

## Tailoring Point Spread Function of Aberrated Two Zone Aperture by Symmetric Apodization

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**Abstract.** The point spread function of the optical system in the presence of defocus and primary spherical aberration with a combination of a Hanning amplitude filter and a Connes amplitude filter was studied. A noted increase in the profile of the point spread function has been achieved. The employment of the Hanning and Connes amplitude pupil functions under a higher degree of primary spherical aberration, and the defocusing effect helps optical systems increase the resolution. The lateral resolution of the central peak is improved by the highest degree of the amplitude apodization parameter  $\beta$ . The presence of the first minimum with zero intensity necessary for the Rayleigh criterion can be used to study two-point resolution.

**Keywords:** Point spread function; Amplitude apodization; Primary spherical aberration; defocus; two-zone aperture; super-resolution.

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Date of Submission: 14-03-2023

Date of Acceptance: 30-03-2023

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### I. INTRODUCTION

Apodization is the process for removal of secondary side-lobes or side-bands in the diffraction field also known as the point spread function (PSF) which is required for any optical system to act as super resolver. And this can be acquired by properly choosing the transmittance of the pupil function of the optical system [7-20], the intensity around the focused fields can be totally suppressed or at least considerably reduced without increasing the dimensions of the pupil.

In our previous work [21], we studied apodization of Hanning complex pupil functions where primary aberrations were only confined to the central region and omitted for the semi annular regions of the pupil. Aberrations considered in the earlier study were confined to the middle region, and therefore the performance is attributed to the pupil mask itself. Therefore, it was not a perfect assumption. Traditionally, apodization across the exit pupil of the system has focused on obtaining the quality output or to improve imaging characteristics. PSFs from the lenses with aberrations are severely distorted and thereby are major concerns in many optical applications. Therefore, generating a smooth intensity profile PSF has been one of the keystones of modern imaging systems underneath aberrations for improving the performance of optical systems employed in numerical applications [1-18]. In this context, the optical system with the proposed apodization technique aims to minimize the aberration effects and to enhance the focusing/resolution power. With the defocusing aberration, the resulting PSF corresponding to the point source is being detected not at the location of the diffraction focus, but at the central peak longitudinally shifted from that point, a point with maximum diffraction intensity implanted into longitudinal expansion of the central peak, and with the energy shifted from the central peak into the side lobe region resulting in a decrease in the central intensity [9, 13, 21]. But, a spherical aberration of the lens distorts the resulting intensity PSF of the point image due to a difference in optical ray paths from the source to the focus, i.e. peripheral rays of the lens are not converging into exact focus related to the paraxial rays. As is evident, there is a decrease in the central peak intensity (known as Strehl ratio), increase in the intensity level of first-order side lobes and the first minima position between the central peak and the first-order side lobe changes a little due to the spherical aberration. Therefore, the defocusing and spherical aberrations are playing a vital role in characterizing as they have severely distorted the PSF optical system. Note that an aberration-free lens pupil forms the Airy PSF in which most of the incident light energy concentrated within the central region or Airy disk and the remaining energy distributed among the side lobes. In such cases, the proposed apodizer is highly efficient in compensating the aberration effects, resulting in high-resolution PSF.

In the current study, the imaging characteristics of the diffracted field of rotationally symmetric optical systems with Hanning and Connes amplitude filters have been investigated in terms of the reduction of secondary side lobes by modifying the two zone aperture with different degrees of amplitude apodization  $\beta$  using defocus and primary spherical aberrations. To study the imaging properties and to design the optical systems, the knowledge of the PSF is an important parameter. The present study provides a significant contribution to the two point resolution studies. We know that by employing suitable apodization function, the

point spread function in the maximum out-of-focus image plane can be modified according to the axial shape requirements. A suitable aperture of shading is very helpful to correct the Seidel aberration effect in the image plane of the optical system. Based on the investigations done in the two zone apodization process, it can be inferred that the Hanning amplitude filter in the outer zone and Connes amplitude filter in the inner zone could be the solution for modifying the point spread function of the optical system under the strong combined influence of defect-of-focus and primary spherical aberration. In the present study, we studied the two zone aperture with the second order Hanning and Connes amplitude mask, to modify the distribution of light radiation in the focal plane of an aberration made optical systems.

## II. THEORY AND FORMULATION

The current study is to study the effect of the Hanning and Connes amplitude filter on the optical system which is under the combined influence of high Seidel aberration and maximum defect of focus. The Connes filter is placed in the inner zone and Hanning filter is placed in the outer zone of the two zone pupil of the apodized optical system. The point spreadfunction is subjected to a higher degree of defocusing and primary, secondary and tertiary wave aberration effect. The general expression for diffraction field of two amplitude filters is given by

$$S(Z) = 2 \int_0^a f_1(x) J_0(Zx) x dx + 2 \int_a^1 f_2(x) J_0(Zx) x dx \quad (1)$$

Where  $f_1(x)$  is Connes amplitude pupil function,  $f_2(x)$  is Hanning amplitude pupil function of the optical system;  $Z$  is the dimension less variable which forms the distance of the point of investigation from the centre of the diffraction field; and  $J_0(Zx)$  is the zero order Bessel function of the first kind; ' $x$ ' is the reduced radial coordinate on the exit-pupil of aberrations influenced optical system.

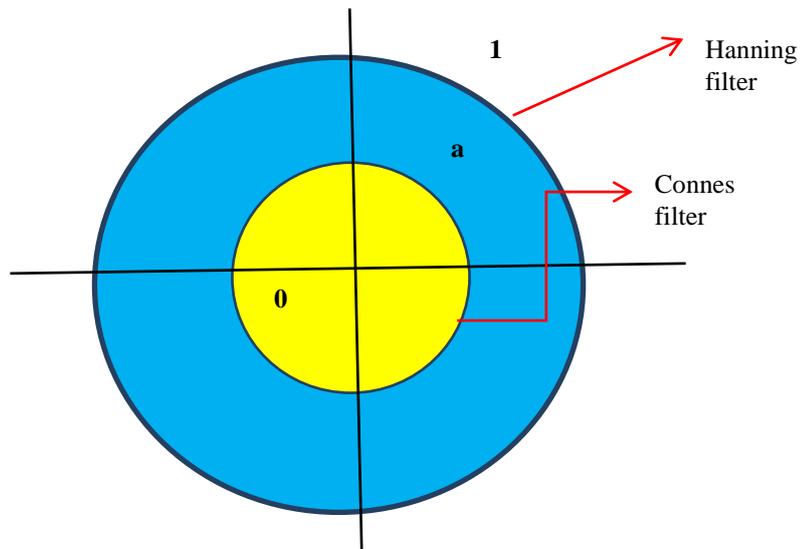


Fig. 1. Two Zone Aperture

Figure 1 shows the two zone aperture with radius ' $r$ ' and ' $a$ ' is the partition parameter for the central zone. Connes filter is introduced for the central zone, i.e., from 0 to  $a$  and Hanning amplitude filter is applied for the outer zone, i.e., from  $a$  to 1.

The generalized expression for the amplitude impulse response of the pupil function in the presence of higher degree of primary spherical aberration and defocusing equation (1) can be written as

$$S(\phi_d, \phi_s, Z) = 2 \int_0^a f_1(x) \exp \left[ -i \left( \phi_d \frac{x^2}{2} + \frac{1}{4} \phi_s x^4 \right) \right] J_0(Zx) x dx + 2 \int_a^1 f_2(x) \exp \left[ -i \left( \phi_d \frac{x^2}{2} + \frac{1}{4} \phi_s x^4 \right) \right] J_0(Zx) x dx \quad (2)$$

Here  $\Phi_d$ ,  $\Phi_s$  are the defect-of-focus and the primary aberration parameters respectively. In current study, the pupil functions we have considered are Connes amplitude filter and Hanning filter of second order respectively which can be represented by

$$f_1(x) = (1 - \beta^2 x^2)^2 \quad (3)$$

$$f_2(x) = \cos(\pi\beta x) \quad (4)$$

Where ' $\beta$ ' is the amplitude apodization parameter controlling the non-uniform transmission of the pupil function.

The intensity PSF  $B(Z)$  which is the measurable quantity can be obtained by taking the squared modulus of  $S(Z)$ . Thus,

$$B(Z) = |S(Z)|^2 \quad (5)$$

### III. RESULTS AND DISCUSSION

The investigations on the effect two zone apertures shading on the images of point objects formed by coherent optical systems apodized by the Hanning in the inner zone and Connes amplitude filter in the outer zone in the presence of defect-of-focus and higher order spherical aberrations have been evaluated using the expression (5) by employing Matlab simulation. The intensity distribution  $B(Z)$  in the images of point objects has been obtained for different values of dimensionless diffraction variable  $Z$  varying from  $-15$  to  $+15$ . The image intensity parameters such as central maxima, first minima, first maxima, second maxima and second minima have been studied for various values of apodization, aberrations and aperture transmission parameters.

The apodization parameter  $\beta$  varies from 0 to 1 in steps of 0.2. With  $\beta = 0$  the optical system is said to be unapodized optical system.  $\beta = 0$ , corresponds to the Airy case (perfect lens) and for the values of  $\beta \neq 0$  represents the apodized system. The influence of defect-of-focus ( $\phi_d$ ) on the optical system aberrated with the primary spherical aberration ( $\phi_s$ ) is investigated analytically for various degrees of the apodization parameter  $\beta$ .

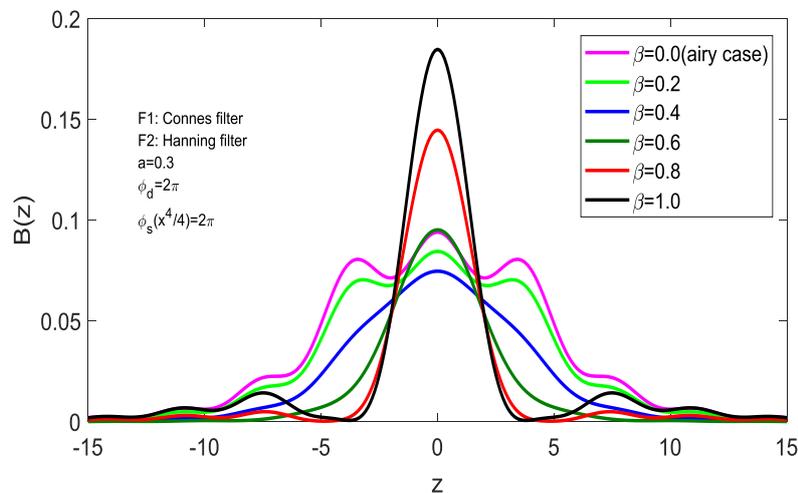


Fig. 2

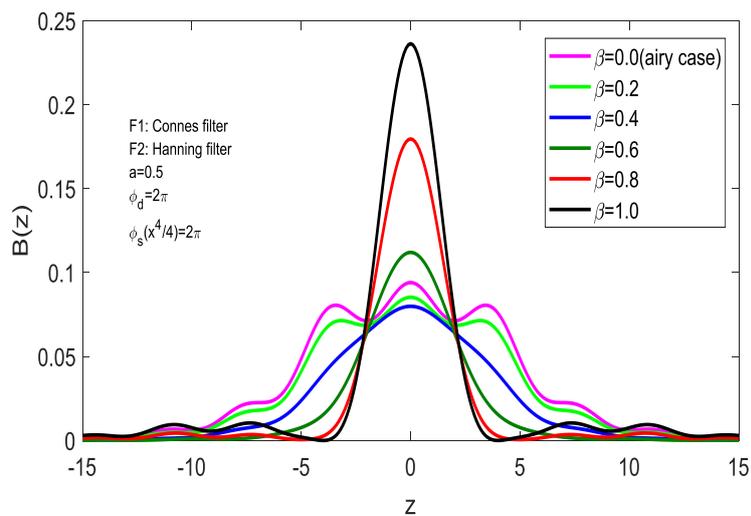


Fig. 3

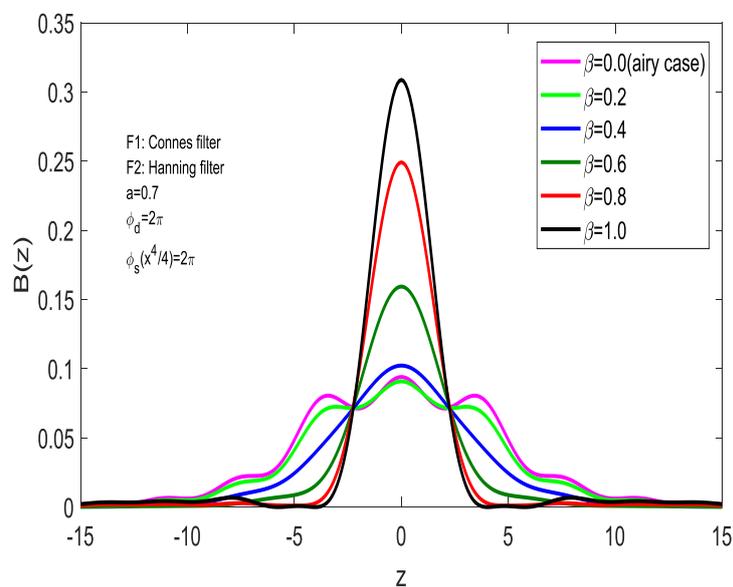


Fig. 4

Figure 2-4. Variation in the Axial Shape of the Point Spread Function of Two Zone Aperture for Different Degrees of Connes Filter (inner zone) and Hanning Filter(outer zone)Apodization Varying Limits of 0.3, 0.5 and 0.7

From figures 2 to 4, It is observed that for  $\beta = 0$  (Airy), in the presence of high degree spherical aberration ( $\phi_s=2\pi$ ) the peak intensity of the central maximum is decreased for the maximum out-of-focus plane ( $\phi_d=2\pi$ ). The Airy PSF lost its axial shape or resolution and non-zero first minima. Similar pattern are noticed in the case of  $\beta = 0.2$ . For  $\beta = 0.4$  and  $0.6$ , the main peak intensity starts to increase, whereas for  $\beta = 0.8$ , the first minima and the side-lobes on the both sides of the main peak reaches to zero intensity and the intensity of the main peak is considerably improved. It helps in detection of the direct image of the faint companion in every direction around the bright companion, known as two-point resolution studies. In the presence of defocusing effect and primary spherical aberration, as the degree of apodization increases from 0.4 to 1 (as shown in the Figures 5-7), there exists a consistent improvement in the lateral resolution of the main peak .It is evident that for highest degree of apodization ( $\beta = 1$ ), the central light flux exhibit maximum intensity compared to that of Airy case ( $\beta = 0$ ) and along with zero intensity in the first minima is measured, resulting in super resolved point spread function. For the highest degree of amplitude apodization ( $\beta = 1$ ). From Figure 7 it is concluded that the lateral resolution of the aberrations made PSF is technically improved by placing Connes filter in the first zone (0-0.7) and Hanning filter (0.7-1) in the second zone amplitude apodization under maximum defocus and primary spherical aberration.

IV. CONCLUSION

From Table 1 and Figure 5, we can conclude that in the presence of high degree of defocus ( $\phi_d = 2\pi$ ) and primary spherical aberration ( $\phi_s = 2\pi$ ) it is found that employing combination of Connes amplitude filter (inner zone from 0 to 0.7) and Hanning amplitude filter (outer zone from 0.7 to 1) there is a significant improvement in the resolution of PSF for higher values of amplitude apodization ( $\beta = 1$ ). The maximum suppression of first dark ring is at  $\beta = 1$ , as the axial shape and the lateral resolution of the PSF is modified into the required module. On the whole it can be concluded that, the two zone aperture with combination of Connes (inner zone) and Hanning (outer zone) amplitude apodization filters will make the system more effective in resolving the intensity of PSF with suppressed side lobes under the combined influence of defocusing effect and the primary spherical aberration.

**Table 1.** Maxima Minima Position and Values of the Point Spread Function of two zone aperture for different degrees of Connes Filter (inner zone) (0-0.7) and Hanning Filter (outer zone) (0.7-1) Apodization for Various Values of Defocus and Primary Spherical Aberration

	$\beta$	c. max		f. min		f. max		s. min	
		pos	val	pos	val	pos	val	pos	val
a=0.3 $\phi_d=2\pi$ $\phi_s(r^4/4)=2\pi$	0.0	0	0.094	2.0175	0.0713	3.4266	0.0806	10.0084	0.0062
	0.2	0	0.0845	2.1325	0.0675	3.2037	0.0704	10.1108	0.0047
	0.4	0	0.0747	13.6255	0.0004	14.4355	0.0004	---	---
	0.6	0	0.0952	8.8769	0.0005	9.9344	0.0006	12.4625	0.0001
	0.8	0	0.1447	4.8609	0.0002	7.4024	0.005	9.3493	0.002
	1.0	0	0.1848	3.7888	0.0006	7.4835	0.0143	9.673	0.0056
a=0.5 $\phi_d=2\pi$ $\phi_s(r^4/4)=2\pi$	0.0	0	0.094	2.0175	0.0713	3.4264	0.0806	10.0075	0.0062
	0.2	0	0.0853	2.1287	0.0687	3.1789	0.0714	10.1548	0.0047
	0.4	0	0.0799	13.7475	0.0004	14.1625	0.0004	---	---
	0.6	0	0.112	8.4851	0.0009	9.7213	0.0011	12.59	0.0002
	0.8	0	0.1796	4.9906	0.0005	7.2854	0.0035	8.8747	0.002
	1.0	0	0.2363	3.9907	0	7.3451	0.0105	9.1378	0.0056
a=0.7 $\phi_d=2\pi$ $\phi_s(r^4/4)=2\pi$	0.0	0	0.0941	2.0177	0.0713	3.4264	0.0806	10.0073	0.0062
	0.2	0	0.0909	2.3067	0.0715	3.0494	0.0725	3.0494	0.0725
	0.4	0	0.1023	13.6941	0.0004	14.6655	0.0005	---	---
	0.6	0	0.1594	---	---	---	---	---	---
	0.8	0	0.2491	4.876	0.0011	7.5842	0.0029	9.4088	0.0016
	1.0	0	0.3087	4.0329	0	4.8212	0.0008	5.7595	0

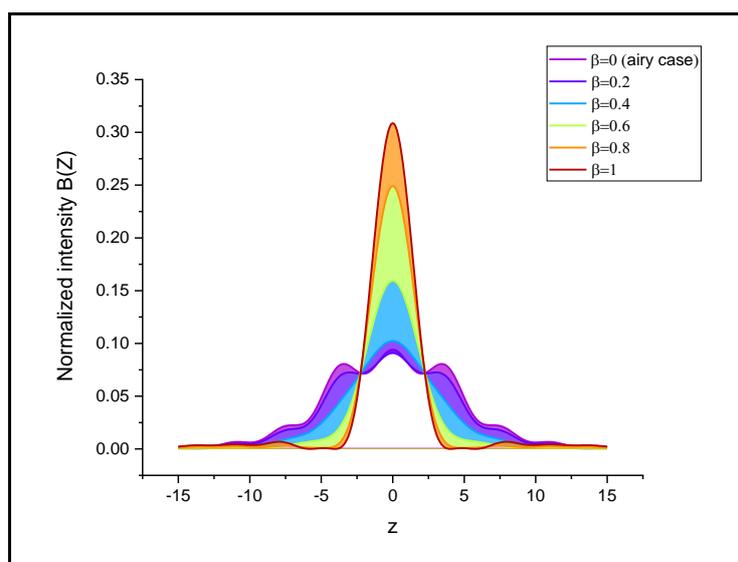


Fig. 5

Fig. 5. Shape of the PSF for various values of apodization parameter  $\beta$  at maximum defocus and primary spherical aberration when  $a=0.7$

### Acknowledgements

We are grateful to Optics research group, Department of Physics, University College of Science, Osmania University, Hyderabad and their contributed publications.

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Salkapuram Vidya Rani, et. al. "Tailoring Point Spread Function of Aberrated Two Zone Aperture by Symmetric Apodization." *IOSR Journal of Applied Physics (IOSR-JAP)*, 15(2), 2023, pp. 37-42.