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Hyperspherical Three Body Calculations For The Exotic ${}_{\Lambda\Lambda}^{6}$He Hypernucleus

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Abstract: *Hyperspherical three body calculations are performed to study and review the various properties of the exotic double Λ hyper nucleus ${}_{\Lambda\Lambda}{}^{6}$He. The ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus is considered as a three body system consisting of two Λ hyperons coupled to an α core. Different Λ- Λ and Λ - α realistic and phenomenological potentials are used in the calculations. Using the hyperspherical formalism, a complete symmetric and mixed symmetric wave functions are introduced. Contributions from partial waves are also taken into account in the calculations. The Fabre optimal subset is adopted to obtain fast and good convergence for the calculated binding energy using the renormalized Numerov method. The obtained results are then compared with recent experimental value as well as those calculated using other methods. The calculated ${}_{\Lambda\Lambda}{}^{6}$He binding energies are in good agreement with the new experimental one reported by Nagara event.*
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I. Introduction

The study of the structure and properties of light exotic hyper nuclei has become an area of special and particular interest to nuclear physists due to its important role in providing information about hyper- nucleon and hyper- hyper interactions. Such information is crucial for understanding the properties of multi strange hyper nuclei .The single Λ hyper nuclei (which consists of Λ hyperons coupled to a core nucleus) have been extensively studied and investigated due to the existence of many experimental data for various single Λ hyper nuclei over almost the whole mass table [1]-[3]. However the situation is different in the case of double Λ hyper nuclei where only three spices of double Λ hyper nuclei ${}_{\Lambda\Lambda}{}^{6}$He and ${}_{\Lambda\Lambda}{}^{10}$Be, ${}_{\Lambda\Lambda}{}^{13}$B have been experimentally discovered and identified [4]-[8]. The most interesting recent event ,Nagara event [9], that reported a new unambiguous value ($B_{\Lambda\Lambda}$= 7.25± 0.19 ${}^{+0.18}_{-0.11}$ MeV) for the binding energy of the double Λ ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus has triggered renewed interest in the physics of double Λ hyper nuclei especially for those light species. This Nagara event[9] suggested a fairly weak Λ- Λ interaction which is compatible with and supported by the scattering length $a_{\Lambda\Lambda}$ ≈ - 0.8 fm [10],[11] and smaller in magnitude than the Λ-N interaction [12]. The previous old value ($B_{\Lambda\Lambda}$= 10.80± 0.6 MeV) for the double Λ hyper nucleus ${}_{\Lambda\Lambda}{}^{6}$He implied a fairly strong Λ- Λ interaction which is stronger than the Λ-N one ($a_{N\Lambda}$ ≈ - 2.0 fm) deduced from studying the single Λ hyper nuclei and odds with the one Boson exchange model [13]. This old value of the ${}_{\Lambda\Lambda}{}^{6}$He binding energy was considered as dubious one by the hyper nuclear physics community. The weak Λ- Λ interaction suggested by Nagara event has triggered the interest of the hyper nuclear physists to theoretically investigate the double Λ hyper nucleus ${}_{\Lambda\Lambda}{}^{6}$He and explore the new $B_{\Lambda\Lambda}$ binding energy in the ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus.

Since the ${}_{\Lambda\Lambda}{}^{6}$He serves in most applications as the primary normalizing datum for extracting the phenomenological Λ- Λ interactions it is desirable to improve as much as possible the calculation aspects of the ${}_{\Lambda\Lambda}{}^{6}$He binding energy evaluation in order to gain confidence in such extractions. We therefore compare our hyperspherical harmonics method (HH) calculations of the ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus with other approaches such as the cluster models Faddeev calculations, coupled channel and variational methods [14]-[20] . The observed separation energy of a Λ hyperons in ${}_{\Lambda\Lambda}{}^{6}$He is 7.6 MeV [2] ,which is evidently smaller than that for nucleon ≈ 20 MeV in ${}_{\Lambda\Lambda}{}^{6}$He . Consequently the ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus falls behind the α particle in energetic stability and no Λ Λ bound state has been reported. The ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus is composed of an α particle and two Λ hyperons . The structure of the α particle in the hyper nucleus assumed to be not distributed by the existence of other particles ,so the distributions of the α particle inside the ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus still keeps the mass form of the high energy electron scattering with the α particle.

It is the aim of our work to investigate and review the different properties of the exotic ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus using hyperspherical harmonics (HH) three body calculations. The ${}_{\Lambda\Lambda}{}^{6}$He hyper nucleus is considered as a three body system consisting of two Λ hyperons coupled to an α core assuming internal structure, stability and compactness of the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus . Different realistic and phenomenological Λ-Λ and Λ-α interactions are used in the calculations. The Fabre optimal subset [28],[29] is adopted to obtain fast and good convergence for the calculated binding energy using the renormalized Numerov method. [35],[36].The obtained results are then compared with those obtained using other methods of calculations as well as the recent experimental one reported by Nagara event [9],[21]. We employ the hyperspherical harmonics expansion (HH) method to solve such a three-body system. This method is a powerful tool for the ab initio solution of the few-body Schrodinger equation for a given set of interaction potentials among the constituent particles. This method has been used for bound states in atomic [22],[23], nuclear [24]-[33] and particle physics [25]. Attempts have been made to use it in scattering problems as well [34]. In the (HH) method, the wave function describing a system of N particles (in the center of mass system) is expanded in terms of a complete set of orthonormal functions of 3N-4 variables. The expansion coefficients are functions of a single variable that represents the length of 3N-3 dimensional vector. By substituting the wave function expansion into the Schroedinger equation describing the system, one obtains an infinite set of coupled differential equations for the expansion coefficients. The resulting set of coupled differential equations can be solved numerically by the renormalized Numerov method [35],[36] or the hyperspherical adiabatic approximation [27]. A multi pole potential is also obtained by expanding the two body interaction on a complete set of hyperspherical harmonics. This multi pole potential is very helpful and useful when used in the Schroedinger equation. As for the three-body system, the angular harmonics are functions of five angular variables. In order to determine the potential matrix of the three-body Schroedinger equation, the matrix elements of the multi pole potentials between a pair of such hyperspherical harmonics were calculated. The symmetry of the system under study rules out some harmonics from appearing in the set of coupled equations. Further, the centrifugal barrier terms occurring in the set of coupled equations grow considerably with higher harmonics. One can therefore, truncate this infinite set [28]-[29] and work with a finite set (Fabre optimal subset) of coupled differential equations or a corresponding one dimensional integral equation. The (HH) method is essentially an exact one and more reliable than other methods. It involves no approximation except for a possible truncation of the expansion basis. By gradually expanding the expansion basis and checking the rate of convergence, any desired precision in the binding energy can, in principle, be achieved. However, the number of coupled differential equations and, therefore, the complexity in the numerical solution increases rapidly as the expansion basis is increased by including larger hyper-angular-momentum quantum number. The numbers of equations that have to be retained in any calculation using the (HH) method will, of course, depend on the nature of the potential used.

In the present work, the Fabre optimal subset [28],[29] was used to obtain a converged set of coupled differential equations in a single variable, namely, the hyper-radius. By numerically solving these equations, the eigenvalues and eigenfunctions of the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He wave function were determined. In section 2 the (HH) method is presented including the different equations used in our calculation. In section 3 the numerical work and results are presented while the discussion and conclusion are given in section 4.Section 5 is devoted for references.

II. Theoretical Work

Let the position vectors of the two Λ particles denoted by r_1 and r_2, respectively, and their masses by m_Λ.The position vector of the α particle is denoted by r_3 and its mass by m_α.The total mass of the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He is $M=2m_\Lambda + m_\alpha$ and the mass of α particle is taken to be $m_\alpha = 4m$ and that for the lambda Λ particle is taken to be $m_\Lambda = 6/5m$, where m is the nucleon (proton or neutron) mass. The Jacobi coordinates set used by Clare and Levinger [37] was chosen here:

$$\eta = \alpha(r_1 - r_2) \tag{1}$$

$$\xi = \beta(r_1 + r_2 - 2r_3) \tag{2}$$

where $$\alpha = \sqrt{\frac{3m_\Lambda}{2(2m_\Lambda + m_\alpha)}} \quad \textit{and} \quad \beta = \alpha\sqrt{\frac{m_\Lambda}{2m_\Lambda + m_\alpha}}$$

The inter-particle separation are expressed as:

$$r_{12} = \frac{\eta}{\alpha}, \quad r_{13} = \frac{1}{2\beta}\xi + \frac{1}{2\alpha}\eta, \quad \text{and} \quad r_{23} = \frac{1}{2\beta}\xi - \frac{1}{2\alpha}\eta \tag{3}$$

Now, the hyper-spherical coordinates ρ and θ are introduced, where ρ is the hyper-radius and θ is the hyper-spherical angle. As a result, the following relations are obtained

$$\eta = \rho \sin\theta \qquad , \qquad \xi = \rho\cos\theta \qquad , \qquad \rho^2 = \eta^2 + \xi^2 \tag{4}$$

$$\text{with} \qquad \tan\theta = \frac{\eta}{\xi} \qquad (0 \le \theta \le \frac{\pi}{2}) \tag{5}$$

The non relativistic Schroedinger equation for the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He, after separating out the motion of the center of mass, can be written as :

$$\left\{ \frac{-\hbar^2}{2\mu}(\nabla_\eta^2 + \nabla_{\xi)}^2 + \sum_{i \succ j} V(r_{ij}) - E \right\} \Psi(\eta, \xi) = 0 \tag{6}$$

where μ is chosen to be μ = M/3 and $V(r_{ij})$ is the two-body central potential. Therefore, the Schroedinger equation for the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He system expressed in terms of the hyper-spherical coordinates becomes:

$$\left\{ \frac{-\hbar^2}{2\mu}\nabla^2 + V(\rho) - E \right\} \Psi(\rho) = 0 \tag{7}$$

Expanding the wave function Ψ(ρ) on the HH basis, $Y_{[L]}$ (Ω), gives

$$\Psi(\rho) = \rho^{\frac{-5}{2}} \sum_{[L]} Y_{[L]}(\Omega) U_{[L]}(\rho) \tag{8}$$

where Ω is a set of 5 angular coordinates describing the position of a point at the surface of the unit hypersphere. $U_{[L]}(\rho)$ are the renormalized hyper radial partial wave functions, [L] stands for the set of quantum numbers including spin and isospin defining the state of grand orbital L. Substituting the expansion (8) into Eq.(7), yields an infinite set of second order coupled differential equations written as:

$$\{\frac{-\hbar^2}{\mu}[\frac{d^2}{d\rho^2} - \frac{(L+2)^2 - \frac{1}{4}}{\rho^2}] - E\} U_{[L]}(\rho) + \sum_{[L]} \langle Y_{[L]}(\Omega) V(\rho) Y_{[L']} \rangle U_{[L']}(\rho) = 0 \tag{9}$$

which is subsequently truncated in order to be treated numerically. This can be done by using the Fabre optimal subset [28],[29]. As a result, the infinite set of coupled equations is transformed into a finite set of coupled ones to be solved numerically. For the case of central potential, the ground state of the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He nucleus is described by even number of the grand orbital momentum L= 2K+l, due to the parity conservation. Then, the finite set of coupled equations, for orbital momentum l = 0, becomes:

$$[T_{2K} - E] U_{2K}(\rho) + \sum_{K'} (-)^K V_{2K''}^{2K'}(\rho) U_{2K'}(\rho) = 0 \tag{10}$$

with the kinetic energy operator and the potential matrix elements being expressed as:

$$T_{2K} = \frac{-\hbar^2}{\mu}[\frac{d^2}{d\rho^2} - \frac{(2K+2)^2 - \frac{1}{4}}{\rho^2}] \tag{11}$$

and

$$V_{2K''}^{2K'}(\rho) = \sum_{K'''} C_{2K''}^{2K'}(K'', 0, C) V_{2K''}(\rho) \tag{12}$$

respectively. The geometrical coefficients, $C_{2K''}^{2K'}(K'', 0, C)$, appearing in Eq.(12) couple the set of equations with the main equation for which K = 0 for each component of the central components of the two-body potentials. Explicit expressions for these coefficients are given in Ref. [28]. The multi-pole potentials, $V_{2K''}(\rho)$, given in Eq.(12) introduce the multi-poles of the central parts of the two-body potential and are expressed as :

$$V_{2K''}(\rho) = \sum_{i \succ j} V_{2K''}^{ij}(\rho) \tag{13}$$

with $$V_{2K}^{ij}(\rho)=\frac{16}{\pi}\int_0^\infty dx\sqrt{1-(\frac{x}{\rho})^2}\,x^2V(x^2)\,{}_2F_1(-K,K+2,\frac{3}{2},(\frac{x}{\rho})^2) \tag{14}$$

where V(x) = V(r_{ij}) is the two-body potential. In the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He the two body potentials refers to the Λ–Λ and the Λ-α interactions. Equation (14) may be written as:

$$V_{2K}^{ij}(\rho)=\frac{16}{\pi}\int_0^1 du(1-u^2)^{\frac{1}{2}}u^2V(\rho u)\,{}_2F_1(-K,K+2,\frac{3}{2},u^2) \tag{15}$$

with V(ρu) = V(r_{ij}) , which is more useful in our numerical calculations

III. Numerical Calculations and Results

In order to carry out the numerical calculations, the set of coupled differential equations represented by Eq. (10) were written in matrix form as:

$$\left\{[\mathbf{I}]\frac{\mathbf{d}^2}{\mathbf{d}\rho^2}+[\mathbf{Q}]\right\}\mathbf{U}(\rho)=\mathbf{0} \tag{16}$$

where [I] is the unit matrix and the column vector U(ρ) contains the partial waves $U_{2K}(\rho)$ as its components. Also the matrix element [Q] is given by:

$$[\mathbf{Q}(\rho)]=\frac{\mu}{\hbar^2}\{\mathbf{E}[\mathbf{I}]-\mathbf{V}_{\mathbf{eff}}(\rho)\} \tag{17}$$

The components of the effective potential matrix is given by:

$$\mathbf{V}_{\mathbf{eff}}(\rho)=\mathbf{V}_{\mathbf{2K}}^{\mathbf{2K'}}(\rho)+\frac{\hbar^2}{\mu}\left[\frac{\mathbf{2K(K+1)}+\frac{\mathbf{15}}{\mathbf{4}}}{\rho^2}\right] \tag{18}$$

where $\mathbf{V}_{\mathbf{2K}}^{\mathbf{2K'}}(\rho)$ is given by Eq.(12).

The renormalized Numerov method [35],[36] was then used to solve [28]- [33] the set of coupled equations (16). In order to study the convergence of the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He energy eigenvalue the Fabre optimal subset [28],[29] have been used, using a different Gaussian forms of the realistic and phenomenological Λ-Λ and Λ-α interactions. The convergence of $_{\Lambda\Lambda}{}^{6}$He eigenvalue have been attained by including terms up to K=18. Therefore in studying the convergence of the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He binding energy we have solved K coupled equations for K=1,2,......,$K_{\{max\}}$. Also the decrease in hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He binding energy was noted by increasing the number of terms in the HH expansion of the wave function. We have solved K=18 coupled equations in order to obtain the hyper-nucleus $_{\Lambda\Lambda}{}^{6}$He binding energy values for the considered realistic Λ-Λ and Λ-α interactions.

In calculating the binding energy of the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus we have used different types of realistic and phenomenological two body potential of Gaussian form for Λ- Λ as well as the Λ -α interactions.

For the Λ- α interactions: We have used four forms of Λ -α interactions Dalitz and Downs DI(ρDG), and DII(ρDG) ,Gibson et al. GI(ρDG) and GII. (ρDG) which are obtained by folding the Λ –N potential into the double Gaussian form nucleon density distribution of an α particle (ρ_{DG}) and are given in table II of Reference [38].

For the Λ- Λ interactions: It is well known that experimental information concerning the two body Λ- Λ potentials could be obtained only from the Λ- Λ binding energy $B_{\Lambda\Lambda}$ in the observed double Λ hyper nuclei, $_{\Lambda\Lambda}{}^{6}$He, for example. On the other hand the importance for the theoretical indication for the OBE part of the Λ- Λ interactions is given by using the SU(3) invariance of the coupling constants. Therefore we have considered three different type [39]-[41] of two body Λ- Λ interactions to study the effect of these of interactions on the $_{\Lambda\Lambda}{}^{6}$He binding energy . The first is a realistic one introduced by Singnan et al.[39] contains only attractive and repulsive parts with parameters determined by fitting the experimental value of the binding energy of the two Λ's hyperons in the

$_{\Lambda\Lambda}{}^{6}$He hyper nucleus. The second one is phenomenological one proposed by Nijmegen group [41]. For the third Λ-Λ interaction we have considered the soft core one suggested by Hiyama et al.[40]. All the considered types of Λ-Λ interactions $_{\Lambda\Lambda}{}^{6}$He are of Gaussian shape and given by the following relation:

$$V^{\Lambda\Lambda}(r) = \sum_{i=1}^{6} V_i \exp(-\frac{r^2}{d_i^2})$$

where the parameters V_i and d_i are given in table (1).

Table (1)

Potn.	i	1	2	3	4	5	6
Ref[39]	V_i MeV	-69.6	165	-	-	-	-
	d_i Fm	1.04	0.6	-	-	-	-
Ref[40]	V_i MeV	-8.967	-226.8	880.7	-	-	-
	d_i Fm	1.5	0.9	0.5	-	-	-
Ref[41]	V_i MeV	-21.34	-187.0	10850	0.1932	32.17	2035
	d_i Fm	1.342	0777	035	1.342	0777	035

Table (1). Parameters for the different types of Gaussian shape Λ-Λ interactions

In our study we have calculated Λ- Λ binding energy $B_{\Lambda\Lambda}$ in double Λ hyper nuclei, $_{\Lambda\Lambda}{}^{6}$He for the different types of the Λ-Λ and Λ-α interactions . In order to get confidence in our calculations we compare our obtained results with those calculated using other methods of calculations such as Faddeev, variational calculations. In table (2) we present the calculated binding energy $B_{\Lambda\Lambda}$ in double Λ hyper nucleus, $_{\Lambda\Lambda}{}^{6}$He for the different types of the Λ-Λ and Λ-α interactions considered. In table (3) a comparison is presented between our results and other methods of calculations.

. Table (2)

Λ-α [38] / Λ-Λ	DI(ρDG)	DII(ρDG)	GI(ρDG)	GII(ρDG)	Exp.[9]
Singnan [40]	7.790	7.063	7.791	6.852	7.620
Hiyama [41]	7.636	7.635	7.636	7.638	
Nijmeg. [42]	7.264	8.081	7.264	7.225	

Table (2) Calculated binding energy $B_{\Lambda\Lambda}$ (MeV) in double Λ hyper nucleus, $_{\Lambda\Lambda}{}^{6}$He for different types of the Λ-Λ and Λ-α interactions

Table (3)

Methods	$B_{\Lambda\Lambda}$ (MeV)	Methods	$B_{\Lambda\Lambda}$ (MeV)
HH [27]	10.894	G Matrix[43]	9.500
Faddeev [11]	11.123	Present	7.636
Variational[41]	9,800	Experiment.[9]	7.620

Table (3) The $B_{\Lambda\Lambda}$ (MeV) in double Λ hyper nucleus, $_{\Lambda\Lambda}{}^{6}$He calculated with different methods is listed and compared with the present as experimental one

IV. Discussion and Conclusions

In calculating the binding energy $B_{\Lambda\Lambda}$ of the double Λ hyper nucleus, $_{\Lambda\Lambda}{}^{6}$He we have applied the exact hyperspherical (HH) method which is a powerful and reliable method of three and many body calculations. In order to get a good and fast convergence of the binding energy ,Fabre optimal subset [28],[29] has been used in solving a finite set of coupled differential equations. Different types of realistic and phenomenological the Λ-Λ and Λ-α interactions have been considered to study the properties of the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus and the effect of using various interactions on these properties. Solving up to K_{max} = 18 coupled equations we have obtained for the binding energy $B_{\Lambda\Lambda}$ values which are in good agreement with the recent experimental one (7.62 MeV) reported by Nagara

[9].Our calculated binding energy $B_{\Lambda\Lambda}$ for the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus are presented in table (1) for the considered types of realistic and phenomenological the Λ-Λ and Λ-α interactions. It is shown form table (1) that our results are in good agreement with the experiment for all types of Λ-Λ interactions considered , specially for those calculated using soft core Λ-Λ interactions suggested by Hiyama et al.[40]. It is also shown from table (1) that the binding energy $B_{\Lambda\Lambda}$ for the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus are not affected by using various types of Λ-α interactions [39]. In order to get confidence in our method of calculations we present in table (2) a comparison of our results with those calculated with other methods such as Faddeev [11], variational [40], G matrix [41], and even HH [27] methods .It is shown form table (2) that our results are more accurate and in good agreement with the recent experiment than those calculated with these methods. The first four partial hyperradial waves of the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus are given in Fig.(1),Fig(2),and Fig(3) for the different types of Λ-Λ interactions considered. As a matter of fact the figures indicate the fairly weak strength of the Λ-Λ potentials compared with the Λ-N interaction and agree with the new report about the fairly weak binding value of the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus and also the suggestion that no Λ-Λ bound state is found.

Finally we conclude that in reviewing the current status of the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus we have obtained $B_{\Lambda\Lambda}$ binding energy values ($B_{\Lambda\Lambda}$= 7.636 MeV) in good agreement with the recent reported experimental one ($B_{\Lambda\Lambda}$= 7.620 MeV).We also conclude that the $_{\Lambda\Lambda}{}^{6}$He hyper nucleus is loosely bound nucleus due to the weak strength of the Λ-Λ potentials compared with the Λ-N one, a result which is confirmed and supported by the recent Nagara report [9].

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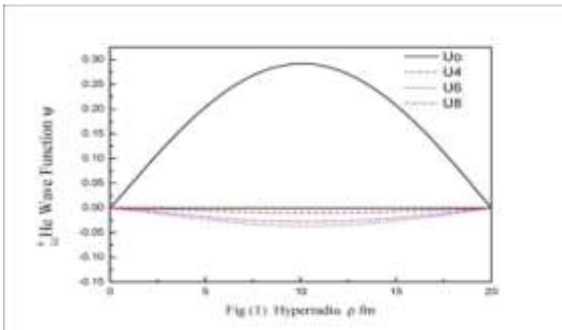

Fig.1. First four hyperradial partial waves $U_{(2K)}$ (with K = 0,,2,3,4) for the hypernucleus $_{\Lambda\Lambda}{}^{6}$He generated by Singnan et al [39] Λ-Λ interaction.

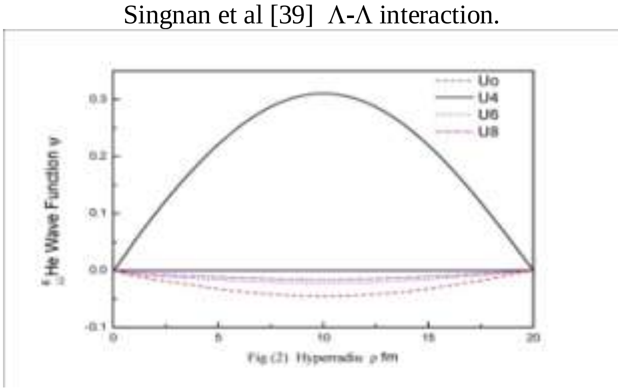

Fig.2. First four hyperradial partial waves $U_{(2K)}$ (with K = 0,,2,3,4) for the hypernucleus $_{\Lambda\Lambda}{}^{6}$He generated by soft core Hiyama et al. [40] Λ-Λinteraction

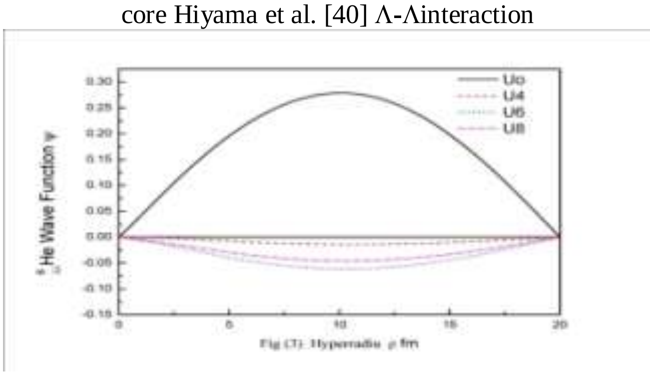

Fig.3. First four hyperradial partial waves $U_{(2K)}$ (with K = 0,,2,3,4) for the hypernucleus $_{\Lambda\Lambda}{}^{6}$He generated by double folded Nijmegen group [41] Λ-Λinteraction.