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Analytical Solution of Projectile Motion in Mid-Air with Quadratic Resistance Law Using Taylor Series Method

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Abstract: *Real-world systems are characterized by a high degree of uncertainty due to external influences such as friction and a lack of accurate information about the system's conceptual circumstances. It has studied the effects of air resistance on a projectile moving (quadratic resistance law) in two dimensions (2D). Equations of motion in this situation may be solved across a wide range of projectile angles, while some solutions have been found for tiny projectile angles using perturbation theory. The technique can easily determine the Taylor series expansion for a resistive material by applying a quadratic force concerning the velocity using the differential equation of the trajectory. The 45-degree angle of projection has more range as compared to other angles of projection using the Taylor series approach. Diverse angles of projection are subsumed to achieve a broad analytic solution. The Taylor series approach is distinct from a perturbation procedure that is applied to problems, free from minute factors. Finally, an analytical solution to this issue is obtained and the Taylor series approach analyses the validity of that solution. The technique proved successful in retrieving the solution in a basic method, videlicet, utilizing the well-established precise formulations of the trajectory equation in a vacuum and a resistant medium with linear velocity.*
Key Word*:Projectile motion, Quadratic resistance law, Power series, Taylor series method, Equation of the projectile, Drag force.*

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I. Introduction

Projectile velocity is the speed of an object hurled (projected) into the air. The object only meets the force of gravity after being propelled into the air by another force. Projectile and trajectory are terms used to describe the movement of an item. It meets an opposing force, known as air resistance, in the air which diminishes its speed [1]. A projectile's mobility can be used in a variety of ways during combat, including mortars, tanks, fighter planes, and submarines. Furthermore, archery, badminton, baseball, cricket, shot put, hammer throw, and discus throw are also examples of projectiles in sports. The velocity of a projectile is subjected to gravity in conjunction with the resistive force (drag) that is exerted by the different space media [2-3].

Point masses are used to represent the motion of a bullet or any other substance that is found in gases as well as liquids. It is also necessary to completely overlook considerations about shape, orientation, and rotation due to the absence of spatial extensions. Therefore, the point mass utilized in this case is expected to feel drag as it moves through the surrounding medium, which is certainly caused by the object's extension. Additionally, the authors assume that lift forces have a lesser magnitude than drag forces. As a result, the words ‘projectile’, ‘body’, and ‘object’ are all used interchangeably to describe the model of a point mass subject to gravity and drag. Moreover, the term ‘fluid’ refers to both liquids and gases [4][5]. A classic mechanic system follows well-recognized equations of action, such as the Hamilton equations and Newton’s second law, which are used in many fields of motion. It is also a predictable uncertainty that is introduced into a mechanical system due to the absence of control over the starting circumstances of the system. Hydraulic actuators are frequently chaotic, which means that they are extremely sensitive to changes in their environment and their beginning circumstances, resulting in a lack of predictability of the system [6]. It is possible to generate a (non-trivial) statistically distributed set of outcomes by advancing the system forward in time which has created a defined distribution curve of starting conditions. Essentially, this recommends the initial circumstances that led to the outcomes are engaged in from the statistical characteristics of the results of a traditional procedure. This is an inverse issue, which can be theoretically handled utilizing Bayes' theorem and communication theoretical approaches from the perspective of probability theory [7]. For the linear law of moderate resistance at slower revolutions, for short trip periods, or split angle trajectory regimes: low, and high, while traveling at high speeds, a restricted set of physical constants has been incorporated. Both traditional procedures and modern methodology are used in combination with one another in a collaborative effort to provide analytical solutions. Some solutions, involving approximation, make use of specific functions, such as the Lambert function, to achieve their results. It follows that deriving the description of projectile motion from basic approximation analytical formulae under quadratic gas resistance has significant methodological and instructional value [8]

I.I Equation Of Projectile Motion

The projectile is subject to the combined effects of gravity F and air resistance R as depicted in Figure 1. Once a projectile travels at a speed of V, its velocity is squared, and the air resistance force is directed in the opposite direction as delineated in Figure 1 [9].

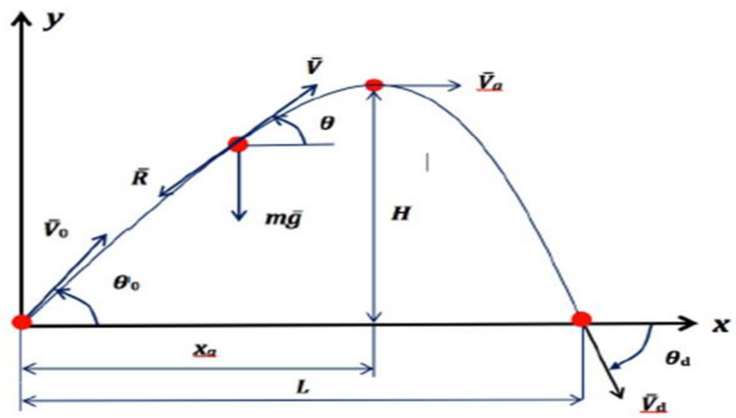

Figure 1. Parameters of projectile motion.

For simplifying the future calculations, the drag force would be expressed as,

$$R = mgKv^2 \qquad (1)$$

Where,
m → projectile's mass
g → the acceleration caused by gravity
K → the proportionality factor.
In most studies, authors use projections on the Euclidean geometry axis as the frame of reference to analyse the velocity of a projectile.
Ballistics makes extensive use of projectile motion equations projected along the natural axes [10]. It has the following form,

$$\frac{dV}{dt} = -g\sin\theta - gkV^2, \frac{d\theta}{dt} = -\frac{g\cos\theta}{V}, \frac{dx}{dt} = V\cos\theta, \frac{dy}{dt} = V\sin\theta \qquad (2)$$

Where,
V → aiming reticule speed
θ → angle between the parallel and horizontal axes of its trajectory
x and y are the aiming reticule cartesian coordinates.

$$k = \frac{\rho_\alpha c_d S}{2mg} = \frac{1}{V_{term}^2} = \text{constant}, \qquad (3)$$

Where,
c_d → drag factor for a sphere
ρ_α → density of the air
S → cross-sectional size of the object
V_{term} → terminal velocity.
It should be noted that the system consists of four equations, the first two equations represent the vector law of motion which is projected onto the trajectory using a tangent and main normal to the trajectory, respectively.

These are kinematic equations that relate the x- and y-axes of the fluid flow vector missiles to variances in velocity [11].
There are three quadratures in the pre-determined solution of the system, which are characterized by an analytic dependency of the speed on the angle of the trajectory.

$$V(\theta) = \frac{V_0 \cos\theta_0}{\cos\theta\sqrt{1+kV_0^2\cos^2\theta_0(f(\theta_0)-f(\theta)}}, f(\theta) = \frac{\sin\theta}{\cos^2\theta} + \text{In}\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right) \quad (4)$$

$$x = x_0 - \frac{1}{g}\int_{\theta_0}^{\theta} V^2 d\theta, y = y_0 - \frac{1}{g}\int_{\theta_0}^{\theta} V^2 \tan\theta d\theta, t = t_0 - \frac{1}{g}\int_{\theta_0}^{\theta} \frac{V}{\cos\theta} d\theta \quad (5)$$

Where,
V_0 → initial value of velocity
θ_0 → slope of each trajectory
t_0 → time's minimal state
x_0 and y_0 → locations of the projectile's initial positions.

Moreover, $t_0 = x_0 = y_0 = 0$. The treatise on calculus demonstrates how the formula was derived. It is impossible to express integrals in terms of elementary functions to calculate the variables t, x, and y, either integrate the system numerically or use the definite integrals [11]. Calculations for the integral were done in a basic function over angle θ. For this research, the integral of an elementary function must be calculated with the required precision across the full period in which the variable changes θ: $[-\pi/2 \leq \theta \leq \theta_0]$.

II. Literature Review

The following study expands on the previous analytical solutions of projectile motion in midair with the quadratic resistance law using the Taylor series approach. Several researchers have explained their findings as seen below.

Pantaleone et al., (2022) [12] analyzed that the inertia of an object travelling through a fluid is greater than the inertia of an object with similar dimensions moving in a vacuum. The sphere is near a solid border and has a low density, the additional mass has a significant impact on its acceleration. The acceleration induced by gravity can be tested by bouncing a beach ball vertically on the floor. A larger amount of extra mass is present at a closer distance from the floor; hence fewer accelerations are observed. A sphere's extra mass is calculated using these measurements as a function of its distance from the floor. In an ideal fluid, a curve is not affected by the size of the ball and tends to follow the same path.

Landon et al., (2021) [13] examined the analytical solutions for the resistive force that acts on a body's motion. Furthermore, its squared velocity is investigated, and a useful analytic solution is formulated. The solution is a time-dependent function that is applicable in scenarios, close to or distant, from the initial state's initial value. A reduction principle is employed to enable us to manage scalar objects in the first step of the analytic process. The researchers looked at both limit behaviors, near and far, from the starting point. Another option is to use heuristics and approximate the result using existing knowledge. A reliable integration method is provided based on the analytic solution.

Xu et al., (2021) [14] stated The Dynamic Cavity Expansion (DCE) model, an analytical model, is often used to predict concrete's resistance to missile impact. It has been suggested that reactive powder concrete (RPC) has a modified Griffith strength criterion, an easy-to-solve form that accurately expresses RPC's tension, compress, and triaxle forces. Material models for RPC are constructed using the specified strength parameter in conjunction with the Murnaghan equation of state. Additionally, numerical, and analytical methods are used to determine the resistance to cavity expansion. The results are nearly identical using the same material response. A mathematical solution is developed by deploying the strain softening in the DCE model. The 122.6 MPa RPC targets were tested using 14.5 mm diameter ogival nose projectiles at impact velocities of 286 to 942 m/s. The penetration depth was performed using the model. The current model's predictions correspond well with the outcomes of tests and simulations. RPCs with compression strength ranging from 67.5 to 140 MPa were also calculated using the approach and by comparing the results of prior empirical equations, the current way was found to be very effective.

Turkyilmazoglu et al., (2020) [15] evaluated that at the projectile's maximum height, the velocity is all considered when calculating Magnus effects on projectiles thrown from a fixed point. The analytical perturbation solutions are possible when both quadratic forces of friction and Magnus are present simultaneously in the governing motion equation. A quadratic force with a continuous magnetic drag force is applied and can represent the entire solution. Projectile characteristics can be estimated and optimized using the formulae described herein. Lastly, examples are given and examined to demonstrate the exactness of the mathematical model compared to the supplied formulas' accuracy.

Ahmed et al., (2020) [16] averred that meteor craters, pile driving, shards, chunks, and debris plummeting from the tall tower are archetypes of potential hazards. The circumstances surrounding the impact are currently receiving a great deal of attention. Studies have examined the impact and penetration abilities of different types of soil media. Once subjected to traveling at fast speeds and plunging to deep levels, the target soil media, particularly in heterogeneous particle soil mediums, neither behave distinctly as solids nor as liquids. Methods to forecast penetration responses of soil media-impacting items are reviewed in detail, including analytic and empirical approaches as well as experimental and computational approaches. It has presented fundamental research into soil media penetration issues. The report identifies potential future avenues for soil media absorption research.

Yuetal., (2019) [17] assessed that the tracking guidance laws are constructed utilizing feedback linearization and optimum control theory for missile targets. Feedback linearization is subsumed for the precise linearization strategy, along with its formulation and necessary criteria since missile motion equations are typical non-linear models. Additionally, it is employed to correct the non-linear standard for the trajectory of the target missile by creating the non-linear equations regulating its longitudinal motion. A linearized model is produced, and a guiding rule for the desired trajectory is established by using optimal control theory. A computer model is created to assess the tracking performance of the guidance law, and a path is traced using the model under various wind disturbances and manoeuvring trajectories. Tracking precision, anti-jamming strength, and superior trajectory tracking ability are all demonstrated by the feedback linearization-based trajectory tracking guiding legislation.

Bakhtiyarov et al., (2018) [18] presented the traditional way of determining the path taken by a weighted object in a resistive material in an instructive manner. Derivative chains may be used to simplify a trajectory differential equation into its cartesian form, which can then be used to simplify the problem further. Assuming a quadratic force is given to a moving item, the trajectory's Taylor series expansion provides an easy way to figure out where the trajectory is going in space. Additionally, a numerical comparison is made between the truncated polynomial approximants and the solution is recently provided through homotropy analysis.

Montecinos et al., (2018) [19] examined that the kinematics, and dynamics of predetermined physical processes, have served as a basis for the knowledge of the universe. The authors have concentrated on this scenario and discussed the application of inference approaches to the problem of establishing the statistical features of classical systems when the initial circumstances are unclear. On the matter concerned with the falling objects, the authors incorporate common knowledge of the total horizontal range across numerous realizations and describe the notion of Maximum Entropy (MaxEnt) inference and how it must be incorporated. Besides reducing initial uncertainty, MaxEnt's conditional probability distribution for starting condition probabilities and projectile path distribution provide further insights. The vast range and simplicity of the applications of this approach, showcase it to be a powerful tool for revisiting a wide range of physics issues from classrooms to cutting-edge research.

Lee et al., (2016) [20] proposed a pseudo-analytic technique to estimate the ideal launching circumstances subjected to a sounding rocket that operates with a constant mass stream of fuel in a conventional configuration environment to maximize the height of the rocket. It is difficult to get analytical solutions for the effects of thrust, gravitational force, and aerodynamic drag of the non-linear nature of the fundamental formula. The piecewise expression of semi-solutions is found when a constant controller output is added to the governing equation. It is implemented to make the speeds inherent in the energy equation analytic by making them integral to the governing equation. The entire range of possible rocket flight is subdivided into minuscule intervals in which either the drag factor or the angular velocity can be pronounced constant in each interval to analyze rocket flight in a standard atmosphere. It is possible to make accurate forecasts of the launching velocity and altitude using the pseudo-analytic approach, and these results accord well with the predicted results of the same parameters. A characteristic equation is availed to reliably forecast the ideal flow rate for maximizing height during the burn-out stage or at apogee. This equation functions with high accuracy.

De Celis et al., (2018) [21] intended that the accuracy of ballistic projectiles is based on position and attitude determination. Precise rotation determination in airplanes is a time-consuming and costly operation since they are often calculated using strap-down sensors like fiber optic gyros. Since authors must neglect huge accelerations solely during the initial phases of a ballistic projectile's flight, these gyroscope determining devices are particularly expensive in ballistic projectiles. Hybrid attitude determination techniques and gravity vector estimating methods are being used in a new way to enhance ballistic missile Navigation, Guidance, and Control (NGC). Furthermore, it assesses methods with a gravity vector estimating method. Measurements from accelerometers and Semi-Active photodetectors are melded. The calculation of gravity vectors is dependent on the flight mechanics as well as the aerodynamics of a kinetic projectile, which includes a highly non-linear behavior that is difficult to model. However, it can be applied to any aircraft and subsequently incorporated into an orientation determination method if the data is extrapolated. Modified proportionate navigation methods, along with previously existing control methods, are used to navigate. The offered technique is evaluated on a genuine non-linear model airplane simulation to demonstrate the veracity of provided algorithms.

Turkyilmazoglu et al., (2016) [22] explored that there is no reliable solution that represents the entire physical event that occurs when a strong resistance force is applied. The fundamental goal is to find an elegant logical approximation for the most essential engineering elements of the projectile's dynamical behavior. This behavior also involves its trajectory. Some analytic explicit formulations are developed for this purpose. When the projectile is at its peak altitude, the projectile's zenith, arrival time, and flying range can be estimated. In particular, the suggested formulae are neither bound to the initial velocity and discharge angle of the object nor are they restricted towards the drag coefficients of the medium. This is an extremely important characteristic. The use of existing approximations in conjunction with flight data makes it feasible to get the information about the flight and to achieve the image of the trajectory with high accuracy, without the need to represent the whole set of regulating motion equations computationally.

A wide range of authors, who used different techniques and presented their discoveries, can be seen in Table 1.

Table 1. Comparison of literature review

Author	Technique	Outcome
Pantaleone et al., (2022) [12]	Integration	A sphere's extra mass is calculated using the observed accelerations as a function of its distance from the floor.
Landon et al., (2021) [13]	Integration	The solution is a time-dependent function that is applicable in scenarios, close to or distant from the initial state's initial value.
Xu et al., (2021) [14]	Dynamic cavity expansion (DCE)	Reactive powder concrete (RPCs) with compression strength ranging from 67.5 to 140 MPa is calculated using the approach and through a comparison of the results of the suggested model.
Turkyilmazoglu et al., (2020) [15]	Analytical perturbation	Projectile characteristics can be estimated and optimized using the formulae described.
Ahmed et al., (2020) [16]	Analytical method	Methods to forecast penetration responses of soil media-impacting items are reviewed in detail, including analytic and empirical approaches as well as experimental and computational approaches
Yu et al., (2019) [17]	Feedback linearization model	A model is created to assess the tracking performance of the guidance law, and a path is traced using the model under various wind disturbances and maneuvering trajectories.
Bakhtiyarov et al., (2018) [18]	Taylor series	A numerical comparison is made between the truncated polynomial approximants and the solution is recently provided through homotropy analysis.
Montecinos et al., (2018) [19]	Maximum Probabilistic (MaxEnt) inference	The vast range and simplicity of the applications of the approach, showcase its ability to be a powerful tool for revisiting a wide range of physics issues from classrooms to cutting-edge research.
Lee et al., (2016) [20]	Pseudo-analytic	A characteristic equation is availed to reliably forecast the ideal flow rate for maximizing height during the burn-out stage or at apogee. This equation functions with high accuracy
De Celis et al., (2018) [21]	Hybrid method	A novel strategy to improve ballistic missile navigation, guidance, or control merges hybrids. Furthermore, it assesses methods with a gravity vector estimating method.
Turkyilmazoglu et al., (2016) [22]	Analytical method	By the analytical method, the projectile's maximum height, arrival time, and flying range are developed when the projectile is at its peak altitude.

III. Background Study

A modification of the direct Taylor expansion technique,is implemented by the authors in which too many linear as well as non-linear equations are relevant in science and mathematics. The findings of the approach are compared with those obtained from other algorithms. Consequently, it is demonstrated that the Taylor series approach accords well with other current algorithms in terms of accuracy and simplicity of application whilst requiring a much lower computing time in comparison [23].

IV. Problem Formulation

It is possible to have a projectile velocity with quadratic air resistance described as follows in a non-dimensional manner:

$$\frac{du(t)}{dt} + f(t)u(t) = 0, \tag{6}$$

$$\frac{du(t)}{dt} + f(t)u(t) + 1 = 0, \tag{7}$$

Where,

$$u(t) = \frac{\hat{u}(\hat{t})}{v_t} = \hat{u}(\hat{t})\sqrt{\frac{\alpha}{g}} \tag{8}$$

$$v(t) = \frac{\hat{v}(\hat{t})}{v_t} = \hat{v}(\hat{t})\sqrt{\frac{\alpha}{g}} \tag{9}$$

$$t = \frac{\hat{t}}{v_t/g} = \hat{t}\sqrt{\alpha g}, \tag{10}$$

$$f(t) = \sqrt{u^2(t) + v^2(t)}, \tag{11}$$

$$v_t = \sqrt{\frac{g}{\alpha}}, \tag{12}$$

Where,
$v_t \rightarrow$ terminal velocity
$\hat{u}(\hat{t}) \rightarrow$ horizontal initial velocity
$\hat{v}(\hat{t}) \rightarrow$ perpendicular initial velocity
$g \rightarrow$ acceleration of gravity
$u(t) \rightarrow$ velocity of an object
$v(t) \rightarrow$ coefficient of kinematics viscosity of the air
$f(t) \rightarrow$ Force
$\alpha \rightarrow$ a. constant.
The initial circumstances are as follows:

$$u(0) = \frac{\widehat{U_0}}{v_t} = U_0, v(0) = \frac{\widehat{V_0}}{v_t} = v_0 \tag{13}$$

V. Research Methodology

Some linear and non-linear multivariable calculus with applications in physics are discussed here using a Taylor series variation, which is applied to the equations. For an arbitrary angle of projection, the authors use the Taylor series approach in the study and attempt to resolve the problem of a falling object with quadratic air resistance. It has been successful in obtaining an analytic solution to this issue, which is stated as a power series. The solution is obtained by constructing the zeroth-order displacement equation using differential calculus, the quadratic equations of a direction of motion, and the solution of these equations.

V.I Taylor Series Method For Solving The Projectile Motion

$$\frac{dy}{dx} = f(x, y) \tag{14}$$

The differential equation (14), has $\frac{d^2y}{dx^2} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx}$ that is $y'' = f_x + f_y f'$.

Differentiating this successively, we can derive the equation of y''.

Substituting $x as x_0$ and $y as$ 0, the values of $(y')_0, (y'')_0, (y''')_0$can be achieved. Hence, the Taylor series can be represented as,

$$y = y_0 + (x - x_0)(y')_0 + \frac{(x-x_0)^2}{2!}(y'')_0 + \frac{(x-x_0)^3}{3!}(y''')_0 + \cdots \tag{15}$$

This equation presents the estimated values for each variable x which merges on discovering the number y_1 for x from equation (15), and the values of y, $y'$and so on may be assessed at x_1. Then, y must be extended to include x_1 and x_2. It is possible to broaden the scope of the solution to include more than just the convergence of series.

V.II Power Series Expansion Of The Trajectory

The numerical form of the trajectory's Taylor series growth is very important for obtaining it simply, namely,

$$Y(x) = \sum_{k=0}^{\infty} \frac{a_k}{k!} X^k \tag{16}$$

Where a_k = Y(k) (0) signifies the evaluation of $Y(x)$ at the zeroth-order spatial derivative, denoted by $a = y$. The basic circumstances are as follows,

$$Y(0) = 0 \tag{17}$$

$$Y^{'}(0) = tan\theta_0 \tag{18}$$

$$Y^{''}(0) = -\frac{1}{V_{0,x}^2} \tag{19}$$

Where,
$V_0 \rightarrow$ initial velocity
$\theta_0 \rightarrow$ angle between velocity and axis
The coefficient $a_3$is depicted by the expression as

$$a_3 = Y^{'''}(0) = 2Y^{''}(0)\sqrt{1 + [Y^{'}(0)]^2} = -\frac{2}{V_{0,x}^2 cos\,\theta_0} \tag{20}$$

However, to get the coefficient a_4, both the left and right sides of the equation must be viewed about x, which means that a cos θ0 must be substituted for a value of unity in cases when minor slopes exist.

$$Y^{(4)} = 2\frac{Y^{'''}(1+Y^{'2})+Y^{'}Y^{''}}{\sqrt{1+Y^{'2}}} \tag{21}$$

After reorganizing and simplifying, the coefficient a_4 can be derived as

$$a_4 = -\frac{4}{V_{0,x}^4}\left(V_0^2 - \frac{sin\,\theta_0}{2}\right) \tag{22}$$

Whereas the short-time calculation would require the equation,

$$Y_{st}^4(0) = -\frac{4}{V_{0,x}^2} \tag{23}$$

Every term in the equation's power series expansion might theoretically be evaluated by using this method (16). Analytical expressions for the factors a_k get more complicated when their expansion is truncated.

V.III Results

Figure 2 below has considered the solution of the Nth-order approximate explanations for equations (6) and (7) from t = 0 to t_1. This figure demonstrates that the current solution is associated with the Taylor series solution for different angles of projection. The angles are defined by utilizing a minimum and maximum value with their coordinates and are between the x-axis and y-axis by using the Taylor series method. There are many angles described in Figure 2 such as 30-degree, 45-degree, 60-degree, and 75 degrees. An Nth-order polynomial approximation is provided for each curve, with N, being the lowest truncation order of the power series that allows for an acceptable agreement (at least at a visual level) with V_0 = 14m/s and many other values of the beginning angle 0. The line signifies the present solution and the balls on the line representthe Taylor series.

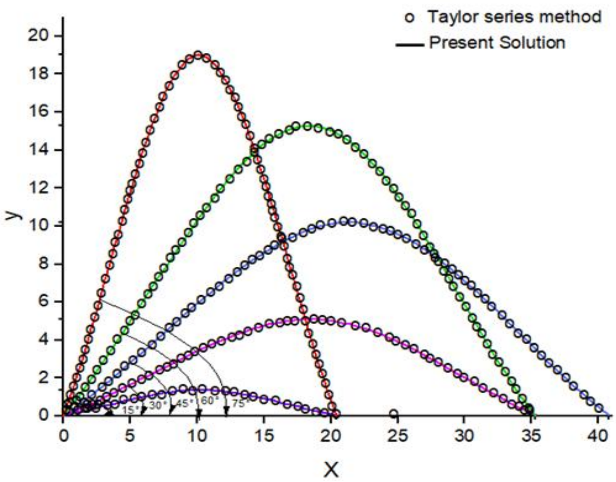

Figure 2. Projectile paths for various angles of projection.

VI. Conclusion And Future Scope

The Taylor series approach is used to solve the issue of 2D projectile motion with the quadratic resistance law for a high angle of projection. The solution must be found by creating the zeroth-order deformation equations for both the governing differential equations and the algebraic equation of the velocity vector. A simple power series is used to represent the solution. A simple explanation of the issue of determining its trajectory has been put forth for a body thrown in any direction and subject to gravity while traveling through a resistive medium. The primary goal of this explanation is to keep the mathematical complexity to a minimum. It has been concluded that the 45-degree angle has more range as compared to other angles. The technique proved successful in retrieving the solution in a basic method, videlicet utilizing the well-established precise formulations of the trajectory equation in a vacuum and a resistant medium with linear velocity. The projectile issue has the potential to be investigated in a myriad of ways in the future. The study has concentrated on straightforward impact functions, for example, investigating the links between the optimum angle and the altering conceptual circumstances, which, under more sophisticated impact function scenarios, would be fascinating.

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