*IOSR Journal of Applied Physics (IOSR-JAP)*
*e-ISSN: 2278-4861.Volume 13, Issue 6 Ser. II (Nov. – Dec. 2021), PP 42-45*
*www.iosrjournals.org*

# Equilibrium in Spring Mass System

Alaa Ahmed Abdelgadir Mustafa[1] Ehssan Omer Adam Ahmed[2]
*(1) Northern Border University ,Faculty of Science and Arts, Department of Mathematics*
*(2) Northern Border University ,Faculty of Science and Arts, Department of Mathematics*

---------------------------------------------------------------------------------------------------------------------------------------
Date of Submission: 13-12-2021 Date of Acceptance: 28-12-2021
---------------------------------------------------------------------------------------------------------------------------------------

**Abstract**
A body is said to be in equilibrium[1] if it is at rest or moving with uniform velocity. Inother words if the linear and angular acceleration of a body are zero, the body is said tobe in Equilibrium. or we can say that when two or more forces act on a body such thattheir resultant or combining effect on the body is zero and the body retains its stateof rest or of uniform motion then the body is said to be in equilibrium. Introduction A spring is regarded [1]as weightless, as always obeying Hooke's law and as having zerocross-sectional area so that it always lies along a straight line. Our reason for using such an abstraction instead of a real spring is that we believe it reproduces the main phenomena of oscillatory motion involving a spring while reducing considerably the mathematical complexity. Then the connection between force and motion is given by

$$F = m\frac{d^2r}{t^2}$$

A dot over a letter will usually be employed to indicate a derivative with respect to $t$.

Thus $\ddot{r}$and $\dot{r}$ signify

$\frac{dr}{dt}$and $\frac{d^2r}{dt^2}$ respectively. In this notation(1) can be written as

$$F = m\ddot{r}$$

Suppose now that in the time $\delta t$ the particle moves from the point$r$ to the point$r + \delta r$.
Then , the work done by the force, is defined by

$$\delta W = F\delta r$$

i.e. the scalar product of the force and displacement. If $F$ and $\delta r$ have components$(\delta X, \delta Y, \delta Z)$ and (x, y, z) respectively parallel to the Cartesian co-ordinate axes

$$\delta W = X\delta x + Y\delta y + Z\delta z$$

If we divide (3) by $\delta t$ and proceed to the limit as $\delta t \to t$ we see that$\frac{dw}{dt}$, the rate at which work is done, is given by $t$

$$\dot{W} = F\dot{r}$$

Another [1]important quantity is the kinetic energy $T$ which is defined as one-half theproduct of the mass and the square of its velocity, i.e.

$$T = \frac{1}{2}m\dot{r}^2$$

Substitute for $F$ in (4) from (2); then

$$w = m\dot{r}\ddot{r} = \dot{T}$$

from . [2]Hence, the rate of change of kinetic energy of the particle is equal to the rateat which work is done by the force, applied to it.
Let us assume that during the motion of the particle it moves from $r_0$ to $r_1$. The workdone during this motion is, after integration of (3),

$$w = \int_0^1 \mathrm{Fdx}$$

In general, $W$ will depend [2] upon the path followed from $r_0$ to$r_1$ and different valueswill be obtained for different paths. It may happen, however, that $W$ depends only on $r_0$ and $r_1$ and that the path between them is irrelevant. The force is then said to be conservative and we define the potential energy $V$ by $V = -W$. Integration of(6) then gives

$$T + V = constant$$

for a conservative force, [2]the sum of the kinetic and potential energies is constant
. All the preceding ideas can be generalized at once to a system containing n particles. Let the masses and positions of the particles be and respectively. Then

$$F_i = m_i\ddot{r}_i$$

where $F_i$, [4]is the total force acting on the ith particle. It includes not only the ex- ternal forces originating from sources outside the system but also the internal forces (whether of electrical, gravitational or other nature) [2]due to the other particles of thesystem.
The work done is defined as the sum of the separate contributions from each of the particles so that

$$\delta w = \sum_{i=1}^{n} F_i \delta r_i$$

The kinetic energy $T$ [3]of the system is also defined as the sum of the individual kineticenergies of the particles; thus

$$T = \sum_{i=1}^{n} \frac{1}{2} m_i \dot{r}_i^2$$

In an analogous manner to that used in deriving it can be proved that

$$\dot{W} = \dot{T}$$

If all the forces,[3] both internal and external, are conservative the system is said to beconservative. In a conservative system a potential energy

$$V = -W$$

can be defined and

$$T + V = constant$$

i.e. in a conservative system the sum of the kinetic and potential energies is constant. [5]It should be remarked that even if the forces are not strictly conservative, the sum of the kinetic and potential energies may be constant. This will happen if the forcesacting consist partly of conservative forces and partly of forces which do no work
. Since the forces which do no work [5]do not contribute to the energy equation this equation takes exactly the same form as if all the forces were conservative.
In general, the forces which combine together to do no work can be excluded and the potential energy of the system calculated as the sum of the remaining separatepotential energies.
Thus the contributions of certain internal forces will cancel and they can be ignored in the system as a whole.
For the internal forces which keep two particles a constant distance apart whether theybe gravitational or due to the tension in a taut string or rigid rod, the natural forces between two bodies smoothly pivoted together.
Two of the most important conservative forces from our point of view are gravity andthe tension of a

spring. We consider only local gravity which produces a constant acceleration g vertically downwards. The potential energy of a particle at height $z$ isthen $mgz$.
As regards the spring we imagine it to be weightless and to have a uniform tension throughout its length.
It is represented diagrammatically by a wiggly line . The natural length a of the spring is defined as the length at which the tension is zero.
It will be assumed that the [5]spring obeys Hooke's law of elasticity that the tensionis proportional to the extension
. Then, when the length of the spring is $a + x$, the tension $F$ is given by

$$F = kx$$

The constant $k$ [4]is known as the stiffness of the spring. $F$ is positive when $x$ is positiveand negative when $x$ is negative; this displays the tendency of the spring to return to its natural1ength whether it be stretched or compressed.

The tension in [4]an elastic string is also given by(8) when $x$ is positive,$ka$ then being known as the modulus of elasticity, $i.e. tension = modulus\ Xextension/original\ length,$ the modulus being independent of the string length. But, when $x$ is negative, the forcein the string is zero since the string cannot support compression
If $k$ be large it is still possible for the tension to assume moderate values provided that$x$ is small We can visualize an extreme case [5] in which $k$ tends to infinity while $x$ tendsto zero in such a way that $kx$ remains finite and non-zero
Idealizing in this way we obtain an inextenszoble string which can support tension without change of length.
The work done by the tension when the length of a spring is changed from a $toa + x4$ is
$\int_0^1 -k\xi d\xi = -\frac{1}{2}kx^2$
This shows that the spring force is conservative with potential energy $-\frac{1}{2}kx^2$ –

### 0.0.1The equilibrium of a conservative system with one de- gree of freedom

A configuration of a dynamical system is said to be a position of equilibrium if, whenthe system is placed at rest in that position, it remains at rest.
Since a particle subject to a force will accelerate according to (2) equilibrium is possibleonly if the total force acting on each particle is zero.
This criterion is rather cumbersome to apply [6]in practice and a more convenient onewill now be derived.
In the first place we shall consider only those conservative systems whose position is completely specified as soon as one quantity q is known, i.e. systems of one degree of freedom.
The position vector [6]I of any particle will then be a function of q alone. Consequently

$$\dot{r}_i = \frac{dr_i}{dt} = \frac{dr-i}{dq}\frac{dq}{dt} = \frac{dr_i}{dq}\dot{g}$$

Therefore, formula(7) for the kinetic energy becomes

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i \left(\frac{dr_i}{dq}\right)^2 \dot{q}^2 = \frac{1}{2}M(q)\dot{q}^2$$

where

$$M(q) = \sum i = 1^n m_i \left(\frac{dr_i}{dq}\right)^2 M$$

M is a function of $q$ only since each$r_i$ is such a function. Furthermore[6], the kineticenergy can never be negative so that $M(q) > 0$ whatever the value of q.

Since the system is conservative $T + V$ is constant, i.e.
$\frac{1}{2}M(q)\dot{q}^2 + V = constant$

Now $V$ involves only and so is a function of q onlyConsequently, if we take a time derivative of (9) and cancel a common factor t $\dot{q}$, we obtain

$$M\ddot{q} + \frac{1}{2}\frac{dM}{dq}\dot{q}^2 + \frac{dV}{dq} = 0$$

this is permissible since the equation is valid as for all $\dot{q}$
Let $q = \alpha$ be a position of equilibrium. Then when $q = \alpha$ and

$$\dot{q} = 0$$

there must be no acceleration, i.e. $\ddot{q}$
It follows from (10) that we must have

$(\frac{dV}{dq})_q = \alpha = 0$

Conversely, [3]suppose that(11) is true. Then (10) shows that when $q = \alpha$ and $\dot{q} = 0$ we must have $\ddot{q} = 0$ since $M \neq 0$ But this is not enough to show that the system doesnot move for the third or higher derivative of $q$ might be nonzero at $q = \alpha$ However, a time derivative of (10) gives

$$M\dddot{q} + 2\frac{dM}{dq}\dot{q}\ddot{q} + \frac{1}{2}\frac{d^2}{dq^2}\dot{q} = 0$$

which shows that

$\dddot{q}$ [4] vanishes when both $\dot{q}$ and $\ddot{q}$ do. Taking another time derivative

we see that the fourth derivative is zero when the preceding ones are and, continuing the process, we can see that all derivatives of $q$ vanish so that it must maintain a constant value.
The system is therefore in equilibrium.
What we have demonstrated is that a necessary and sufficient [?]i condition for $q = \alpha$ to be a position of equilibrium for a conservative system of one degree of freedom is

$$(\frac{dV}{dq})_q = \alpha = 0$$

Conculation
We shall consider the [4]simple problem of a particle of mass m suspended by a springof stiffness k from a fixed platform . Both the spring tension and the force of gravityare conservative so that the system is conservative. Let the length of the spring be$a+q$,a being the natural length.
Then the potential energy of the spring is $\frac{1}{2}kq^2$ and that of the mass is$-mg(a + q)$.

Hence

$$V = \frac{1}{2}kq^2 - mg(\alpha + q)$$

$$\frac{dV}{dq} = kq - mg$$

Thus there is one position of equilibrium given by

$$q = \frac{mg}{q}$$

In this position the spring tension exactly balances the force of gravity so that there isno net force on the particle.

## Bibliography


[1]. Applying general equilibrium Shoven, John B and Whalley, John, 1992, Cam-bridge university press
[2]. Informational equilibrium Riley, John G 1979, JSTOR
[3]. A classificatory note on the determinateness of equilibrium Kaldor, Nicholas,1934, JSTOR
[4]. Foreword: Law as equilibrium Eskridge Jr, William N and Frickey, Philip P,1994, HeinOnline
[5]. Equilibrium unemployment theory Pissarides, Christopher A, 2000, MIT press
[6]. Institutional equilibrium and equilibrium institutions Shepsle, Kenneth A, Polit-ical science: The science of politics, 51, 1986


Alaa Ahmed Abdelgadir Mustafa. "Equilibrium in Spring Mass System." *IOSR Journal of Applied Physics (IOSR-JAP)*, 13(6), 2021, pp. 42-45.