IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 14, Issue 2 Ser. II (Mar. – Apr. 2022), PP 18-22
www.iosrjournals.org

An Empirical Analysis of a Particle in an Infinite Square Well Potential by ELZAKI Transform with Eigen Energy Values and Eigen Functions

Arun Prakash Singh,
Associate Professor, Department of Physics
Hindu College .Moradabad

Dinesh Verma
drdinesh.maths@gmail.com

Abstract: *One of the branches of physics is a Quantum mechanics and is a theory of fundamental which gives details the nature of atoms and subatomic particles at the least scale of energy. In mechanics, physical problems are explained by algebraic and analytic methods. Through Elzaki Transform we are finding the Complementary solutions of one dimensional Schrodinger's time - independent wave equation for a particle in an infinite square well potential. This paper, we are talking about the Eigen energy values and Eigen functions of a particle in an infinite square well potential. We will find that general solutions of one dimensional Schrodinger's time - independent wave equation for a particle in an infinite square well potential have very interesting characteristics that each Eigen function is associated with a particular value of energy (Eigen energy value) of the particle confined inside an infinite square well potential.*

Key words: *Eigen functions, Eigen values, infinite square well, Elzaki Transform.*

Date of Submission: 08-04-2022 Date of Acceptance: 25-04-2022

I. Introduction:

Elzaki Transform approach has been applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 13, 14, 15]. It also comes out to be very effective tool to analyze the problems in physical sciences[16,17,18,19,20,21,22,23,24,25,26].The problems in physical sciences are generally solved by adopting Laplace transform method or matrix method or convolution method or calculus method. Quantum mechanics is one of the branches of physics and is a fundamental theory which gives details the nature of atoms and subatomic particles at the least scale of energy and also depicts the behavior of matter and energy on the scale of atoms and subatomic particles or waves [27,28,29,30,31,32,33,34,35,36,37,38]. In this mechanics, physical problems are solved by algebraic and analytic methods. By applying Elzaki Transform to the one dimensional Schrodinger's time - independent wave equation, we can obtain the Eigen energy values and Eigen functions of a particle in a square well potential of infinite height. To define Eigen values and Eigen functions, suppose that an operator Ô is operated on the function g and results the same function g multiplied by some constant α i.e. Ô g = α g. In this equation, Eigen function is g of operator Ô and α is the Eigen value of the operator Ô related with the Eigen function g and the equation is known as Eigen value equation. If Eigen functions selected from a special class of functions. For instance, in bound state problem, all wave functions and their derivatives must be continuous, single valued and finite everywhere. They have to also vanish at infinity. Such functions are called as well-behaved functions. As an instance, to illustrate the Eigen value of an operator, consider the operator $(\frac{d}{du})$ operating on a well – behaved function e^{-6u}. The result is

$$\frac{d}{du}(e^{-6u}) = -6e^{-6u}$$

Comparing this equation with standard Eigen value equation Ô g = α g, we find that (-6) is the Eigen value of operator$(\frac{d}{du})$ associated with the Eigen function e^{-6u}

II. Definitions

Elzaki Transform

If the function $\hbar(y)$, y ≥ 0 is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(y)$ is given by

$$E\{\hbar(y)\} = \bar{\hbar}(q) = q\int_0^\infty e^{-\frac{y}{q}}\hbar(y)dy.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\{y^n\} = n!\,q^{n+2}, where\ n = 0,1,2,..$
- $E\{e^{ay}\} = \frac{q^2}{1-aq},$
- $E\{sinay\} = \frac{aq^3}{1+a^2q^2},$
- $E\{cosay\} = \frac{aq^2}{1+a^2q^2},$
- $E\{sinhay\} = \frac{aq^3}{1-a^2q^2},$
- $E\{coshay\} = \frac{aq^2}{1-a^2q^2}.$

Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{q^n\} = \frac{y^{n-2}}{(n-2)!}$ $, n = 2, 3, 4 \ldots$
- $E^{-1}\{\frac{q^2}{1-aq}\} = e^{ay}$
- $E^{-1}\{\frac{q^3}{1+a^2q^2}\} = \frac{1}{a}\sin ay$
- $E^{-1}\{\frac{q^2}{1+a^2q^2}\} = \frac{1}{a}\cos ay$
- $E^{-1}\{\frac{q^3}{1-a^2q^2}\} = \frac{1}{a}\sin hay$
- $E^{-1}\{\frac{q^2}{1-a^2q^2}\} = \frac{1}{a}\cos hay$

Elzaki Transform of Differential coefficient

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $E\{h'(y)\} = \frac{1}{q}E\{h(y)\} - q\,h(0)$

  $or E\{\hbar'(y)\} = \frac{1}{q}\bar{\hbar}(q) - q\,\hbar(0),$
- $E\{\hbar''(y)\} = \frac{1}{q^2}\bar{\hbar}(q) - \hbar(0) - q\hbar'(0),$

  $and so on$

III. Methodology

Infinite Square well potential:

With a minimum, a potential well is a potential energy function. If a particle is left in the well and the total energy of the particle is less than the height of the potential well, we state that the particle is attentive in the well. By classical mechanics a particle attentive in the potential well can vibrate back and forth but cannot go away the well. In quantum mechanics, such a trapped particle is known as bound state. Considering a particle confined to the region 0 < u < c. it can move freely with in the region 0 < u < c, but subject to strong forces at u = 0 and u = c. Therefore, the particle never cross to the right to the region u > c, or to the left of u < 0. It means V = 0 in the region 0 < u < c, and rises to infinity at u = 0 and u = a. This situation is called one – dimensional potential box .Therefore, an infinite square well potential V (u) (shown in figure 1) is defined as

V (u) = 0 for 0 < u<c

= ∞ for u ≤ 0 and u ≥ c.

The time - independent Schrodinger'sequation in one dimension is written as:

$$D_u^2\emptyset(u) + \frac{2m}{\hbar^2}\{E - V(u)\}\emptyset(u) = 0 \ldots\ldots\ldots (I) \qquad D_u \equiv \frac{\partial}{\partial u}$$

In equation (I), $\emptyset(u)$ is probability wave function of the particle and V (u) is the potential energy. Now from equation (1),

For a particle inside an infinite square well potential,

V(u) = 0

Substitute this value of potential in equation (1), we get
$D_u^2\emptyset(u) + \frac{2m}{\hbar^2}E\,\emptyset(u) = 0$ …… (II)
Where u belongs to [0, c] with boundary conditions $\emptyset(0) = \emptyset(c) = 0$.
For convenience, let us substitute
$\frac{2m}{\hbar^2}E = b^2$…………… (III)
Therefore, equation (II) becomes
$D_u^2\emptyset(u) + k^2\emptyset(u) = 0$ …… (IV)
Taking Elzaki Transform of equation (IV), we get
$E[D_u^2\emptyset(u)] + b^2E[\emptyset(u)] = 0$
This equation gives
$\frac{1}{q^2}\overline{\emptyset}(q) - \emptyset(0) - q_u\emptyset(0) + b^2\overline{\emptyset}(q) = 0$... (V)
Applying boundary condition: $\emptyset(0) = 0$, equation (V) becomes,
$\frac{1}{q^2}\overline{\emptyset}(p) - qD_u\psi(0) + b^2\overline{\emptyset}(q) = 0$
Or $\frac{1}{q^2}\overline{\emptyset}(q) + b^2\overline{\emptyset}(q) = qD_u\emptyset(0)$ … … … (VI)
In this equation, $D_u\emptyset(0)$ is some constant.
Let us substitute $D_u\emptyset(0) = H,$
Equation (VI) becomes
$\frac{1}{q^2}\overline{\emptyset}(q) + b^2\overline{\emptyset}(q) = q\,H$
Or $\overline{\emptyset}(q) = \frac{Hq}{(\frac{1}{q^2} + b^2)}$
$\overline{\emptyset}(q) = \frac{Hq^3}{(1 + q^2b^2)}$……….. (VII)
Taking inverse Elzaki transform of equation (VII), we get

$\emptyset(u) = \frac{H}{b}\sin(b\,u)$ ……….. (VIII)

Applying boundary condition: $\emptyset(c) = 0$, equation (8) gives $\frac{H}{b}$ Sin (b c) = 0
Since A cannot be equal to zero because for H = 0, $\emptyset(z) = 0$. This means that particle is not present inside the infinite square well potential which is not possible.
Therefore, sin (bc) = 0
Or bc = n π, where n is a positive integer.
Or $b = \frac{n\pi}{c}$ ------------- (IX)
Substitute the value of b from equation (IX) in equation (VIII), we get

$\emptyset(u) = \frac{H}{\frac{n\pi}{c}}\sin(\frac{n\pi}{c}u)$………. (X)
Where A is normalization constant.

Eigen energy values:
Substitute the value of b from equation (IX) in equation (III), we get
$\left(\frac{n\pi}{c}\right)^2 = \frac{2mE}{\hbar^2}$
Solving, we get $E = \frac{n^2\pi^2\hbar^2}{2mc^2}$
Replacing E by E_n for different values of quantum number n, we have
$E_n = \frac{n^2\pi^2\hbar^2}{2mc^2}$………. (XI)
This equation presents energy quantization is a consequence of restricting a micro particle to a certain region. The minimum possible energy crazed by the particle inside the infinite square well potential is known as Ground State or Zero Point Energy. The energy of the particle inside the infinite square well potential will be minimum at n = 1. This is because if n = 0, then $\emptyset_0(u) = 0$ everywhere inside the infinite square well potential and then probability density inside the infinite square well potential, $|\emptyset_0(u)|^2 = 0$, which means that particle is not present inside the infinite square well potential. Hence E = 0 is not allowed. i.e. the particle does not have zero total energy inside the infinite square well potential, Therefore, it does not be at rest inside the infinite square well potential quantum mechanically.
For n=1, $E_1 = \frac{\pi^2\hbar^2}{2mc^2}$
This presents the Ground State or Zero Point Energy of particle in an infinite square well potential.
Determination of constant A:

Since the probability density between u = 0 and u = c, is one, because the particle is somewhere within this boundary, that is inside the infinite square well potential. Hence applying normalization condition, we can write

$\int_{u=0}^{u=a} \emptyset(u)\ \emptyset(u)^{*} du = 1$ ……..(XII)

Where $\emptyset(u)^{*}$ is the complex conjugate of $\emptyset(u)$.

Using equation (X) in equation (XII), we can write

$\left(\frac{H}{\frac{n\pi}{c}}\right)^2 \int_{u=0}^{u=c} \sin^2\left(\frac{n\pi}{c}u\right) du = 1$

Or $\left(\frac{H}{\frac{n\pi}{c}}\right)^2 \int_{u=0}^{u=c} \frac{1}{2}\left[1 - \cos\left(\frac{2n\pi}{c}u\right)\right] du = 1$

Solving the integration and arranging, we get

$H = \frac{n\pi}{c}\left(\frac{2}{c}\right)^{1/2}$ ………… (XIII)

Normalized wave function (Eigen Functions):

Substitute the value of H from equation (XIII) in equation (X), we get

$\emptyset(u) = \left(\frac{2}{c}\right)^{1/2} \sin\left(\frac{n\pi}{c}u\right)$

Replacing $\emptyset(z)$ by $\emptyset_n(u)$ for different values of quantum number n, we can write

$\emptyset_n(u) = \left(\frac{2}{c}\right)^{1/2} \sin\left(\frac{n\pi}{c}u\right)$……..(XIV)

This equation provides the Eigen functions or normalized wave function of the particle in an infinite square well potential. Since $\emptyset_n(z)$ is normalized, therefore the value of $|\emptyset_n(u)|^2$ is usually positive and at a given u is equal to the probability density of finding the particle there. At u = 0 and u = c, the boundaries of the infinite square well potential,$|\emptyset_n(u)|^2 = 0$. Inside the infinite square well potential, The probability density of the particle being present, can be different for different quantum numbers. For example, $|\emptyset_1(u)|^2 = \frac{c}{2}$, maximum in the middle of the box and $|\emptyset_2(u)|^2 = 0$, minimum in the middle of the box.i.e. a particle in the lowest energy state at n = 1 is most likely to be in the middle of the box while a particle have n = 2 in the coming higher energy state at is never there quantum mechanically.

For n = 1, $\emptyset_1(u) = \left(\frac{2}{c}\right)^{1/2} \sin\left(\frac{\pi}{c}u\right)$…….. (XV)

This equation (XV) gives the ground state wave function of the particle in an infinite square well potential.

***Conclusions**:* This paper an effort is complete to get the Eigen energy values and Eigen functions of the particle in an infinite square well potential via solving Schrodinger time - independent equation for the particle in an infinite square well potential via Elzaki transform. We found a remarkable characteristic of the Eigen functions of one dimensional Schrodinger's time - independent wave equation for a particle in an infinite square well potential that each Eigen function is connected with a particular Eigen energy value of the particle restricted inside an infinite square well potential.

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Arun Prakash Singh, et. al. "An Emorical Analysis of a Partical In an Infinite Square Well Potential by Elzaki Transform with Eigen Energy Values and Eigen Functions." *IOSR Journal of Applied Physics (IOSR-JAP)*, 14(02), 2022, pp. 18-22.