mouvement des corps célestes en général"* is available http://eulerarchive.maa.org/ as E112 , does not give F =ma but it gives F =2ma . The equation F =ma is given in paper *Novi Commentarii academiae scientiarum Petropolitanae* 20, 1776, pp. 208-238 as E479.

(b) Euler did not use Newton's second law of motion in while deriving F =ma. It is independent way of deriving F =ma by Euler.

14.0 Updated or modified form of Newton's second axiom (axiomata) or law of motion.

In Newton's second law of motion in mathematical form (F =ma) acceleration (a) is very significant term. Acceleration was illustrated earlier by Galileo in 1638 in his book *Dialogue Concerning Two New Sciences*, in Day Three pages 133-134,146. But Galileo did not apply acceleration in explanation of motion like uniform motion as page 195. Also, like Galileo, Descartes (1644) and Huygens (1673) did not use acceleration in interpretation of motion and used Galileo's law of inertia in their laws. Like this Newton also used law of inertia in first law of motion. Newton did not quote or discuss acceleration (dV/dt) in the Principia (1686,1716,1726).

Thus F =ma was never written by Newton. Herman has given equation F =mdV/dt in 1716, and Newton did not acknowledge it as equation for second law of motion in third and final edition of the Principia. Also in book Miscellanea at page 67, J Jennings defined momentum and velocity in eqs.(16,17) . Both have given equations without using Newton's second law of motion.

Unfortunately, names of these original discoverers (Euler and Hermann) of F =ma, occur nowhere in the literature (textbooks or reference books) when F =ma is mentioned. In view of scientific ethics their names should be quoted along with F =ma. It must be investigated how and when F =ma follows from Newton's second law of motion? It is explained in sections (4.6).

However F =ma follows from modified form of second law of motion consistently with due regard to original discoverer Jacob Hermann (1716) and Leonhard Euler (1775). Both scientists were Swiss and distant relative to each other.

(i) Newton defined laws of motion in 1686. Newton neither mentioned about acceleration nor wrote F =ma. It is justified from statements of scientists. Author has justified it by various statements of scientists in the discussion. It was attributed to Newton by following or succeeding scientists. Now when we try to derive F=ma from the definition of second law of motion; then inconsistent assumptions have to be made.

(ii) According to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann.

In the *Phoronomia* at page 59 , Hermann has given equation

G = MdV : dT or G = MdV/dT

G is weight or force of gravity .

Cohen has stated that it represents equation for second law of motion as

F = mdV/dt (1)

(iii) Leonhard Euler (1736, 1749,1752 , 1765, 1775) gave different equations relating to force and acceleration (d^2x/dt^2). Euler did not use definition of Newton's second law of motion. In 1775, Euler specifically derived F =ma as shown in section (5.0). Euler **[18]** did not use definition of Newton's second law of motion in deriving his equations relating to force, mass and acceleration. In fact Newton had not used word acceleration throughout his life.

$$P=\int dM \frac{d^2x}{dt^2},\ Q=\int dM \frac{d^2y}{dt^2},\ R= \int dM \frac{d^2z}{dt^2} \qquad (53)$$

Or in general, $F = \int dM \frac{d^2s}{dt^2} = ma$ (2)

(iv) The equation based on definition of Newton's second law of motion (change in motion i.e. velocity is proportional to impressed force is given by

$F \propto kdV$

F =kdV (11)

where F is impressed force , V is velocity and k is constant of proportionality. This equation is obtained just by method in which law of gravitation is obtained ($F \propto m_1m_2$, $F \propto 1/r^2$; and $F \propto dV$). But this equation is not quoted in the standard references.

(v) Walter Willian **Rouse Ball's equation [19].** Cambridge based British Mathematician in his book *An Essay on Newton's "Principia."* (London: Macmillan and Co., 1893) stated at page 77 Newton's law different way. Rouse Ball did not agree with assumption

change in motion = rate of change of motion with respect to time

mv-mu = d/dt(mv-mu) (5)

The magnitude, units and dimensions both sides of eq.(5) are different. Thus gave partially changed equation.

The partially changed definition (division by time) of Rouse Ball is
*The change in momentum **[per unit of time]** is always proportional to moving force impressed and takes place in direction in which force is impressed.*
If change in momentum occurs in 10 seconds
m[v-u]/t ∝F or F∝m[v-u]/10
F= k (mv-mu)/50 (48)
Walter William Rouse Ball (1893) assumed that motion is momentum(mV).

(v) Cohen (1999) did not agree with Rouse Ball's alteration in definition and stated at page 111 that
" It apparently never occurred to him to try to find out what Newton meant , rather than to introduce , per unit of time."
As Ball divided with time (after changing the definition of second law of motion), Cohen also divided with time dt (constant in magnitude) in right hand side without changing the left hand side but arbitrarily.
How Rouse's Balls division by total time is invalid and Cohen's division by time dt is valid ? Further Cohen's mathematical steps are arbitrary as he has also multiplied Right Hand Side with mass m.

Inconsistent Solution given by I Bernard Cohen

Cohen started from eq.(11) i.e. F =kdV and regarded motion as velocity V. It is first equivalent form of second law of motion. Cohen's fourth equivalent form of second law of motion is F $=k_2$mdV/dt or F= k_2 d^2x/dt^2 (k_2 =unity), it is used as equation for second law of motion since centuries. Cohen's third equivalent form of second law of motion is just acceleration, F =KdV/dt (K=unity). Likewise, Cohen's second equivalent form is just change in momentum. F = k_1d(mV).

Cohen gave four equivalent (equal **in value, amount, function, meaning, etc.**) equations as, F =kdV, F= k_1d(mV), F = KdV/dt, F = k_2d(mV)/dt. These equations are obtained inconsistently. F = md^2x/dt^2 was used as an equation for second law of motion for centuries at that time; but Cohen gave four equivalent equations thus,
kdV = k_1d(mV) = KdV/dt = k_2d(mV)/dt. (54)
These four equivalent equations are obtained inconsistently by dividing and multiplying right hand sides of equations, and leaving left hand sides unchanged. It is unscientific. Cohen neither calculated values of constants of proportionality, k, k_1, K and k_2 nor gave any method to determined their values. Cohen neither gave need nor advantage of equivalent forms or equations of second law of motion. Cohen neither justified need nor advantage of second law of motion. So, this is only speculative mathematical interpretation.
The various perceptions of Rouse Ball, Cohen and other scientists are shown in Table III.

Table:III The arbitrary mathematical description of Newton's Second Law of Motion by I Bernard Cohen

Sr. No	Scientist	Statement	Multiplication	Division	Equations	Conclusion
1	Rouse Ball	Change in momentum **per unit time**	No arbitrary multiplication	No arbitrary division	F =dp/dt = ma	Gives one equation F =ma
2	I Bernard Cohen	Original equation F =kdV	Arbitrary multiplication by m ; F=k_1d(mV)	Arbitrary division by dt F = KdV/dt F = k_2 d(mV)/dt	F=kdV F = KdV/dt F=k_1d(mV) F = k_2 d(mV)/dt	Four equations arbitrarily written?
3	Newton	Change in motion ∝ force	No multiplication	No division	No equation was given	Equation follows F =kdV but completely neglected.
4	Following scientists	Changed definition in terms of acceleration.	No multiplication	No division	F =ma	Newton never wrote F =ma.
5	Ajay Sharma	Rate of change of momentum equals force	No multiplication	No division	F =ma	F=ma follows genuinely

Note: It is basic rule of science that new perception is given by scientists so that better results are obtained from the old doctrines.

Newton did not give any equation for second law of motion. The following scientists associated F =ma with second law of motion. Then to derive F =ma from second law scientists assumed; change in momentum = rate of change of momentum [mv-mu = d/dt (mv-mu)], which is practically objected by W W Rouse Ball (1893). Consequently, Rouse Ball suggested change in definition of law as 'Change in momentum per unit time' it gave mathematical equation F =ma.

After more than 100 years, I Bernard Cohen objected to Rouse Ball's perception and in method (involving arbitrary division and multiplication) gave four equations [F =kdV, F = KdV/dt, F=k_1d(mV) and F = k_2 d(mV)/dt] instead of F =md^2x/dt^2 or F = dp/dt or F =ma. It the rarest example of arbitrariness. The best way to obtained equation F =ma is modification and generalization as given in section (14) by the author.

14.1 Genuine updated or modified form of Newton's second law of motion. The definition which gives directly F =ma

The axiom is a statement or principle that is generally accepted to be true, but need not be so. Thus by definition axiom can be extended. In 1893, Walter William Rouse Ball has changed its definition. Also, I Bernard Cohen gave four equivalent equations for Newton's second law of motion. Here logically the definition of Newton's second law of motion is modified.

It is undeniable truth that F =ma is the most extensively used equation in physics, it is basis of Einstein's mass energy equivalence ($E_{rest} = M_{rest}\, c^2$). The units and dimensions of physical quantities are based on it. So it is inseparable part of physics. Thus it has to be a part of science even as postulate. It means modified form of second **axiom** (a statement or principle that is generally accepted to be true, but need not be so) or law of motion is considered purposely as it leads to consistent results. The modification is just extension of original form of Newton's second law of motion.

14.2 Newton's second law of motion in modified form.

At page 19 of *the Principia* Newton has given axioms or laws of motion. Thus, Newton has primarily given axioms, so they can be altered if need arises and results can be checked. Science us not rigid but adaptable. The modified form of second law of motion is

"*The rate of change of quantity of motion* (Quantitas motus) *with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

or

"*The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

In original form Newton's second law of motion, Newton has used words 'motus' not 'Quantitas motus' or 'motion' not 'Quantity of motion'. Newton did not mention 'Quantitas motus' is motus and 'Quantity of motion' is motion.

Before this Newton has clearly mentioned 'Quantitas Materae' as

"It is this quantity that I mean hereafter everywhere under the name of body or mass"

Thus Newton clearly stated that he would regard 'quantity of matter' as body or mass.

14.3 Advantages of modified form of second law of motion

When the original form of Newton's second law of motion is modified **;** then all the inconsistencies in obtaining F=ma are removed as discussed above in various sub-sections. F =ma genuinely follows from modified or extended form the second law. **Also it highlights the wisdom of the legend and his magical touch on the words when he formulated second law of motion.**

So, various inconsistent issues are over whether

(i) Now we need not consider

(a) 'motion' means 'quantity of motion' (Quantitas motus) i.e. mV (momentum) or velocity.

Motion = mV (4)

(b) 'simple difference in motion' means 'rate of change of motion w.r.t. time'

mv-mu = d/dt (mv-mu) (5)

It is justified as both Left Hand Side and Right Hand side have different units (kgm/s & kgm/s^2) , dimensions (MLT^{-1} & MLT^{-2}) and magnitudes. These basic issues of physics /science cannot be ignored just to obtain pre-supposed F =ma from definition of second law of motion. The logics are supreme in science.

Left Hand Side, eq.(5)	Right Hand Side , eq.(5)
Units : kgm/s	Units : kgm/s^2
Dimensions : MLT^{-1}	Dimensions : MLT^{-2}
Magnitude : mv-mu	Magnitude : d/dt (mv-mu)

Thus equality of equation (5) is not justified , hence the equation is arbitrary.

So these inconsistencies do not come in picture when Newton's second law is generalized.

(ii) Now Walter William Rouse Ball's proposition of addition of 'per unit of time' need not be considered.

(iii) I Bernard Cohen's four equivalent forms F =kdV, F= k_1d(mV), F = KdV/dt, F = k_2d(mV)/dt for second law

of motion become irrelevant. We need not consider their equality as
$kdV = k_1d(mV) = KdV/dt = k_2d(mV)/dt$
The reason is that we are capable of obtaining F =ma, when its original definition of Newton's second law of motion when it is logically modified.
(iv) Now we need not have to assume value of constant of proportionality is unity to define units of force.

14.4 F =ma easily obtained from modified form of Newton's second law of motion.

Mathematically,
Motive impressed force = rate of change of 'momentum with time'.
= rate of change 'quantity of motion or Quantitas motus with time.'

$$F = d(mv - mu)/dt \quad (55)$$

$$F = dp/dt \text{ or } F = ma = mdV/dt = md^2x/dt^2 \quad (1)$$

Thus with modified form of second law of motion we exactly get F =ma , without any scientific inconsistency and with due respect to the legend Sir Isaac Newton. Thus, applicability of Newton's second law of motion are harmoniously extended without any inconsistency. Science is like lighting one lamp from the other.

The Original and updated or modified forms of Newton's second law of motion are shown in Table III

Table VI Comparison of original and generalized form of law

Sr. No	Term in Newton's original form of the law	Term in Newton's generalized form of the law	Difference between original and generalized form
1	Motion	Quantity of motion (Quantitas motus) **m**V or momentum	'Motion' is changed to 'Quantity of motion', mV
2	Alteration or difference	Rate of change with time	'Difference' is changed to 'Rate of change of with time'
3	Proportional to	Equal to	'Proportional to' is changed to Equal to
4	Equation implied $F \propto kdV$ F =kdV	Equation implied F = mdV/dt =ma	Equation F =kdV changes to F = mdV/dt =ma
5	Alternation or change in motion is proportional to motive impressed force	Rate of change of momentum (mV) is equal to motive impressed force	The change is clear in two columns.

NOTE: Newton's second law of motion gives equation F =kdV. Newton never wrote F =ma in any way.
Newton did not write acceleration (which is found in F =ma) throughout his life. Acceleration existed 4 years before birth of Newton. Newton did not write F =ma, which is associated with his name over centuries. Newton's modified second law of motion gives equation F =ma.
Even in standard references Newton's second law is altered by authors but altered form is known as Newton's law. Well if law is altered even in speculative form (just small changes are made in it) then it must be called Newton's modified second law by 'particular scientist'; or the 'particular scientist' must associated with altered form of Newton's second law of motion. Otherwise, that scientist must justify that his altered form of law if nothing but Principia's second law of motion as given by Newton.

Table V Chronological Developments of mathematical equation relating to Newton's second law of motion.

Sr. No	Year	Name of scientist	Name of publication	Equation /comments
1	1604	Galileo	No publication Conducted experiments with domestic equipment	No equation
2	1638	Galileo	*Dialogue Concerning Two New Sciences,* p.128	Uniform velocity: equal distances in equal times
3	1638	Galileo	*Dialogue Concerning Two New Sciences* p.195	Law of inertia for resistance free systems

4	1638	Galileo	*Dialogue Concerning Two New Sciences* p. 133-134, 146	Acceleration (a) =Change in velocity /change in time
5	1644,	Renne Descartes,	*Principles of Philosophy* 1644, article II	Law II is given in terms of uniform velocity
6	1673	Christiaan Huygens	*Horologium oscillatorium sive de motu pendularium* (1673) p. 21	Hypothesis I is given in terms of equal velocity
7	1686,1713,1726	Isaac Newton	*Mathematical Principles of Natural Philosophy* (*Principia*).	No equation, No acceleration Newton did not give F =ma
8	?	? Requires historical reviews	Published or unpublished reviews	F =kdV , based on Newton's 2nd law . The derivation like $F = m_1m_2 /r^2$ ($F \propto m_1m_2$,$F \propto 1/r^2$;)
9	1716	Jacob Hermann	*Phoronomia* pp. 59	G = MdV/dT G weight or force of gravity. F =mdV/dt (deduction by Cohen)
10	1736,1749,1752, 1765	Leonhard Euler	Various books and research papers available at http://eulerarchive.maa.org/	Various equations relating to Force, mass and acceleration F =ma/n, F=2ma, F= ma/2g, g=constant
11	1775	Leonhard Euler	Research paper E479 p. 222-223 http://eulerarchive.maa.org/	$F =md^2x/dt^2 = ma$
12	1871	John H Evans and P T Main.	*First Three Sections of Newton's Principia* P. 10	Equation F =ma is not given in 1871
13	1893	W W Rouse Ball	*An Essays on Newton's Principia* p.77	Impressed Force = Change in momentum /total time
14	1972	V V Raman	*The Physics Teacher* March 1972 p.136-137	Second law quoted in altered form (acceleration dependent). First and third law quoted in original form as in the Principia.
15	1972	V V Raman	*The Physics Teacher* March 1972 p.136-137	Assumes to derive F =ma; mv-mu = d/dt (mv-mu) It is completely inconsistent.
16	1972	V V Raman	*The Physics Teacher* March 1972 p.137	Euler should be associated with F =ma
17	1999	I B Cohen	*Isaac Newton: The Principia* p.117	Newton did not given any equation for his laws
18	1999	I B Cohen	*Isaac Newton: The Principia* p.133	Newton did not write second law as fluxion or derivative form
19	1999	I B Cohen	*Isaac Newton: The Principia* p.116	Four equivalent forms of 2nd law of motion Arbitrarily given.
20	1999	I B Cohen	*Isaac Newton: The Principia* p.211	Under estimated and misinterpreted work of Euler regarding F=ma
21	2011	Bruce Pourciau	*American Journal of Physics*	Newton did not write

			p.1015	'acceleration' & rate of change in 2nd law of motion
22	2020	Ajay Sharma	*Newton's generalized form Of second law gives F =ma*	Motion represents velocity V . Evident from section (3.8)
23	2020	Ajay Sharma	*Newton's generalized form Of second law gives F =ma*	Principia's second law does not give F =ma
24	2020	Ajay Sharma	*Newton's generalized form Of second law gives F =ma*	Generalized 2nd Law gives F =ma, Original form F =kdV
25	2020	Ajay Sharma	*Newton's generalized form Of second law gives F =ma*	Should there be equation of force for system full of resistive forces relating, F, V, S and t (usual meanings) F = AM(u+v)S/t, A is coefficient

Note: Newton never wrote F =ma. Newton never wrote acceleration; it was given by Galileo in 1638. Galileo did not apply acceleration to motion of bodies; as in case of uniform velocity at page 195 (law of inertia). Descartes (1644) and Huygens (1673) also expressed their laws in form of law of inertia. Newton's first law of motion is also based on law of inertia.

According to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann. Thus eq. (1) follows from Hermann's direct interpretation as given in his book *Phoronomia* at page 57 published in 1716. Euler derived $F= md^2x/dt^2$ =ma in 1775 it without using Newton's second law of motion in 1736, 1749,1752, 1765. Newton did not write F =ma. But definition of second law implies F=kdV. But F =ma is derived from definition by inconsistent method. Some authors quote definition of the law in textbooks or standard references in different way than given by Newton. If Newton's law is modified or generalized then equation F =ma is consistently obtained.

Every critical discussion also leads to speculative results. Should there be equation of force for system full of resistive forces relating, F, V, S and t (usual meanings). Currently the force depends on acceleration (F depends on u, v and t) and force depends on acceleration only. The correct concepts must be given to coming generations.

Appendix 1

17.2 Impartial Conclusions

Newton has been never given F =ma. Newton ignored acceleration throughout his scientific career. The genuine equation based on second law is F =kdV, which is neglected by scientists. F = ma has been derived by Euler in 1775 (E479 http://eulerarchive.maa.org/). Scientists hurriedly associated F =ma with Newton's second law. When scientists tried to obtain F =ma from second law then two assumptions are made; as genuine equation from second is F =kdV. The motion (basically velocity) is regarded as momentum (mV); as it does not solve the purpose then 'change in motion' is regarded as 'rate of change of momentum'. It is again not consistent as discussed in section (2.1).

In further justification scientists changed the definition of second law i.e. definition of law we find in textbook or standard references is different from that given in the Principia.

Now simple questions are …why genuine form of mathematical equation F=kdV not even quoted in literature? Why Newton's original form of second law is not quoted in the standard reference? Why mv-mu = d/dt (mv-mu) is regarded as true? Why changed form of Newton's second law of motion is quoted as Newton's second law of motion in the standard references? Why definition of first and third laws of motion are quoted same as given by Newton in the Principia, as in textbooks and standard references.
All these inconsistent steps be avoided if definition of second law of motion is modified or generalized. Then we get exact equation for second law as F =ma.

Part I
Galileo (1638), Descartes (1644) Huygens (1673), Newton (1686, 1713,1726) and Euler (1775)

(i) Italian legend Galileo's book Dialogue *Concerning Two New Sciences,* in 1638 may be regarded as first starting point in physics. Galileo perceived, conducted experiments and theoretically interpreted the results in mechanics. Galileo's law of inertia is even basis of Newton's First Law of Motion. Galileo also made significant discoveries. In the same book at page 128 Galileo had defined steady or uniform
By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal.

(ii) Galileo applied the definition of uniform motion and expressed it in form of law of inertia at page 195 in the book Dialogues

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

Galileo's law of inertia had tremendous applications; even Newton's First Law of Motion is nothing but Law of inertia in refined form.

(iii) In this book at pages 133-134 and 145 acceleration is defined and explained as
Linear acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) = dV/dt (27)
The acceleration was not applied in motion of bodies; as applications of uniform motion were quiet simple. The acceleration was applied in motion of bodies when differential and integral calculus was developed.

(iv) Galileo did not explain the motion of bodies in terms of acceleration but in form of uniform motion. In the same book at page 128 Galileo had defined uniform velocity as equal distance travelled by bodies in equal intervals of time.

(v) French scientist Rene Descartes in his book *Principles of Philosophy* published in 1644 , expressed his second law of motion in terms of uniform velocity. Like Galileo, Descartes did not use acceleration to interpret motion of bodies.

(vi) Christiaan Huygens Dutch scientist his book Horologium *oscillatorium sive de motu pendularium* in 1673 in gave his first hypothesis of motion of bodies in terms of equal velocity. Like Galileo (1638) and Descartes (1644), Huygens (1673) did not use acceleration to interpret motion of bodies.

(vii) Sir Isaac Newton published his scientific epic *The Mathematical Principles of Natural Philosophy* (popularly known as the Principia) published in 1686, expressed his first law of motion, impressed force and innate force in terms of uniform velocity or 'moving uniformly forward' (uniform velocity). Like Galileo, Descartes and Huygens, Newton did not use acceleration to interpret motion of bodies.

Newton expressed second law of motion in dynamical form (associated force with this). Newton regarded, 'alteration or change in motion' proportional to impressed force. Cohen [11] at page 116-117 stated that

"Newton did not give any equation for his laws."

Newton did not give any equation for second law of motion. The genuine equation based on this definition is
Impressed force $\propto$ change in velocity (motion).
F =kdV (11)
But prevalent equation since centuries for Newton's second law of motion is
F =ma = mass x acceleration
Also an article published in American journal of Physics [12] (2011) by **Bruce Pourciau** at page 1015 states that –
"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."
The critical analysis of literature also justifies the same. There are inconsistencies with this derivation.

(viii**)** **Newton (1642-1727) and acceleration**

Newton neglected acceleration throughout his life. Acceleration was defined and explained by Galileo in 1638 i.e. 4 years before birth of Newton (1642).

(a) Newton [2] did not mention word acceleration neither in new definitions (I-VIII) i.e. quantity of matter (mass), Quantity of motion (mV), impressed force, the innate force of matter (inertia) , centripetal force, various types of centripetal force nor in already known terms (time, space ,palace and motion). Newton further categorized motion as absolute motion and relative motion. This significant term (acceleration) was not discussed neither in definitions nor known quantities which Newton discussed in the beginning of the Principia pages 1-9.

(b) Newton did not mention acceleration neither in definition of second law of motion nor explanation given in the Principia at page 19. The generalized form of second law is,

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

So acceleration is not mentioned in the definition, it simply involves impressed force, change in motion etc.

Thus, 'alteration or change or difference in motion' is not acceleration.

Final motion (velocity) – Initial motion (velocity) $\neq$ Acceleration (a)

The Methods of Fluxions and Infinite Series

In Newton's terminology fluxions means derivative. It is believed Newton had completed this book in 1671 but published in 1736 i.e. 9 years after his death. In this book Newton did not write acceleration as, a =dV/dt. The book was published 65 years after its completion. The no reason is given in the literature for the delay in publication.

Thus it is evident that Newton (1686, 1716, 1727) like his predecessors, Galileo (1636), Descartes (1644), Huygens (1673); Newton also neglected in applications of acceleration to motion of bodies. All explained motion in terms of uniform motion, regarding Galileo's law of inertia as basis.

After death of Newton, especially in Euler era, acceleration was found very useful physical quantity in differential and integral calculus. Then acceleration was associated with second law of motion as F=ma.

Part II

Jacob Hermann (1716) and Euler (1736, 1749, 1752, 1765 and 1775)

(ix) Thus according to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann. Thus eq. (1) follows from Hermann's direct interpretation as given in his book *Phoronomia* at page 57 published in 1716. Jacob Hermann (1716) has given equation directly [6] at page 59. The same equation is also quoted by Cohen at page 113

G = MdV : dT or G = MdV/dT

G is weight or force of gravity . It can be other form of force F, Cohen has written that it seems different form of Second Law of Motion (in terms of derivative) then we can write

F = mdV/dt (1)

This equation was quoted independently (without any derivation) without using Newton's second law of motion. Newton did not quote Hermann's equation even in last edition of the Principia, as equation for second law of motion.

(x) Leonhard Euler (1736, 1749, 1752, 1765, 1775) gave different equations relating to force and acceleration. In 1775, Euler gave specifically F =ma as shown in section (5.0)

$P=\int dM \frac{d^2x}{dt^2}$, $Q=\int dM \frac{d^2y}{dt^2}$, $R= \int dM \frac{d^2z}{dt^2}$

Or in general, $F = \int dM \frac{d^2s}{dt^2} = ma$ (1)

Truesdell had stated in 1960 that F =ma has been given by Euler . About this Raman [14] has written in The Physics Teacher [March 1972, page 137] of American Institute of Physics that

" Although this remark was made over a decade ago we still find textbooks in which F =ma is called Newton's formula, and which make absolutely no mention of Euler in this context".

So scientific opinion is that Euler's name should have been associated with equation F=ma. Cohen [11] presented the law in such a way that Euler and Hermann had no original contributions regarding this. Further he gave four equivalent forms in speculated way. These are F =kdV, F = KdV/dt, $F = k_1 d(mV)$, $F = k_2\, d(mV)/dt$.

(xi) It is confirmed in section (4.6) that there are no clear scientific evidences when F =ma was associated with Newton's second law of motion. The historical reviews of physics and mathematics are required purposely.

Part III

F=kdV, F =ma and Rouse Ball's change in definition of Newton's second law of motion.

(xii) Newton did not give any equation for second law. The genuine equation for second law is F =kdV which is not discussed. When acceleration was found exceptionally useful term in differential and integral calculus, then scientists associated F =ma with second law of motion.

(xiii) When scientists tried to derive F =ma from definition of Newton's second law of motion, then some inconsistent assumptions were made.

(a) Motion = momentum (mV). In fact, motion is other name for velocity which was prevalent earlier.

(b) Change in motion = rate of change of momentum = d/dt (mv-mu)

But equation is not justified as units, dimensions and magnitude of both Left-Hand Side and Right Hand Side are different.

Left Hand Side	Right Hand Side
Units m/s	Units m/s^2
Dimensions M^0LT^{-1}	Dimensions MLT^{-2}
Magnitude mv-mu	Magnitude $m(v-u)/(t_2-t_1)$

Hence we find that

$mv-mu \neq m(v-u)/(t_2-t_1)$
Also an article published in American journal of Physics [12] (2011) by **Bruce Pourciau** at page 1015 states that –
"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."
Even then here 'rate of change' is associated with derivation of F=ma. Newton ignored 'acceleration' throughout his life but it is associated with F =ma (prevalent form of Newton's second law of motion).
(c) The constant of proportionality k =1 (F=kma), whereas constant of proportionality in universal constant (experimentally measured)
$G = 6.6743 \times 10^{-11}\ m^3 kg^{-1}\ s^{-2}$.
So, there are two different methods for measurement of constants i.e., one is by experiments (as in case of G) and other by assumptions (as in case of k).

Rouse Ball's change in definition of second law of motion

(ivx) Apparently Rouse Ball did not agree with that fact that
Change in motion = rate of change of momentum = d/dt (mv-mu) (5)
The LHS and RHS of equation has different dimensions, units and magnitudes. It is justified above.
Then he introduced phrase in definition of book An *Essay on Newton's "Principia* ***[19]."*** (London: Macmillan and Co., 1893.) stated Newton's law different way.
The change in momentum ***[per unit of time]*** *is always proportional to moving force impressed and takes place in direction in which force is impressed.*
If change in momentum occurs in 10 seconds
$m[v-u]/t \propto F$ or $F \propto m[v-u]/10$

Part IV

Cohen's equivalent forms of equations (**equal in value, amount, function, meaning, etc.**)

(xv) Cohen did not agree with introduction of 'per unit of time' by W W Rouse Ball **[19].** I Bernard Cohen objected Rouse Ball's definition at page 111 as
" It apparently never occurred to him to try to find out what Newton meant, rather than to introduce, per unit of time."
Thus Cohen gave four equivalent forms or equations of second law of motion instead of one equation $F = ma = md^2x/dt^2$. Cohen initiated these equations from genuine form of Newton's second law of motion,
$F \propto dV$ or $F = kdV$ (11)
Thus Cohen assumed that motion is velocity V. Thus, four equivalent forms of equations of second law of motion are I F =kdV, genuine form of second law of motion. First equivalent form of second law of motion.
II F = KdV/dt = dV/dt (acceleration) second equivalent form of Newton's second law of motion (K =1)
III $F= k_1 d(mV) = d(mV)$ change in momentum, third equivalent form of second law of motion (k_1 =1)
IV $F = k_2 d(mV)/dt = m(dV/dt) = ma$ fourth equivalent form of Newton's second law of motion (k_2 =1)
As all forms are equivalent (**equal in value, amount, function, meaning, etc.**)
$F = kdV = k_1 d(mV) = KdV/dt, = k_2 d(mV)/dt$. These equations are obtained inconsistently.
(xvi) Cohen did not give neither need nor advantage of equivalent forms or equations of second law of motion. Cohen did neither give magnitude of k, k_1, K and k_2. The magnitude of constants is determined experimentally e.g. value of universal gravitational constant G is measured as $6.6743 \times 10^{-11}\ m^3 kg^{-1}\ s^{-2}$.
So simply saying that these are four equivalent forms of second law of motion as cited above, whereas since centuries the prevalent form of second law of motion is $F = \int dM \frac{d^2s}{dt^2} = ma$.

Part V

Generalized or modified form of Newton's second law of motion

Newton's second law of motion implies that change in motion is proportional to impressed force. Change in motion is proportional to impressed force i.e. F = kdV. This equation is completely neglected in the existing physics. Also, the definition of the second law of motion is used in altered form such that F=ma is obtained not F =kdV. V V Raman has obtained F =ma from original form of definition of second law of motion by giving inconsistent arguments.
(xvi) Newton's second law of motion as F =ma is integral part of Physics. Einstein's rest mass energy $E_{rest} = M_{rest}\, c^2$, is derived from F =ma. The units are dimensions of various physical quantities are based on it. So, this equation (F=ma) is inseparable part of physics.

(xvii) It is better to have a definition as postulate rather than having inconsistent derivation, having series of inconsistencies one after the other.
(xviii) "*The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

$F = dp/dt = mdV/dt = ma$ (2)

Part VI
Advantages of modified or generalized form of second law of motion

When the original form of Newton's second law of motion is modified then all the inconsistencies are removed as discussed above and equation F =ma is obtained flawlessly and genuinely. Also it highlights the vision of the legend and his magical wisdom on the words when he formulated second law of motion. When it is modified, we get F =ma.

(ixx)So, various inconsistent issues are irrelevant whether

(a) 'simple difference' means 'rate of change w.r.t., time

and

'motion' means 'quantity of motion' i.e., mV (momentum) or velocity.

These need not to be considered.

(b) Also, mv-mu = d/dt (mv-mu)

(c) Now Walter William Rouse Ball's proposition of addition of 'per unit of time'.

(d) I Bernard Cohen's four equivalent forms F =kdV, F= $k_1d(mV)$, F = KdV/dt, F = $k_2d(mV)/dt$ for second law of motion are irrelevant. We need not consider their equality as

$kdV = k_1d(mV) = KdV/dt = k_2d(mV)/dt$

Thus we need not consider above equivalence of four equations of equations of second law of motion. Only then equation F = $\int dM \frac{d^2s}{dt^2}$ = ma is obtained.

The reason is that we are capable of getting F =ma, when its original definition of Newton's second law of motion when it is logically modified.

Part VII
The greatness of Newton beyond expression by words

(xx) Newton has given second law of motion, putting it in dynamical form i.e. related force with change in velocity

Due to conceptual limitations Newton

(a) Newton did not use acceleration throughout his career

(b) Newton did not write F =ma

When differential and integral calculus were developed, acceleration was found exceptionally useful terms. Come what may the followers wrote inconsistently Newton's second law of motion as F =ma. Thus, they associated acceleration with Newton's second law of motion.

(xxi) Newton's law simply implies

Change in motion (velocity) $\propto$ impressed force

or F =kdV

But this equation is neglected in the textbooks and standard references. This equation is based on method of proportionality like law of gravitation.

(xxii) If now we modify or generalize Newton's original second law of motion (1686) within frame its own domain.

The real change in definition:

"*The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Original law : Change in motion (velocity) , v_2-v_1; is proportional to impressed force

Modified or generalized section : rate of change of momentum ; is equal to impressed force

The impressed force is common in original and generalized form

rate of change of momentum = impressed force

$m(v_2-v_1) /t_2 - t_1 = F$

ma = F or F=ma

So, F =ma is obtained from the modified or generalized form which is within extension of original law. Thus, the applications and domain of Newton's second law of motion is extended.

Part IX
The magician of wisdoms and words, thy name is Sir Isaac Newton;

(xxiii) It is confirmed that Newton ignored acceleration (a) throughout his life like other preceding scientists (Galileo, Descartes, Huygens etc.). Newton did not write F =ma. The equation was associated with Newton's second law of motion inconsistently. Hermann's equation seems to give F =mdV/dt in 1716 but Newton did not acknowledge it as equation for second law of motion in third and final edition of the Principia in 1726. Euler derived F $=md^2x/dt^2$ in 1775.

(xxiv) The following scientists inconsistently associated F =ma with Newton's Second Law of Motion and not at all mentioned Hermann and Euler. So, it not mistake or fault of Newton, but these are errors by following scientists. It is strange that why did following scientists did not write F =kdV?

Both Hermann and Euler were born at Basel, in Switzerland (the country which never invaded any other country, so a loving territory) and were distant relatives of each other. Scientifically and ethically names of Hermann should be mentioned with F = ma.

However, F =ma follows from modified form of second law of motion, which is basis of physics now.

(xxv) Physics is based on F =ma. So, if Newton's second law is generalized then it leads to F =ma.
The modified or generalized form of second law of motion leads to F =ma with ease and simplicity. So we should teach correct concepts to our coming generations.

The magician of wisdoms and words, thy name is Sir Isaac Newton.

Appendix II

2.5 Some Historical facts

While interacting with expert scientists and general audience one idea comes to mind. Had there been no science in case Newton's laws were not there. Are Newton's laws responsible for development of science. Newton's first law of motion is just improvised form of law of inertia given by Galileo in *the Dialogue* in 1638. Newton has written second law of motion is in dynamical form i.e. related force with motion or velocity. Newton neither wrote F =kdV nor F =ma. Newton's third law of motion explained in somewhat crude form by Rene Descartes in 1644 in somewhat rudimentary or crude form given in the Principles of Philosophy in 1644. As far as fireworks and missiles are concerned, they were prevalent in system about 650 years before Newton's third law of motion in China. Also, initial form of missiles was used in wars in India.

People were used to cross the rivers using wooden logs without knowing about Archimedes Principle. Science will come to halt without speculations or when speculations are experimentally justified, they become valid scientific theories. So, both are inter-related and separated by experimental verifications. For one person an idea may look wrong, but may appear correct for other.

Further science is relative. Whatever is true now, may not have true in past. Whatever is true now may not be true in future.

(a) Newton's Corpuscular Theory of Light implies speed of light must be more in water than in vacuum. Hence it is abandoned now. Now dual nature of light is considered.

(b) Equation based on Newton's doctrine of speed of sound in air (propagates isothermally) predicts that its value must be 280m/s which is not experimentally justified.

$$v = \sqrt{P/d} \qquad (56)$$

where P is atmospheric pressure and d density of air.

Perre-Simon Laplace, the French scholar and polymath argues that sound waves propagate in air adiabatically, not isothermally as assumed by Newton. Thus Laplace modified Newton equation as

$$v = \sqrt{\gamma P/d} \qquad (57)$$

where γ is specific heats of gas .Thus, Laplace's equation gives speed of sound in air equal to 330m/s. So upgradation of laws is common feature.

(c) For about 2000 years we taught that the Sun revolves around the stationary Earth, even then civilizations progressed. Now we teach Sun is stationary and Earth revolves around the Sun, in both the cases humankind progressed.

(d) Einstein's static Universe. A static universe, also called a "stationary" or "Einstein" universe, was a model proposed by Albert Einstein in 1917. Edwin Hubble's discovery of the relationship between red shift contradicted it by completely demonstrating that the universe is constantly expanding. Thus, theories change with experimental evidences.

Ideas are rare may be wrong. One correct idea is explored after hunting 100s or 1000s wrong ideas. Such type of thinking helps us in promoting science. Anyhow here our main aim is to discuss Newton neither discussed acceleration nor wrote F =ma. Then how F=ma is credited to Newton? Newton's original form of second law of motion as given in the Principia is not quoted in textbooks or standard references. If the law which is quoted in textbooks or standard references is same as Principia's law; then why Principia's original law is not quoted instead altered form. Newton's first and third laws are quoted in the original form in the textbooks and standard references. Further, F =ma is derived by Euler, then why Euler's name is not associated with F =ma (equation can be found in school level textbooks)? So, in progress of science there are no shortcuts, it requires tremendous patience and hard work.

(I) Newton, the most celebrated scientist ever walked on the planet Earth
Newton may be regarded as the greatest ever

Newton worked at the greatest scientific institutions as Lucasian Professor of Mathematics at University of Cambridge, 1669-1703, President of The Royal Society London, 1703-1727. Lifelong bachelor Newton, dedicated his life for science and cause of science. He made discoveries which are basis of science today.

Newton started physics separating it from natural philosophy (philosophical study of nature and the physical universe that was dominant before the development of modern science). At that time there are was no experimental data, no mathematical theories. Even then imagination of the legend was so far sighted it is showing us path even now. For example, dynamical nature of second law of motion or rudiments of law of gravitation and calculus in mathematics.

Newton was the greatest scientist of world whose works are taught right from school level; thus, even greater than Einstein undoubtedly. Also his work has astonishing scientific depth and far reaching importance. When his second law of motion just modified or generalized it leads to F =ma. Even Einstein's rest mass energy equation $E_{rest} = M_{rest} c^2$ is based on F =ma. Newton worked at the greatest scientific institutions (Lucasian Professor of Mathematics at University of Cambridge, 1669-1703, President of The Royal Society London, 1703-1727.

Lifelong bachelor Newton, dedicated his life for science and cause of science. The won all controversies in his favor, be it with Robert Hook or Leibniz relating to law of gravitation or calculus. Newton never wrote F =ma but it is associated with his name. V V Raman has published in an ace pedagogical or academic journal *The Physics Teacher* [14] in March 1972 issue at page 137…

"Although this remark was made over a decade ago we still find textbooks in which F =ma is called Newton's formula, and which make absolutely no mention of Euler's in this context. "

Even then Euler's name is not associated F =ma who is genuine discovered of the equation. Also no body writes F = kdV ,

It highlights the significance of Newton.

Thus according to Cohen [11] at page 113, the first person who related second law of motion with derivative (fluxion means derivative) seems to be Jacob Hermann. Thus eq.(1) follows from Hermann's direct interpretation as given in his book *Phoronomia* at page 57 published in 1716. Further Euler derived F =ma in 1775 i.e. 48 years after death of Newton. However, F=ma can be obtained by slightly modifying definition of Newton's second law of motion.

Then scientists tried to deduce F =ma (the central equation in mechanics) from Newton's second law of motion, keeping scientific logic aside. It is done after death of Newton, so Newton cannot be held responsible for deriving equation F =ma inconsistently. This issue is discussed in section (4.6) i.e. under the heading 'A significant question'. The equation F =ma is very significant as gives impression that all three laws of mechanics (hence science) are coherently inter related and complete; and given by Newton only.

Apparently, scientists from England or admirers or ardent followers of Newton might have attributed to the famous equation F =ma ($F = mdV/dt$ or $F = md^2x/dt^2$) to second law inconsistently and the process is on ever since. This issue is also discussed in section (3.7). Here conceptual similarities and dissimilarities between F =ma and F =kdV are considered. There were some similarities between F =ma and F =kdV. But Newton neither wrote acceleration in the Principia nor equation F =ma. The genuine equation based on definition of Newton's second law of motion is F =kdV.

(II) 3.5 million Britishers ruled 24% land area of the Earth

England definitely had some golden and rarest years in history of science. As Britishers colonized various countries (or British Empire spread) of world, then Newton's laws were also taught there. It is possible Newton's laws reached in many countries before the original Principia. By 1920, British empire covered 24 % of the Earth's land area of world. At that time population of England was just about 35 million. It is anybody's guess that how many people lived in England and how many abroad? Also, it is one's guess how many Britishers remained in transit at particular time. English used local resources and people for the dominance in the particular regions.

Thus, English controlled the world by just brain power or advanced science. At the peak of its power, the phrase

"the empire on which the sun never sets"

was often used to describe the British Empire, because its expanse around the globe meant that the sun was always shining on at least one of its territories.

Further Newton belonged to England a greatest country at that time, even now. It is evident from the book *All the Countries We've Ever Invaded*: *And the Few We Never Got Round To* written by **Stuart Laycock [35].** According to book out of 193 countries that are currently UN member states, Britain has invaded or fought conflicts in the territory of 171. It is irony that F =mdV/dt and F = md^2x/dt^2 or F =ma both equations were given by Swiss scientists (Euler and Hermann) ; and Switzerland has not invaded any country. It is the power of the peace. Euler and Hermann were distant relative to each other (both born in Basel, Switzerland). Also, it is the rarest scientific coincidence in various respects. Further F =ma can be obtained from Newton's second law of motion by modifying it. It is discussed in section (14.2-14.4).

(III) The Principia was exceptionally distinguished book:

In England, Newton's work was introduced in school and college textbooks. Here the oldest possible reference available for discussion, **'First Three Sections of Newton's Principia'** [29]. It is designated as Cambridge School and College Text Books published in 1871 from London. But now the Principia is not included in textbooks, but laws from *the Principia* are taught right from school level. In the book the definition of the law is same as that in the Principia, but F =ma was not quoted at all. At that time the Principia had limited access but now it can be down loaded anywhere via internet **[1,2].** Thus now Newton's Principia is more accessible for critical analysis.

(IV) University of Cambridge and University of Oxford

Now it is important issue to understand that where F=ma was related to Newton first of all. It needs the thorough search of history of scientific literature 18th and 19th century. But apparently it may have been done in Europe particularly in England (where the Principia was taught as textbook for first time). Further more specifically by scientists of university of Cambridge, University of Oxford and luminaries of The Royal Society are mentioned. It is very true that earlier Newton's laws were taught in England. It may be regarded as just speculation unless complete review of literature is made.

The oldest universities of world/Europe exist in England. When once F =ma was credited to Newton's second law, then scientists tried to justify by various methods that F =ma follows from Newton's second law.

(V) The fall of Latin and emergence of other languages: The fall of Latin as principal language definitely affected understanding of scientific literature in Latin. The Principia and works of Euler were published in the Latin. The translation of the Principia started to English from Latin as soon as its final edition was published. Andrew Motte (1696-1734), brother of one of Great Britain's most famous publishers, Benjamin Motte who encouraged Andrew to translate the 1726 edition of the Principia in English. Thus, Benjamin Motte published English translation in 1729. Even now majority of Euler's works are in Latin. It affects the understanding of Euler's work significantly. The scientific literature in Latin became dependent of translators. Thus, the limited understanding of literature in Latin cannot be denied. Euler's nearly 900 papers, articles, notes and books etc. were published in Latin in Russia, Germany and Switzerland.

According to **Professor Gordin [36]**, he wrote in the book *Scientific Babel: How Science Was Done Before and After Global English.* From 1880 to 1910 roughly equal numbers of publications appeared in German, French and English, and German overtook English by 1910. After World War I English became the dominant language. Thus, now texts in Latin are not fully understood now unless translated to other languages. Now Ian Bruce has taken voluntary job of translation of scientific books written in physics by legends in 17th - 18th century from Latin to English. Such efforts be gratefully acknowledged.

2.6 Launching of Satellites

I quote here a true incidence that how useful Newton's laws are? General public have tremendous faith in the laws, that what has happened or what is happening or what would happen is explainable with help of Newton's laws. That it true but scientific temperament always keeps room for slightest doubt so critically analyze the facts. This is scientific temper requires for scientific speculation and critical analysis. Should one be completely satisfied that exhaust (in simple words smoke, spark, gases, fire etc.) from fireworks moves backward and fireworks (rocket) moves ahead. The next part is critically analyze the same what is momentum of exhaust? What is momentum of rocket? Are both always precisely and quantitatively equal or some factors affect both. But there is no experimental data which is experimentally confirms it. It is common observation that fireworks moves in atmosphere leaving behind exhaust. Nobody has quantitatively measured momentum of exhaust (mass x velocity of exhaust) and momentum of rocket (mass x velocity of rocket). It can be measured instantaneously, and can be conducted in vacuum also. There is no such experimental data.

Once I critically talked about Newton's law with my journalist friend. He listened to me carefully and said nothing as it was usual way of our conversation sometimes. He did not have scientific background. But Journalist friend contacted a physicist and when we met next time I got reply.

Journalist friend said,

"The satellites are launched on the basis of Newton's third law of motion. Do you how precisely scientists have

studied it? If Newton's third law were wrong then how our space mission 'pppp'has been successful."
I simply requested him to let me know
"If Newton's third law is correct, then why our space mission 'qqqq' has been failed? Many other scientific phenomena are used in launching of satellites, not just Newton's Third Law of Motion. We study that part of journey of satellite where Newton's third law is applicable. Even there is no experimental data regarding upward movement of fireworks. The quantitative experimental data can be easily measured with fireworks compared to launching of satellite. Or we can form specific artifacts to confirm this point."

So exchange of ideas is always useful. The science is based on speculations, without speculations it is static or superstition. If speculation is experimentally confirmed then it becomes theory. It was just occasional conversation which led me to study more and more historical background of Newton's third law of motion. The journalist has also consulted some senior physicists of the region as it appeared to me.

Finally I told him I have discussed papers on 'Role of shape of body on third law of motion', in international conferences. For example, in American Association of Physics Teachers at Washington on 1st August 2018. Many reputed scientific bodies and scientists are encouraging me to conduct such experiments to study role of shape of bodies. Also, at Indian Science Congress at Bengaluru in January 2020. Thus aspect is also mentioned in section (2.5). Thus, arguments continued till one of us was not tired. I spoke for some increments augmentations in Newton's third law of motion (by modifying it) within domain of Newton, and journalist spoke that is being taught over centuries. Then we really relished cups of tea.
Any how the question of journalist friend is valid even as of now and can be scientifically understood with help of scientific and historical facts.

Rockets were discovered 650 years before Newton's Third law

The rockets were developed by Chinese in 11th century. Initially they were used for amusement purposes. The rockets were used for destructive purposes in 1232 during the Chinese and Mongols war. During the battle of Kai-Keng, the Chinese repelled the Mongol invaders by a bombardment of "arrows of flying fire." These fire-arrows were a simple form of a solid-propellant rocket. The principle was simple as that of launching of satellites or fireworks. The fire, sparks, gases, smoke (exhaust) move backward and rocket moves in forward direction. At that time law of motion was not available for discussion. The rocket move upward defying gravity.

Hyder Ali (the Sultan and de facto ruler of the Kingdom of Mysore in southern India), was an innovator in the military use of rockets, which were used against positions and territories held by the British East India Company during the Anglo-Mysore Wars during 18th century. In Hyder's time the Mysorean army had a rocket corps of as many as 1,200 men, which Tipu Sultan (son of Hyder Ali) increased to 5,000.

At the 1780 Battle of Pollilur, during the second war, Colonel William Baillie's ammunition stores are thought to have been detonated by a hit from one of Hyder's rockets, contributing to the British defeat. Tipu Sultan used aggressive rockets during the Battle of Pollilur. These rockets were more advanced than any that the British East India Company had previously seen, chiefly because of the use of iron tubes for holding the propellant. This enabled higher thrust and a longer range for the missile (up to 2 kilometres (1.2 mi).

After Tipu Sultan's eventual defeat and death in the Fourth Anglo-Mysore War (1798-99). It changed the scenario. Thus British captured a number of Mysorean iron rockets and carried to England for research; they were influential in British rocket development, inspiring the Congreve rocket, which was soon put into use in the Napoleonic Wars.

Rocket technology was developed without knowing Newton's Third Law

The fire arrows were first reported to have been used by the Southern Wu in 904 during the siege of Yuzhang.[1] Chines have developed rocket technology in 11th century i.e. about 650 years before Newton's Third law of motion i.e. action is precisely equal to reaction under all conditions. So rockets were discovered practically centuries before Newton's third law of motion. Newton published the Principia in the Latin in 1686, 1716 and 1726 and translated to English in 1729. Indians were not well versed in the Latin or English to study Newton's Principia, hence third law of motion.

"Thomas Babington Macaulay was the secretary to the Board of control of India during the British rule. He was the secretary under Lord Grey from 1832 to 1833; he is known for his Minute on Indian Education which came out in February 1835. He wanted to teach English to the people of India and not Sanskrit or Persian."

Thus, rocket technology was developed in India and China without knowledge of Newton's third law of motion. So, it is obvious that this technology is developed with experiments and experiences; as Frenchmen known as Montgolfier brother developed hot air balloon in 1783. They were paper manufacturers. Of the two brothers, Joseph who was interested in aeronautics; as early as 1775 he built parachutes, and once jumped from the family house. They invented balloons by curiosity and experience

According to Archimedes principle as aluminum or plastic body falls down as heavier than air. So aeronautical scientists went ahead of Archimedes inventing how heavier bodies can fly? They did not confine their research just to limited to fact that heavier bodies cannot fly. According to Archimedes principle the density of aero plane or helicopter is more than that of air, so both must fall down. Thus, external factors also contribute. So scientific thinking is not just confined to laws which are taught in text books

Ideal Rocket Equation.

Sputnik 1 was the first artificial Earth satellite launched by The Soviet Union into an elliptical low Earth orbit on 4 October 1957. The force driving rocket forward is an example of Newton's Third law of Motion [to every action (force) there is equal and opposite reaction (force)). The gas goes in one direction (say backward) and the rocket in the opposite direction (say forward direction)] . Russian scientist Konstantin Tsiolkovsky derived and published equation in 1903 known as ideal rocket equation. This equation even does not involve acceleration due to gravity as given by

$$v = v_e \ln \frac{M_0}{M} \qquad (58)$$

where v is forward velocity of rocket , mass (M) of rocket at any time, M_o is original mass and v_e is velocity of exhaust. It simply implies as mass decreases the velocity increases. The acceleration due to gravity is not involved in eq.(58), so it is the limitation of equation. Now movement of rocket right from launching to landing is controlled by computerized programs, the quantitative applicability of the third law can be clarified by scientist working at satellite launching center. The rocket propellent are of different types e.g. solid chemical, liquid chemical, cryogenic propellent etc. So different types of propellent have different masses. But according to Konstantin Tsiolkovsky's equation the velocity of rocket depends upon reduction of mass.

The purpose of mentioning these facts is that scientific breakthroughs are wide open; they do not wait for enunciation of law, highly sophisticated laboratories or big sponsors. So human mind should continue with patience, hard work and determination without worrying for the facilities and sponsorships for welfare of mankind. John Logie Baird did not wait for million-dollar sponsors and applaud before invention of television. Baird was even not respected immediately after the biggest invention of the 20th century.
This was little discussion not dealing with Newton's second law of motion, it only provides reader an impetus to continue working for welfare of mankind with whatever resources they have.

The purpose of mentioning Konstantin Tsiolkovsky's equation is that it does not contain important factor 'g' (acceleration due to gravity), so this equation is just initial equation. So, science progresses gradually. However right from moment of launch the satellite is controlled by computer programming.

2.7 Discoverer of television, John Logie Baird was called lunatic when he reported invention for first time.

The scientific innovations have never been easy. In July 1924, John Logie (Scottish inventor who discovered television) received a 1000-volt electric shock, but survived with only a burnt hand, and as a result his landlord, Mr Tree, asked him to vacate the premises. In his laboratory on 2 October 1925, Baird successfully transmitted the first television picture with a greyscale image. Baird went downstairs and fetched an office worker, 20-year-old William Edward Taynton, to see what a human face would look like. Thus Taynton became the first person to be televised in a full range. It shows how alone was J L Baird when he discovered television and hardships he faced? But his quest for knowledge invented television which is extremely useful for humankind not only in entertainment and information but also in medical science.

Looking for publicity, Baird visited the *Daily Express* newspaper to promote his invention. The news editor was terrified and he was quoted by one of his staff as saying:

"For God's sake, go down to reception and get rid of a lunatic who's down there. He says he's got a machine for seeing by wireless! Watch him — he may have a razor on him."

Some discoveries are difficult to make, some discoveries are accepted with difficulty.

Some related information dealing with historical aspects

The University of Cambridge was closed during 1665-1666 due to spread of Great plague of London. During these two years Newton was in the village Woolsthorpe Manor, years of isolation from scientific community. The historians of physics or science point out that Newton made important discoveries during this time, it was period of great scientific creativity in isolation.

During this period Newton is supposed to have discovered Fluxions (Newton's term for derivatives), optics and other significant discoveries like law of gravitation. Thus this period when Newton was isolated from scientific community in 1665-1666, may be called '*Annus Mirabilis or* pl. *anni mirabiles'* for Newton. Einstein published five papers including that of The Special Theory of Relativity in 1905. The year 1905 is called *annus mirabilis* (wonderful year or auspicious year) for Einstein. But Einstein was not in isolation. He worked at patent office six days a week when five epoch-making papers were published. Einstein's son Hans Einstein was born in 1904 and in 1906 Einstein got PhD degree when he submitted thesis second time (the first submission was withdrawn by him). He may have helped him by his wife Mileva Maric in one way or other. Einstein's all five papers and many others also were published un-reviewed i.e. without scientific scrutiny. The papers were published as sent by Einstein, as papers were in short supply at that time and peer review was not standard for *Annal der Physic* (journal in which Einstein published many articles) at that time. But Newton is believed to have drawn important conclusions in this period (1665-1666) and developed later on.

It can summed up as

"In 1665-66 Isaac Newton, is supposed to have made ground-breaking inventions and discoveries in calculus, motion, optics and gravitation. In 1905 when Einstein made his wonderful discoveries was of 26 years (born in 1879) and Newton was three years younger to him in 1665. Einstein's discoveries were published in the same years whereas Newton published his discoveries afterward developing as complete theories. He worked as an examiner at the Patent Office in Bern, Switzerland."

It was in this year 1666 that Isaac Newton was alleged to have observed an apple falling from a tree, and in which he in any case hit upon the law of universal gravitation (Newton's apple). However, Newton himself never mentioned this incidence in his writings. Newton has given various propositions (I-X), regarding law of gravitation in Book III of the Principia [ ]. This incidence is supposed to be taken place in 1665, thus Newton had plenty of time till 1727. Or Newton had 61 years to describe this incidence in any book, article or memoir. But he did not mention.

However, in 1726, Newton shared the apple anecdote with William Stukeley who included in a biography, "Memoirs of Sir Newton's Life" published in 1752 i.e. 25 years after death of Newton.

It is believed that Archimedes found solution of the problem given to him by king to determine the purity of king's crown while taking bath in tub of water. Archimedes ran in streets forgetting that he had not put clothes on body shouting, Eureka …Eureka (i.e. I have found it, I have found it). If this story (2300 years old) is true, then it must be *moments mirabilis* for Archimedes. Also there is no evidence that Archimedes has ever written above incidence. However it is true that very little writings of Archimedes has survived in past nearly 2300 years.

Newton was knighted by April 1705, Queen Anne so he became Sir Isaac Newton. Also Newton has suffered nervous breakdown in 1692 and 1693. In 1697 Newton was appointed as warden of Royal Mint and afterwards Master of Royal Mint. Newton was also a member of the Parliament of England for Cambridge University in 1689 and 1701.

Gottfried Wilhelm (von) Leibniz (1646-1716)

Leibniz was a prominent German polymath and one of the most important logicians, mathematicians and natural philosophers of the Enlightenment discovered the same (fluxions) around in 1773. However, Leibniz had published his similar results 9 years earlier in 1684. Further Newton published his results in 1693 i.e. 9 years after Newton. However, Leibniz claimed that he has discovered calculus independently and published in 1684 whereas Newton published in 1693 after him. Why did Newton did not claim the priority in 1684 and why he claiming the same after 9 years and disputing now?

It sparked controversy between Newton and Leibniz; Newton claimed that he has discovered method of fluxion earlier and Leibniz has stolen his work. Leibniz questioned why did not Newton publish earlier if he had discovered the laws earlier? Why did Newton publish 9 years after him (Leibniz)? Newton's book The Method of Fluxions and Infinite Series was published 9 years after author's death. Newton is supposed to have completed this book in 1671 but The *Methods of Fluxions and Infinite Series* was published in 1736, 9 years after death of Newton. But Newton did not write acceleration as dV/dt in the book, this is the main outcome relevant for current discussion. It is mentioned earlier that Galileo had discovered and explained acceleration at pages 133-134 and 146 of his book *Dialogues Concerning Two New Sciences* published in 1638.

Appendix III
Frequently Asked Questions
Part I
Theme of Discussion

Q.1 What is the theme of discussion, about a law (F =ma) which is the most established law in science. F =ma is used to derive $E_{rest} = M_{rest} c^2$ and used with law of gravitation ($F = Gm_1m_2/r^2$) to draw various conclusions ?

Ans . In one sentence the theme is ……F =ma has never been given by Newton. Acceleration was ignored by Newton throughout his scientific career, however it existed in literature during Newton's time. Galileo has elaborated acceleration in 1638 (4 years before birth of Newton) in his book Dialogue Concerning Two New Sciences at pages 133-134 and 146.

The genuine equation based on Newton's second law is F =kdV, which is ignored by scientists. F =ma (given by Euler in 1775, E479 http://eulerarchive.maa.org/) is credited to Newton's second law arbitrarily. As F =ma is not derivable from Principia's second law of motion ,

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

To derive F =ma from definition of Newton's second law, scientists arbitrarily meant motion as momentum (mV), when it did not solve the purpose then they assumed 'change in motion' equal to 'rate of change of momentum'. The later assumption is not justified as units (kgm/s and kgm/s^2), dimensions (MLT^{-1} , MLT^{-2}) and magnitudes are different. It is discussed in section (2.1).

Further scientists had further a arbitrary step by changing the definition of second law of motion, and associating it with acceleration. Newton has ignored acceleration throughout his scientific career. So the definition of second law of motion is different in textbooks or standard references than in the Principia given by Newton.

Thus remedy to the problems is to change the definition of the law as

"*The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Thus, in this case we get equation F =ma and need not have to make inconsistent and arbitrary assumptions.

Part II

About Newton's Second Law of Motion

Q.2 Is Galileo's law of inertia basis of Newton's laws? How Descartes third law of motion is related with Newton's third Law of motion?

Aristotle

Ans. 2 Aristotle (385-323BC) stated that force is required for movement of body. The table stops as soon force (may be push or pull) ceases to act on it. It is clearly observed even now due to presence of various resistive forces in daily life observations. So it must be domain of applicability of Aristotle's assertion

The concept of *inertia* was alien to the physics of Aristotle. Aristotle, and his peripatetic followers held that a body was only maintained in motion by the action of a continuous external force. Aristotle implied that rest is natural tendency of body, it is disturbed when external force acts on body; and justified in above example. This doctrine was contested between admirers and critics for centuries.

Galileo and other scientists preceding to Newton

Galileo conducted some simple experiments with domestic instruments in 1604. These were published in in the book Dialogues Concerning Two New Sciences in 1638. Here Galileo defined uniform velocity (p.128), uniform acceleration (p.133-134,145) and law of inertia as

Uniform velocity : *By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal.*

Uniform acceleration : *A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed.*

Descartes used Galileo's law of inertia in second law of motion and Christiaan Huygens in first hypothesis in book *Horologium oscillatorium sive de motu pendularium* (1673). All implies bodies move with constant or uniform motion (velocity) under certain conditions. When motion is mathematically expressed then it is nothing but velocity. Newton's perceptions of First Law of Motion is based on Law of inertia.

"*Everybody perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon".*

Thus, Galileo's law of inertia,

"*Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits".*

Thus law may be regarded as basic law of mechanics.

Descartes second law of motion

Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line. (Principles Part II, article 39).

Christiaan first hypothesis

If there is no gravity, and the air offers no resistance to the motion of bodies, then any one of these bodies admits of a single motion to be continued with an equal velocity along a straight line.

Newton's first law of motion

"*Everybody perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon".*

implies that bodies move with uniform velocity in absence of force. These laws are kinematical in nature i.e. do not relates force with motion mathematically.

However, Newton's second law of motion is dynamical in nature i.e. it relates force with motion or velocity. For example, change in motion is proportional to alteration (change) force. F change in motion (velocity). The second law of motion reduces to first law of motion when force is zero.

As far as third law of motion is considered then initially Descartes has given third law in somewhat unpolished form as

When a moving body collides with another, if its power of continuing in a straight line is less than the resistance of the other body, it is deflected so that, while the quantity of motion is retained, the direction is altered; but if its power of continuing is greater than the resistance of the other body, it carries that body along with it, and loses a quantity of motion equal to that which it imparts to the other body. (Principles Part II, article 40).

Whereas Newton's third law of motion is in simple and poetic form

"To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts".

Thus basis of three laws of motion was present at time of the Principia (1686).

Q.3 What is Newton's second law of motion? What is equation based on definition of second law?

Ans.3 Newton has written masterpiece *The Mathematical Principles of Natural Philosophy* (popularly known as *the Principia*) in 1686. The other editions of *the Principia* were published after 1713 and 1726. Newton made various changes in second and third editions of the Principia but the laws of motion remained same for the 40 years. Also, Newton did not change the Definition section of the Principia.

Definition of Newton's second law as given in the Principia.

Galileo's law of inertia, Descartes second law, Huygens first hypothesis and Newton first law of motion are kinematical in nature i.e. they do not relate with force with motion. However, Newton improvised second law of motion to dynamical form i.e. relates force with motion of bodies. Newton's second law of motion

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Genuine equation based on definition of Newton's second law of motion

Newton's second law of motion implies

Alteration or change in motion (velocity) $\propto$ impressed force

F (impressed) = kdV

There are so many evidences that Newton has assumed motion as velocity, it is discussed in sections ( ).

This equation is just like law of gravitation, $F \propto M_1M_2$, $F\propto 1/r^2$

$F \propto M_1M_2 /r^2$ or $F = G M_1M_2 /r^2$

Both the equations i.e. eq.(9) and eq.(11) are obtained by method of proportionality. The equation for law of gravitation is retained by scientists and for second law of motion i.e. F =kdV is ignored by scientists.

Q.4 Is Newton's second law of motion as given in the Principia, quoted in textbooks or standard references?

Ans.4 No , Newton's second law of motion as given in the Principia is rarely quoted in the textbooks.

The law is quoted in the different form so that F =ma may be arbitrarily obtained from it.

It must be clearly noted that Newton's first law of motion and third law of motion as given in the Principia (1686,1713,1726) are quoted in the textbooks and standard references.

But Newton's second law of motion as given in the Principia is not quoted in the standard textbooks , it is quoted in distorted form it is not ethical.

Scientists distort (express in other way than given by Newton) the definition of Newton's second law of motion so that they may get F =ma. But even this does not serve the purpose.

Newton's second law of motion as given in the Principia gives equation

F =kdV

However the distorted form of second law of motion is does not give F =ma, so scientists make arbitrary assumptions.

Part III
Origin of Acceleration

Q.5 Who gave acceleration first of all i.e., who is genuine originator of acceleration? When acceleration was given?

Ans.5 Galileo has given acceleration in his book *Dialogue Concerning Two New Sciences* published in 1638 at pages 133-134, 145. Galileo has defined acceleration, explained experimentally also gave equation in statement form. Mathematically, Linear acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) =dV/dt (27)

Let body starts from the rest then after 1s, 2s, 3s and 4s its velocity will become 1m/s , 2m/s , 3m/s , 4m/s respectively then it will fall with uniform acceleration as

Linear acceleration = change in velocity (v_2-v_1) /change in time (t_2-t_1) = 1m/s^2 (27)

Acceleration was given by Galileo in 1638.

Newton neither gave acceleration nor used it throughout his life of 85 years (1686-1742).

The relevant scientific literature is reviewed right from Galileo's experiments in 1604 to 2011 regarding the development of acceleration as quoted in the American Journal of Physics 2011. Newton's book **The Methods of Fluxions** was written by Newton in 1671 and published in 1736 i.e. 9 years after death of Newton. The acceleration (dV/dt) is not mentioned in *the Principia.*

Q.6 Did Newton quote acceleration given by Galileo?

Ans.6 Newton never quoted acceleration given by Galileo. Newton only quoted Galileo's uniform velocity and law of inertia. The reason is that it is simpler to explain uniform motion than accelerated motion. Initially scientists followed the simpler path. Newton's first law of motion is other form of law of inertia, as quoted in Q.3. Newton ignored acceleration throughout his life for 85 years, even it existed in literature as given by Galileo in 1638.

Q.7 What is status in existing literature regarding Newton's contribution of acceleration and equation F = ma? definition of second law he writes motion not quantity of motion. Cohen at page 111

Ans.7 In the existing literature, it is quoted at different occasions but in discrete way that Newton ignored acceleration (given by Galileo in 1638 i.e., 4 years before birth of Newton) like Descartes and Huygens and did not write F =ma. Even Galileo put forth law of inertia in terms of uniform velocity Here in this compilation all the facts are put together for comprehensive understanding in single volume

Newton did not quote acceleration (as defined by Galileo) throughout his life (1642-1727) for 85 years. Practically Newton ignored acceleration as it was given by Galileo in 1638 i.e. 4 years before birth of Newton. Scientists also believe that credit of discovery of F =ma should be given to Euler (however Jacob Hermann has directly quoted the equation). Even then equation F =ma is credited to Newton inconsistently (Newton did not give F =ma but obtained from second law by arbitrary assumptions). So far in the literature all facts are not put together in any other publication as in this case. So, this is purposeful compilation for highlighting hidden or scattered truth at one front.

Thus, scientists regard F=ma was given by Newton as all information is not available to them. It is simply due to lack of well compiled information, the needful is done here. This is purpose of writing book/monograph. Science is above any precedence; it varies with logical theoretical and experimental results. That is why science is science and superstition is superstition. Even superstitions vary, as at present we may have different superstitions than in earlier days.

(i) Newton did not given acceleration;

An article published in American journal of Physics [10] (2011) at page 1015 states that –

"But there is nothing in the Principia's second law about acceleration and nothing about a rate of change."

It is obvious conclusion. Like Galileo, Descartes, Huygens; Newton also ignored acceleration; and only used uniform velocity in law of inertia. Acceleration was given by Galileo in 1638 i.e. 4 years before birth of Newton.

(ii) Newton did not acknowledge F =mdV/dt in third and final edition of the Principia in 1726.

Cohen [11] at page 113 has correctly written that

"Newton never actually made a formal statement of the second law in the algorithm of fluxions or the calculus."

Fluxion: means derivative**.** It implies Newton never gave statement F =mdV/dt, which is true. Herman has given statement F =mdV/dt in 1716 in his book Phoronomia at page 59, but Newton ignored it completely in third and final edition of the Principia in 1726 as it does not exist.

(iii) Newton did not write any equation for his laws.

Cohen [11] has written at page 117 of that

"Newton did not give equations to his laws."

Like Galileo, Descartes, Huygens; Newton also did not give any equation as equations were not prevalent in those initial days. Newton initiated physics from natural philosophy, where phenomena were explained geometrically in form of propositions and theorems not mathematical equation. We should not expect everything in the beginning at inception of physics. The science has developed gradually, there have been no short cuts in science. The first differential equations of motion for systems having more than two mass-bearing points were published in 1743 by John Bernoulli and by D'Alembert.

(iv) **Name of Euler must be associated with F=ma.**

V V Raman has published in an ace pedagogical or academic journal *The Physics Teacher* [8]in March 1972 issue at page 137…

"Although this remark was made over a decade ago, we still find textbooks in which F =ma is called Newton's formula, and which make absolutely no mention of Euler's in this context. "

This remark was made over Truesdell's paper published in 1960 in *"Archive for History of Exact sciences,* that Euler has given F=ma . Thus, it is obvious conclusion that Euler must be associated with F=ma

V V Raman did not quote in the discussion about Hermann's equation. So scientific views are gradually developed. All these quotations are different publications. Here attempt has been made to put all information together along with detailed and logical explanation. Science is like lighting one lamp from the other.

(v) Definition of second law of motion: The definition of second law of motion implies change in motion. In the explanation Newton stated that -If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. Thus, both in definition and explanation Newton has used change in motion (velocity); but not acceleration as used by scientists above. Thus, arbitrary interpretation is just to obtain F =ma from second law of motion.

Part IV
Origin of F =ma

Q.8 Then how did equation F =ma was originated?

Ans.8 Firstly, Newton did not write F =ma as in that era, it was not prevalent to write mathematical equation. The laws were described philosophically with genuine explanation, may be qualitatively. The propositions, theorems etc. were main modes of interpretation geometrically. It must be noted that Newton's definition of second law of motion as given in *the Principia* gives equation F =kdV.

Also, Newton did not write F=ma; it was related with definition of second law of motion inconsistently by following or succeeding scientists. Had Newton given any equation (but it was not feasible at that time) then issue would have been resolved amicably. There would have been no need for this discussion.

(i) Swiss Jacob Hermann in 1716 has directly given equation in his book *Phoronomia* at page 57

F =mdV/dt

(ii) Swiss Leonhard Euler (distant relative of Jacob Hermann) gave various equation relating force, mass and acceleration in 1736, 1749,1752 and 1765. Finally Euler derived equation in paper *Novi Commentarii academiae scientiarum Petropolitanae* which was published in 1776. However, equation was derived in 1775 as

$$F = \int dM \frac{d^2s}{dt^2} = ma$$

Both have equation of force without using Newton's second law of motion.

Q.9 When did Newton give F =ma? Who associated F =ma with definition of Newton's second law of motion?

Ans.9 Newton never wrote F =ma. Neither in Newton's time nor before there was any tradition or precedence of expressing mathematical equations.

Here acceleration is main physical quantity which was never given or used by Newton (1642-1727) i.e. for long life of 85 years.

It is confirmed in section (4.6) that there are no clear scientific evidences when F =ma was associated with Newton's second law of motion. Who credited F =ma to Newton completely ignoring F =kdV. The historical reviews of physics and mathematics are required purposely.

First Three Sections of Newton's Principia (1871)

When acceleration was found useful differential and integral calculus, then it was associated with Newton's second law of motion. A text book for Cambridge School and College level books titled **First Three Sections of Newton's Principia** was published in 1871. This book does not quote equation for definition of second law of motion as F = ma. Should we conclude that F=ma was associated with F =ma after 1871. It is matter of research of history of physics. The issues have been raised in the discussion. Science is lighting one lamp from the other.

Part V
Inconsistent credit of F =ma to Newton's second law.

Q.10 How and why F =ma was associated with definition of Newton's second law of motion?

Ans.10 Newton did not write any equation with second law of motion; as at that time there was no tradition or precedence to write mathematical equations. The laws were understood in terms of propositions and theorems with help of geometrical methods mainly.

Newton even did not give equation for law of gravitation.

Afterwards the genuine equation of derivation was given as

F =kdV

This derivation is similar to derivation of Newton's law of gravitation ,

$F = G M_1M_2 /r^2$

In both cases scientists use proportionality method.

The equation F =kdV is not mentioned in the textbooks or standard references. Even definition of Newton's second law of motion as given in *the Principia* is not mentioned in the textbooks, as already mentioned in Q. 2. Then methods of differential and integral calculus were developed; and acceleration (dV/dt, d^2x/dt^2) was found useful term. Then Euler's equation ($F = \int dM \frac{d^2s}{dt^2} = ma$) as given in 1775, was useful and was associated as equation for second law of motion hurriedly. Also, similar equation (F =mdV/dt) as directly quoted by Jacob Hermann in 1716.

There is no clear-cut date, place or name of scientist who related F =ma with Newton's second law of motion, is available in the existing literature. So, it is matter of historical research in physics and mathematics. The issue is raised for first time. Also, F =kdV, the genuine equation is not related with second law in standard textbooks.

Q.11 Now scientists have associated F =ma with definition of Newton's second law of motion, it is prevalent over centuries. Then what is problem with this?

Ans.11 The equation based on definition of second law of motion as given by Newton *the Principia* is
F (impressed) = kdV

But this definition is not mentioned in textbooks and standard references. It indicates the law is not properly studied.

Now the equation F =ma is quoted as for Newton's second law of motion but obtained with help of arbitrary assumptions. The equation must logically follow from the definition of the law.

Q.12 How you can prove that F=ma does not follow Newton's second law of motion?

Ans.12

Newton did not give any equation for his second law of motion. This issue should not be misinterpreted. The equation based on Newton's second law of motion is F =kdV. But F =ma is prevalent form or equation of definition of Newton's second law of motion since centuries. The equation F =ma was clearly derived by Euler in 1775 and Hermann directly quoted equation F =mdV/dt. So, scientists hurriedly related F =ma with Newton's second law of motion (as Newton did not given F =ma).

Then scientists tried to obtain F =ma from definition of second law of motion. Then to obtain equation F =ma from definition of second law of motion, under arbitrary assumptions.

F=ma is obtained from Newton's second law of motion under two assumptions. These assumptions are not justified.

(i) **First arbitrary assumption:** The genuine equation based on definition of second law of motion is F=kdV. To obtain F =ma from definition of second law of motion; the motion is regarded as momentum. Newton never called motion as momentum and there are evidences that motion is velocity as discussed in sections (3.1-3.4, 3.8) and other discussion. Thus in arbitrary way,

Impressed Force $\propto$ change in motion or change in momentum (mv-mu)

$F \propto (mv-mu)$ or $F = k\, mdV$

The equation $F = k\, mdV$ is not $F = kma$

The k is regarded as dimensionless constant with magnitude equal to unity (k = dimensionless , magnitude =1) . Like this unit of force is defined. So

So, F is not equal to ma.

However it is justified number of times in sections (3.1-3.4,3.8) and other discussion. that motion is velocity.

(ii) **Second arbitrary assumption:** Further V V Raman [14] has written that it is usual tendency to write *'change in momentum'* equal to *'rate of change of momentum with time'.*

So scientists assume that

Change in motion or change in momentum = rate of change of momentum = $(mv-mu)/(t_2 - t_1)$

$$(mv-mu) = m(v-u)/(t_2-t_1) \qquad (5)$$

But eq.(5) is not justified as units, dimensions and magnitude of both Left Hand Side and Right Hand Side are different.

Left Hand Side		Right Hand Side	
Units	m/s	Units	m/s^2
Dimensions	M^0LT^{-1}	Dimensions	MLT^{-2}
Magnitude	mv-mu	Magnitude	$m(v-u)/(t_2-t_1)$

Thus above equation is not justified. Hence we find that

$mv-mu \neq m(v-u)/(t_2-t_1)$

Now ignoring all the facts, scientists interpreted this equation for F =ma.

$F \propto m(v-u)/(t_2-t_1)$ or $F = k\, m(v-u)/(t_2-t_1) = k\, ma$

The value of constant of proportionality is regarded as unity (to define unit of force).

F = ma

It has been done after death of Newton by other scientists, so it is not fault or mistake of Newton.

Part VI

Motion represents by velocity(V), not by momentum (mV)?

Q.**13 In the literature scientists quote that motion is momentum (mV) , whereas you say motion is velocity (V) . How do you justify deduction? It is very important issue**.

Ans.13 It can be understood on the basis of followings that motion is represented by velocity not by momentum, (i) In Definition I of the Principia Newton has defined quantity of matter (Quantitas Materiae). Further Newton wrote that

"It is this quantity that I mean hereafter everywhere under the name of body or mass"

In Definition I at page 1, Newton clearly stated that he would regard 'quantity of matter' (Quantitas Materiae) as body or mass.

In Definition II at page 2, Newton never wrote in Definition II (few lines down) as he would regard quantity of motion ('Quantitas motus) as motion. However, Newton has defined and explained motion separately with help of examples.

(ii) At page 9, in scholium Newton did not define space, time, place and motion as these are already known. Here Newton further categorized motion in terms of absolute motion and relative motion, both were defined by Newton as velocity. It is explained in section (3.0).

(iii) At page 11 (while explaining motion as velocity) Newton discussed relative motion sailor on moving ship, then motion was regarded as velocity. The reason is that Newton in calculation of relative velocity added and subtracted velocities, not momenta. The velocity of earth is regarded as 10010 *parts* (units of velocity, as we have m/s, now*); towards east*, the ship moves towards west with 10 parts, the sailor walks in ship with velocity 1 part towards east. In Newton's time, units of velocity were not defined (beginning or inception of physics), the units and dimensions [29,30] were defined in 1822.

Now while calculating the relative motion, Newton did not consider momentum of earth (mass of earth x velocity of earth), momentum of ship and momentum of sailors. But Newton considered 10010 parts. 10parts, 1 part as velocities. Newton calculated relative motions equal to 10001 parts or 9 parts or 1 part as velocities of earth, ship and sailor. So, Newton regarded, relative motion (one category of motion) as velocity. So, motion is expressed in terms of velocity not momentum.

(iv) Cohen has given four equivalent forms or equations of second law of motion, the first form or equation is F=kdV. Thus, Cohen has regarded motion as velocity (V) to get equation F =kdV.

(v) Prior to Newton, Galileo has defined uniform motion, acceleration when these are put in mathematical form these are in terms of velocity. Thus, motion is nothing but velocity. It is evident from section (2.15).

(vi) The motion or movement are old terms for velocity. Now motion is not a physical quantity as it does not possess symbol, units and dimensions. The velocity is represented by V, but motion is not represented by m, the symbol m represents length (meter). The symbol, units and dimensions for velocity are V, m/s and M^0LT^{-1}.

(vii) In mathematical equation velocity is taken not motion e.g.

v = u+at (final velocity = initial velocity +at , not final motion = initial motion +at) P =FV (Power = force x velocity). $V_e = \sqrt{2gR}$ (it is known as escape velocity not escape motion). Thus, mathematically we always use velocity. So motion is always represented by velocity, not by momentum.

(viii) Newton's second law of motion is regarded as central law of motion i.e. it reduces to first law when no external force acts in the system (F=0),

$F = m(v-u)/(t_2 - t_1)$

$0 = m(v-u)/(t_2 - t_1)$

or u =v or initial velocity = final velocity

Thus Newton's first law of motion is expressed in terms of velocity. Now the definition of the can be understood as

"*Everybody perseveres in its state of rest, or of uniform motion (uniform velocity) in a right line, unless it is compelled to change that state by forces impressed thereon".*

(x) Newton himself has stated that the velocity produced by force is directly proportional to force in further interpretation to Newton's second law of motion in Book II of the Principia, Proposition XXIV, Theorem XIX [12] as given below.

"For the velocity, which a given force can generate in a given matter in a given time, is as the force and the time directly, and the matter inversely. The greater the force or the time is, or the less the matter, the greater velocity will be generated. This is manifest from the second law of motion."

Thus Newton himself related force with velocity, thus obviously motion is velocity when considered with definition of the law in the Principia.

(ix) Further, Newton himself considered vertical motion of bodies as velocity i.e. in case of falling bodies in Proposition XLI, General Scholium of the Book III of the Principia and Scholium of Corollary VI at page 31 of the Principia

So practically motion is velocity (V), Newton has defined quantity of motion as mV and motion at page 9 in different ways. There are so many other evidences that motion or movement is noting but velocity. Scientists regard motion as momentum to derive F =ma from second law of motion, which is arbitrary.

When they are unable to get F =ma under this arbitrary assumption (motion and momentum); then they regard 'change in momentum' equal to 'rate of change of momentum' which is further more arbitrary. So, these inconsistencies and arbitrary assumptions are made that scientists may get F =ma, from second law of motion (and ignorance of F =kdV may be justified). F =ma is obtained if Newton's second law is modified or generalized.

Part VII
Walter William Rouse Ball and I Bernard Cohen

Q.14 What **the definition of Newton second law of motion as changed by Walter Willian Rouse Ball?**

Ans.14

W W Rouse Ball did not agree with eq. (5) and has written book *An Essays on Newton's Principia* in 1893 at page p.77,

*The change in momentum **[per unit of time]** is always proportional to moving force impressed and takes place in direction in which force is impressed.*

If change in momentum occurs in 10 seconds

$M[v-u]/t \propto F$ or $F \propto M[v-u]/10$

Cohen did not agree with introduction of 'per unit of time'. I Bernard Cohen objected Rouse Ball's definition at page 111 as

" It apparently never occurred to him to try to find out what Newton meant, rather than to introduce, per unit of time."

But the solution given by Cohen is not logical, as it is simply speculative mathematical exercise. Thus Cohen gave four equivalent forms of second law of motion [F =kdV, F = k_2d(mV)/dt, F = KdV/dt F = KdV/dt, F= k_1d(mV)] instead of one equation (F =ma or F =mdV/dt or F =md^{2}x/dt^2)

Thus to give four equivalent forms or equations for second law of motion instead of one equation (F =ma or F =mdV/dt or F =md^{2}x/dt^2), cannot be regarded as logical solution. Cohen has objected to Rouse Ball's proposition.

Q.15 What are Cohen's four equivalent forms of second law of motion? What is their need and advantage?

Ans. 15 Cohen's four equivalent forms or equations (equal in value, amount, function, meaning, etc.) are arbitrarily written. Cohen started from genuine form of second law of motion F =kdV; then obtained other equations by dividing right hand sides by 'dt' (constant time) or multiplying right hand side with mass m. The left-hand sides of equations are not touched at all. So, this mathematical step is completely arbitrary. This aspect is described in sections (11.2).

Cohen did not state the need of arbitrary equations. Also he did not mention the advantages of arbitrary equivalent forms or equations [F =kdV, F = k_2d(mV)/dt, F = KdV/dt F= k_1d(mV)], these are suggested instead of one equation (F =ma or F =mdV/dt or F =md^{2}x/dt^2).

Part VIII
Generalized or modified form of second law of motion (2020)

Q.16 What is need or requirement of modification or generalization of second law of motion ? What is generalized form of second law of motion?

Ans.16 The genuine form of second law of motion is F=kdV, but it is not quoted in the textbooks. F =ma is the prevalent equation for second law of motion since centuries. F=ma is inseparable part of science as units and dimensions of force are based on it. Einstein's Rest Mass Energy equation $E_{rest} = M_{rest} c^2$ is based on F =ma.

So, we should try to obtain it from second law of motion by modifying or generalizing it. The equation F =ma can be obtained from definition of second law of motion when modified or generalized. Both the original and modified equations are shown below.

Original form of Newton's second law of motion (1686)

"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Mathematical equation. F =kdV

Modified form of Newton's second law of motion (2020)

"*The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

Mathematical equation F =ma

Impressed Force $\propto$ rate of change of momentum with time

F $\propto$ (mv-mu) /(t_2-t_1) or F = (mv-mu) /(t_2-t_1) = ma

So we should teach correct concepts to our coming generations.

Q.17 What are the remaining issues about second law of motion which require discussion

Ans. 17 The significant issue left for further discussion is that who related F =ma with Newton's second law of motion? When it is done? Why it is done? Also why genuine equation (F =kdV) based on second law of motion is neglected? Was ever it was considered? This equation (F =kdV) was derived by method of proportionality like that of F =Gm_1m^2/r^2.

The definitions of second law of motion are given in textbooks or standard references which are not given by Newton. It is neither scientifically consistent not logical. This issue needs to be discussed.

Q.18 Do your regard yourself a critic of Newton?

Ans.18. No, I am not critic but I am a student of Newtonian doctrines.

Newton is exceptional – exceptional brain ever walked on the Earth. I regard him higher than Galileo and Einstein.

The genuine equation for Newton's second law of motion is F =kdV, which is not studied The equation F = ma is credited to Newton which does not follow from his law at all.

When we just modify Newton's second law of motion, we get F =ma, this is the beauty as it is basis of science.

"*The rate of change of momentum with time is equal to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

The magician of wisdoms and words, thy name is Sir Isaac Newton

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