

Energy Attenuation Mechanism of a Carrier Wavepropagating in a Viscous Fluid

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Abstract: For the first time, we derived a constituted carrier wave equation which is the result of superposing a 'parasitic wave' on a 'host wave'. The attenuation mechanism of the several properties of the carrier wave produced by the two interfering waves as it propagates in a viscous fluid is effectively studied using simple differentiation technique. In this work, we subjected the constituted carrier wave to some basic boundary conditions with a multiplicative factor as a constraint in order to determine quantitatively the basic intrinsic characteristics of the two interfering waves which were initially not known. Initially, the carrier wave and its several properties show a blurred spectra characteristics followed by a gradual depletion of the wave form. The initial blurred nature of the resulting spectra is an indication of the resistance of the intrinsic parameters of the 'host wave' to the destructive tendency of the interfering 'parasitic wave'. The subsequent depleting behavior of the carrier wave indicates the predominance of the 'parasitic wave'. After this time, a steady decay process resulting to a gradual reduction and weakening in the initial strength of the intrinsic parameters of the 'host wave' becomes prominent. The constituted carrier wave becomes monochromatic in nature since each component of the wave packet have different phase velocity in the medium, the modulation propagation number of the components of the carrier wave changes in the medium and consequently the group velocity changes. This study reveals that when a carrier wave is undergoing attenuation under any circumstance, it does not consistently come to rest; rather it shows some resistance at some point during the decay process, before it is finally brought to rest.

Keywords: 'Host wave', 'parasitic wave' carrier wave, characteristic angular velocity, group angular velocity, phase velocity.

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I. Introduction

Vibration is the cause of all that exists and vibration produces wave. Consequently, all forms of matter and their corresponding characteristics can be described by vibration. Some waves in nature behave parasitically when they interfere with another one. Such waves as the name implies has the ability of transforming the initial characteristics and behaviour of the interfered wave to its own form and quality after a given period of time. Under this circumstance, all the active constituents of the interfered wave would have been completely eroded and the resulting wave which is now parasitically monochromatic, will eventually attenuate to zero, since the 'parasitic wave' does not have its own independent parameters for sustaining a continuous existence.

Interference effect that occurs when two or more waves overlap or intersect is a common phenomenon in physical wave mechanics. When waves interfere with each other, the amplitude of the resulting wave depends on the frequencies, relative phases and amplitudes of the interfering waves. The resultant amplitude can have any value between the differences and sum of the individual waves [1]. If the resultant amplitude comes out smaller than the larger of the amplitude of the interfering waves, we say the superposition is destructive; if the resultant amplitude comes out larger than both we say the superposition is constructive.

When a wave equation ψ and its partial derivatives never occur in any form other than that of the first degree, then the wave equation is said to be linear. Consequently, if ψ_1 and ψ_2 are any two solutions of the wave equation ψ , then $a_1\psi_1 + a_2\psi_2$ is also a solution, a_1 and a_2 being two arbitrary constants. This is an illustration of the principle of superposition, which states that, when all the relevant equations are linear we may superpose any number of individual solutions to form new functions which are themselves also solutions [2,3].

The interference of one wave say 'parasitic wave' y_1 on another one say 'host wave' y_2 could cause the 'host wave' to decay to zero if they are out of phase. The decay process of y_2 can be gradual, over-damped or critically damped depending on the rate in which the amplitude of the 'host wave' is brought to zero.

However, the general understanding is that the combination of y_1 and y_2 would first yield a third stage called the resultant wave say y , before the process of decay begins. In this work, we refer to the resultant wave

as the constituted carrier wave (CCW). Several properties of the CCW, are the amplitude, group velocity, total phase angle, etc., and they are also expected to attenuate with time during the propagation, since they make up the CCW.

A carrier wave in this wise, is a corrupt wave function which certainly describes the activity and performance of most physical systems. Thus, the reliability and the life span of most active systems are determined by the reluctance and willingness of the active components of the ‘host wave’ to the destructive influence of the ‘parasitic wave’.

Any actively defined physical system carries along with it an inbuilt attenuating factor such that even in the absence of any external influence the system will eventually come to rest after a specified time. This accounts for the non-permanent nature of all physically active matter.

If the wave function of any given active system is known, then its characteristics can be predicted and altered by means of anti-vibratory component. The activity and performance of any active system can be slowed down to zero-point ‘dead’ by means of three factors: (i) Internal factor (ii) External factor, and (iii) Accidental Factor.

The internal factor is a normal decay process. This factor is caused by aging and local defects in the constituent mechanism of the matter wave function. This shows that every physically active system must eventually come to rest or cease to exist after some time even in the absence of any external attenuating influence. The internal factor is always a gradual process and hence the attenuating wave function is said to be under-damped.

The external factor is a destructive interference process. This is usually a consequence of the encounter of one existing well behaved active wave function with another. The resultant attenuating wave function under this condition is said to be under-damped, over-damped or critically-damped, depending on how fast the intrinsic constituent characteristics of the wave function decays to zero.

The accidental factor leads to a sudden breakdown and restoration of the active matter wave function to a zero-point. In this case, all the active intrinsic parameters of the matter wave function are instantaneously brought to rest and the attenuation process under this condition is said to be critically-damped. Generally, we can use the available information of the physical parameters of a wave at any given position and time to determine the nature of its source and the initial characteristics at time $t = 0$, more also, to predict the future behaviour of the wave.

The initial characteristics of a given wave with a definite origin or source can best be determined by the use of a sine wave function. However, for the deductive determination of the initial behaviour of a wave whose origin is not certain, the cosine wave function can best be effectively utilized. However, the reader should permit the lack of adequate references in this paper, since there is no author who had obviously thought in this line before now.

The organization of this paper is as follows. In section 1, we discuss the nature of wave and interference. In section 2, we show the mathematical theory of superposition of two incoherent waves. The results emanating from this study is shown in section 3. The discussion of the results of our study is presented in section 4. Conclusion and suggestions for further work is discussed in section 5. The paper is finally brought to an end by an appendix and a few lists of references.

1.1 Research methodology

In this work, we superposed a ‘parasitic wave’ with inbuilt raising multiplier λ on a ‘host wave’ which also contain an inbuilt lowering multiplier β . The attenuation mechanism of the carrier wave which is the result of the superposition is thus studied by means of simple differentiation technique.

II. Mathematical theory of superposition of waves

Let us consider two incoherent waves defined by the non - stationary displacement vectors

$$y_1 = a\beta \cos(\vec{k}\beta \cdot \vec{r} - n\beta t - \varepsilon\beta) \quad (2.1)$$

$$y_2 = b\lambda \cos(\vec{k}'\lambda \cdot \vec{r} - n'\lambda t - \varepsilon'\lambda) \quad (2.2)$$

Where all the symbols retain their usual meanings. In this study, (2.1) is regarded as the ‘host wave’ whose propagation depends on the raising multiplier or inbuilt multiplicative factor $\beta (= 0 \dots 1)$. While (2.2) represents a ‘parasitic wave’ with an inbuilt multiplicative factor $\lambda (= 0, 1, 2, \dots, \lambda_{\max})$. The inbuilt multipliers are both dimensionless and as the name implies, they are capable of gradually raising the basic intrinsic parameters of both waves respectively with time. Now let us add the two waves given by (2.2) and (2.1) as follows.

$$y = y_1 + y_2 = a\beta \cos(\vec{k}\beta \cdot \vec{r} - n\beta t - \varepsilon\beta) + b\lambda \cos(\vec{k}'\lambda \cdot \vec{r} - n'\lambda t - \varepsilon'\lambda) \quad (2.3)$$

Suppose, we assume that for a very small parameter ζ , the below equation holds,

$$n'\lambda = \zeta + n\beta \quad (2.4)$$

$$y = a\beta \cos(\bar{k}\beta \cdot \bar{r} - n\beta t - \varepsilon\beta) + b\lambda \cos(\bar{k}'\lambda \cdot \bar{r} - n\beta t - \zeta t - \varepsilon'\lambda) \quad (2.5)$$

Again in (2.5), we assume that for a small and negligible parameter ζ the below relation holds.

$$\varepsilon'_1 = \zeta t + \varepsilon'\lambda \quad (2.6)$$

$$y = a\beta \cos(\bar{k}\beta \cdot \bar{r} - n\beta t - \varepsilon\beta) + b\lambda \cos(k'\lambda \cdot \bar{r} - n\beta t - \varepsilon'_1) \quad (2.7)$$

For the purpose of proper grouping we again make the following assumption:

$$\bar{k}\beta \cdot \bar{r} = \bar{k}'\lambda \cdot \bar{r} = \xi \quad (2.8)$$

$$(\bar{k}\beta - \bar{k}'\lambda) \cdot \bar{r} = \xi \quad (2.9)$$

$$y = a\beta \cos((\xi - n\beta t) - \varepsilon\beta) + b\lambda \cos((\xi - n\beta t) - \varepsilon'_1) \quad (2.10)$$

We can now apply the cosine rule for addition of angles to re-evaluate each term in (2.10), that is,

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \quad (2.11)$$

$$y = a\beta \left\{ \cos(\xi - n\beta t) \cos \beta \varepsilon + \sin(\xi - n\beta t) \sin \beta \varepsilon \right\} + b\lambda \left\{ \cos(\xi - n\beta t) \cos \varepsilon'_1 + \sin(\xi - n\beta t) \sin \varepsilon'_1 \right\} \quad (2.12)$$

$$y = \cos(\xi - n\beta t) \left\{ a\beta \cos \beta \varepsilon + b\lambda \cos \varepsilon'_1 \right\} + \sin(\xi - n\beta t) \left\{ a\beta \sin \beta \varepsilon + b\lambda \sin \varepsilon'_1 \right\} \quad (2.13)$$

For technicality, let us make the following substitutions so that we can further simplify (2.14).

$$A \cos E = a\beta \cos \beta \varepsilon + b\lambda \cos \varepsilon'_1 \quad (2.14)$$

$$A \sin E = a\beta \sin \beta \varepsilon + b\lambda \sin \varepsilon'_1 \quad (2.15)$$

$$y = A \left\{ \cos(\xi - n\beta t) \cos E + \sin(\xi - n\beta t) \sin E \right\} \quad (2.16)$$

$$y = A \cos \left\{ \xi - n\beta t - E \right\} \quad (2.17)$$

$$y = A \cos \left\{ (\bar{k}\beta - \bar{k}'\lambda) \cdot \bar{r} - n\beta t - E \right\} \quad (2.18)$$

The simultaneous nature of (2.14) and (2.15) would enable us to square though them and add the resulting equations term by term. After a careful algebraic simplifications we arrive at the equation

$$A = \sqrt{a^2 \beta^2 + b^2 \lambda^2 + 2ab\beta\lambda \cos((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t)} \quad (2.19)$$

$$y = \sqrt{a^2 \beta^2 + b^2 \lambda^2 + 2ab\beta\lambda \cos((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t)} \times \cos((\bar{k}\beta - \bar{k}'\lambda) \cdot \bar{r} - n\beta t - E) \quad (2.20)$$

Upon dividing (2.16) by (2.15), we get that

$$\tan E = \frac{a\beta \sin \beta \varepsilon + b\lambda \sin \varepsilon'_1}{a\beta \cos \beta \varepsilon + b\lambda \cos \varepsilon'_1} \quad (2.21)$$

$$E = \tan^{-1} \left(\frac{a\beta \sin \beta \varepsilon + b\lambda \sin(\varepsilon'\lambda - (n\beta - n'\lambda)t)}{a\beta \cos \beta \varepsilon + b\lambda \cos(\varepsilon'\lambda - (n\beta - n'\lambda)t)} \right) \quad (2.22)$$

Hence (2.20) is the resultant wave which describes the superposition of the 'parasitic wave' on the 'host wave'. As the equation stands, it represents a resultant wave equation in which the effects of the constitutive waves are additive in nature. However, suppose we assume that the effects of the constitutive waves are subtractive and with the view that the basic parameters of the 'host wave' are constant with time, that is, $\beta = 1$ and leave its variation for future study, then without loss of dimensionality we can recast (2.20) and (2.22) as

$$y = \sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda))} \times \cos((\bar{k} - \bar{k}'\lambda) \cdot \bar{r} - (n - n'\lambda)t - E) \quad (2.23)$$

where we have redefined the amplitude and the total phase angle as,

$$A = \sqrt{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda))} \quad (2.24)$$

$$E = \tan^{-1} \left(\frac{a \sin \varepsilon - b\lambda \sin((n - n'\lambda)t - \varepsilon'\lambda)}{a \cos \varepsilon - b\lambda \cos((n - n'\lambda)t - \varepsilon'\lambda)} \right) \quad (2.25)$$

Equation (2.23) is now the required constitutive carrier wave (CCW) equation necessary for our study. As the equation stands, it is only the variation in the intrinsic parameters of the 'parasitic wave' that determines

the activity of the physical system which it describes. Henceforth, we have agreed in this study, that the initial parameters of the 'host wave' are assumed to be constant and also they are initially greater than those of the 'parasitic wave'. By definition: the modulation angular frequency is given by $(n - n'\lambda)$, the modulation propagation constant is $(\bar{k} - \bar{k}'\lambda)$, the phase difference δ between the two interfering waves is $(\varepsilon - \varepsilon'\lambda)$, the interference term is given by $2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda))$, while waves out of phase interfere destructively according to $(a - b\lambda)^2$ and waves in-phase interfere constructively according to $(a + b\lambda)^2$. In the regions where the amplitude of the carrier wave is greater than either of the amplitude of the individual wave, we have constructive interference that means the path difference is $(\varepsilon + \varepsilon'\lambda)$, otherwise, it is destructive in which case the path difference is $(\varepsilon - \varepsilon'\lambda)$. If $n \approx n'$, then the average angular frequency say $(n + n')/2$ will be much more greater than the modulation angular frequency say $(n - n')/2$ and once this is achieved, then we will have a slowly varying carrier wave with a rapidly oscillating phase.

2.1 The calculus of the total phase angle E of the carrier wave function

Let us now determine the variation of the total phase angle with respect to time t . Thus from (2.25),

$$\frac{dE}{dt} = \left(1 + \left(\frac{a \sin \varepsilon - b\lambda \sin((n - n'\lambda)t - \varepsilon'\lambda)}{a \cos \varepsilon - b\lambda \cos((n - n'\lambda)t - \varepsilon'\lambda)} \right)^2 \right)^{-1} \times \frac{d}{dt} \left(\frac{a \sin \varepsilon - b\lambda \sin((n - n'\lambda)t - \varepsilon'\lambda)}{a \cos \varepsilon - b\lambda \cos((n - n'\lambda)t - \varepsilon'\lambda)} \right) \quad (2.26)$$

$$\frac{dE}{dt} = \left\{ \frac{(a \cos \varepsilon - b\lambda \cos((n - n'\lambda)t - \varepsilon'\lambda))^2}{(a \cos \varepsilon - b\lambda \cos((n - n'\lambda)t - \varepsilon'\lambda))^2 + (a \sin \varepsilon - b\lambda \sin((n - n'\lambda)t - \varepsilon'\lambda))^2} \right\} \times \frac{d}{dt} \left(\frac{a \sin \varepsilon - b\lambda \sin((n - n'\lambda)t - \varepsilon'\lambda)}{a \cos \varepsilon - b\lambda \cos((n - n'\lambda)t - \varepsilon'\lambda)} \right) \quad (2.27)$$

After a lengthy algebra (2.27) simplifies to

$$\frac{dE}{dt} = Z \quad (2.28)$$

where we have introduced a new variable defined by the symbol Z and which is given by

$$Z = (n - n'\lambda) \left(\frac{b^2 \lambda^2 - ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)}{a^2 + b^2 \lambda^2 - 2ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)} \right) \quad (2.29)$$

This is the characteristic angular velocity of the constituted carrier wave. It has the dimension of *rad./s*. Also the variation of the total phase angle E with respect to the wave number is given by

$$\frac{dE}{d(k - k'\lambda)} = t \frac{d}{d(k - k'\lambda)} (n - n'\lambda) \left(\frac{b^2 \lambda^2 - ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)}{a^2 + b^2 \lambda^2 - 2ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)} \right) \quad (2.30)$$

2.2 Evaluation of the group angular velocity (ω_g) of the carrier wave function

The group velocity is a well-defined but different velocity from that of the individual wave themselves. This is also the velocity at which energy is transferred by the wave [4]. When no energy absorption is present, the velocity of energy transport is equal to the group velocity [5]. The carrier wave function is a maximum if the spatial oscillatory phase is equal to 1. As a result

$$\cos(r((k - k'\lambda) \cos \theta + (k - k'\lambda) \sin \theta) - (n - n'\lambda)t - E) = 1 \quad (2.31)$$

$$r(\cos \theta + \sin \theta) - t \frac{d}{d(k - k'\lambda)} (n - n'\lambda) - \frac{dE}{d(k - k'\lambda)} = 0 \quad (2.32)$$

$$r(\cos \theta + \sin \theta) = t \frac{d}{d(k - k'\lambda)} (n - n'\lambda) + \frac{dE}{d(k - k'\lambda)} \quad (2.33)$$

$$r(\cos \theta + \sin \theta) = t \frac{d}{d(k - k'\lambda)} (n - n'\lambda) + t \frac{d}{d(k - k'\lambda)} (n - n'\lambda) \left(\frac{b^2 \lambda^2 - ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)}{a^2 + b^2 \lambda^2 - 2ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)} \right) \quad (2.34)$$

$$v_g = \frac{r}{t} = \frac{d}{d(k - k'\lambda)} \frac{(n - n'\lambda)}{(\cos \theta + \sin \theta)} \left\{ 1 + \left(\frac{b^2 \lambda^2 - ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)}{a^2 + b^2 \lambda^2 - 2ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)} \right) \right\} \quad (2.35)$$

$$v_g = \frac{r}{t} = \frac{d\omega_g}{d(k - k'\lambda)} \quad (2.36)$$

which is the basic expression for the group angular velocity, where

$$\omega_g = \frac{(n - n'\lambda)}{(\cos \theta + \sin \theta)} \left(\frac{a^2 + 2b^2 \lambda^2 - 3ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)}{a^2 + b^2 \lambda^2 - 2ab\lambda \cos((\varepsilon + \varepsilon'\lambda) - (n - n'\lambda)t)} \right) \quad (2.37)$$

is the group velocity of the carrier wave which has the dimension of *radian/s*. Although, ω_g and Z has the same dimension, but where Z depends on time, ω_g is dependent upon the spatial frequency or wave number (k).

2.3 Evaluation of the phase velocity (v_p) of the carrier wave.

The phase velocity denotes the velocity of a point of fixed phase angle [5]. At any instant of the wave motion the displacements of other points nearby change also and there will be one of these points, at $x + \delta x$ say, where the displacement $y(x + \delta x, t + \delta t)$ is equal to the original displacement $y(x, t)$ at point x . Now from (2.23) the carrier wave is a maximum when the spatial oscillatory phase ϕ is equal to one.

$$\phi = \cos((\vec{k} - \vec{k}'\lambda) \cdot \vec{r} - (n - n'\lambda)t - E) = 1 \quad (2.38)$$

$$(\vec{k} - \vec{k}'\lambda) = (k - k'\lambda)_x i + (k - k'\lambda)_y j + (k - k'\lambda)_z k \quad (2.39)$$

$$\vec{r} = xi + yj + zk \quad (2.40)$$

If we assume that the motion is constant in the z-direction and the wave vector mode is also the same for both x and y plane, then (2.40) becomes

$$\vec{r} = r \cos \theta i + r \sin \theta j \quad (2.41)$$

where $\theta = \pi - (\varepsilon - \varepsilon'\lambda)$ is the variable angle between y_1 and y_2 , please see appendix for details. Hence

$$\cos((k - k'\lambda)r \cos \theta + (k - k'\lambda)r \sin \theta - (n - n'\lambda)t - E) = 1 \quad (2.42)$$

$$((k - k'\lambda)r \cos \theta + (k - k'\lambda)r \sin \theta - (n - n'\lambda)t - E) = 0 \quad (2.43)$$

$$((k - k'\lambda) \cos \theta + (k - k'\lambda) \sin \theta) dr - (n - n'\lambda) dt - \frac{dE}{dt} dt = 0 \quad (2.44)$$

$$((k - k'\lambda) \cos \theta + (k - k'\lambda) \sin \theta) dr - (n - n'\lambda) dt - Z dt = 0 \quad (2.45)$$

$$((k - k'\lambda) \cos \theta + (k - k'\lambda) \sin \theta) dr = ((n - n'\lambda) + Z) dt \quad (2.46)$$

$$v_p = \frac{dr}{dt} = \left(\frac{(n - n'\lambda) + Z}{(k - k'\lambda)(\cos \theta + \sin \theta)} \right) \quad (2.47)$$

This has the dimension of *m/s*. Since our argument is equally valid for all values of r , (2.47) tells us that the whole sinusoidal wave profile move to the left or to the right at a speed v_p .

2.4 Evaluation of the oscillating angular frequency ($\dot{\theta}$) of the carrier wave.

The variation of the spatial oscillatory phase of the carrier wave with respect to time gives the oscillating frequency ($\dot{\theta}$). Hence, from (2.43)

$$(k - k'\lambda)r \left(\frac{d}{dt} (\cos \theta + \sin \theta) \right) - (n - n'\lambda) - \frac{dE}{dt} = 0 \quad (2.48)$$

$$(k - k'\lambda)r \left(\frac{d \cos \theta}{d\theta} \frac{d\theta}{dt} + \frac{d \sin \theta}{d\theta} \frac{d\theta}{dt} \right) - (n - n'\lambda) - Z = 0 \quad (2.49)$$

$$(k - k'\lambda)r (\cos \theta \dot{\theta} - \sin \theta \dot{\theta}) = ((n - n'\lambda) + Z) \quad (2.50)$$

$$v_\theta = \dot{\theta} = \left(\frac{((n - n'\lambda) + Z)}{(k - k'\lambda)r(\cos \theta - \sin \theta)} \right) \quad (2.51)$$

The unit is per second (s^{-1}). Thus because of the tethered nature of the elastic pipe the constituted carrier wave can only possess oscillating radial velocity and not oscillating angular velocity.

2.5 Evaluation of the radial velocity (v_r) of the carrier wave.

The variation of the spatial oscillatory phase of the carrier wave with respect to time gives the radial velocity (v_r). Hence, from (2.43)

$$(k - k'\lambda) \frac{dr}{dt} (\cos \theta + \sin \theta) - (n - n'\lambda) - \frac{dE}{dt} = 0 \quad (2.52)$$

$$(k - k'\lambda) \dot{r} (\cos \theta + \sin \theta) - (n - n'\lambda) - Z = 0 \quad (2.53)$$

$$v_r = \dot{r} = \left(\frac{((n - n'\lambda) + Z)}{(k - k'\lambda)(\cos \theta + \sin \theta)} \right) \quad (2.54)$$

This has the same unit as the phase velocity which is m/s .

2.6 Evaluation of the velocity of the ‘carrier wave’

Let us now evaluate the velocity with which the entire constituted carrier wave moves with respect to time. This has to do with the product differentiation of the non-stationary amplitude and the spatial oscillatory cosine phase.

$$\begin{aligned} v = \frac{dy}{dt} &= (a - b\lambda)^2 (n - n'\lambda) \sin((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \times \\ &\left((a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \right)^{\frac{1}{2}} \times \\ &\cos((k - k'\lambda)r(\cos \theta + \sin \theta) - (n - n'\lambda)t - E) + \\ &\left((a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \right)^{\frac{1}{2}} \times \\ &\left\{ (n - n'\lambda) + Z - V_r (k - k'\lambda)(\cos \theta + \sin \theta) - V_\theta (k - k'\lambda)r(\cos \theta + \sin \theta) \right\} \times \\ &\sin((k - k'\lambda)r(\cos \theta + \sin \theta) - (n - n'\lambda)t - E) \quad (2.55) \end{aligned}$$

$$\begin{aligned} v &= (a - b\lambda)^2 (n - n'\lambda) \sin((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \times \\ &(a^2 - b^2\lambda^2)^{\frac{1}{2}} \left(1 - \frac{2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda))}{(a^2 - b^2\lambda^2)} \right)^{-\frac{1}{2}} \times \\ &\cos((k - k'\lambda)r(\cos \theta + \sin \theta) - (n - n'\lambda)t - E) + \\ &(a^2 - b^2\lambda^2)^{\frac{1}{2}} \left(1 - \frac{2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda))}{(a^2 - b^2\lambda^2)} \right)^{\frac{1}{2}} \times \\ &\left\{ (n - n'\lambda) + Z - V_r (k - k'\lambda)(\cos \theta + \sin \theta) - V_\theta (k - k'\lambda)r(\cos \theta + \sin \theta) \right\} \times \\ &\sin((k - k'\lambda)r(\cos \theta + \sin \theta) - (n - n'\lambda)t - E) \quad (2.56) \end{aligned}$$

Upon using Binomial expansion on the fractional terms and stopping at the second term we get

$$\begin{aligned}
 v = & \frac{(a-b\lambda)^2 (n-n'\lambda)}{(a^2-b^2\lambda^2)^{\frac{1}{2}}} \sin((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \times \\
 & \cos((k-k'\lambda)r(\cos\theta + \sin\theta) - (n-n'\lambda)t - E) + \\
 & \frac{(a-b\lambda)^4 (n-n'\lambda)}{(a^2-b^2\lambda^2)^{\frac{3}{2}}} \cos((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \times \cos((k-k'\lambda)r(\cos\theta + \sin\theta) - (n-n'\lambda)t - E) + \\
 & (a^2-b^2\lambda^2)^{\frac{1}{2}} \left\{ (n-n'\lambda) + Z - V_r (k-k'\lambda)(\cos\theta + \sin\theta) - V_\theta (k-k'\lambda)r(\cos\theta + \sin\theta) \right\} \times \\
 & \sin((k-k'\lambda)r(\cos\theta + \sin\theta) - (n-n'\lambda)t - E) - \frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)^{\frac{1}{2}}} \cos((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \times \\
 & \left\{ (n-n'\lambda) + Z - V_r (k-k'\lambda)(\cos\theta + \sin\theta) - V_\theta (k-k'\lambda)r(\cos\theta + \sin\theta) \right\} \times \\
 & \sin((k-k'\lambda)r(\cos\theta + \sin\theta) - (n-n'\lambda)t - E) \quad (2.57)
 \end{aligned}$$

For the velocity of the constituted carrier wave to be maximum we have to ignore all the oscillating phases, so that

$$v_m = \frac{(a-b\lambda)^4 (n-n'\lambda) + (a^2-b^2\lambda^2)^2 (n-n'\lambda)}{(a^2-b^2\lambda^2)^{\frac{3}{2}}} \quad (2.58)$$

The unit is m/s .

2.7 Evaluation of the energy attenuation equation

In natural systems, we can rarely find pure wave which propagates free from energy-loss mechanisms. But if these losses are not too serious we can describe the total propagation in time by a given force law $f(t)$. The propagating constituted carrier wave in an elastic pipe containing a viscous fluid is affected by two major factors: (i) the damping effect of the mass of the surrounding fluid (ii) the damping effect of the dynamic viscosity of the elastic walls of the pipe in response to the wave propagation [5]. Let us consider a carrier wave propagating through an elastic pipe of a given elasticity Q , if the fluid in the pipe has a mass m and viscosity μ , the dissipation of the carrier wave-energy if the fluid is Newtonian, would obey the equation

$$f(t) = \mu \frac{d^2 y}{dt} + Q y^2 \quad (2.59)$$

$$f(t)dt = 2\mu y dy + Q y^2 dt \quad (2.60)$$

For the carrier wave to have a maximum value then the spatial oscillating part is ignored such that

$$y_m^2 = (a^2 - b^2\lambda^2) - 2(a-b\lambda)^2 \cos((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \quad (2.61)$$

$$f(t)dt = 2\mu y_m dy_m + Q \left(a^2 - b^2\lambda^2 - 2(a-b\lambda)^2 \cos((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \right) dt \quad (2.62)$$

$$\int f(t)dt = 2\mu \int y_m dy_m + Q \int \left((a^2 - b^2\lambda^2) - 2(a-b\lambda)^2 \cos((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \right) dt \quad (2.63)$$

$$impulse = 2\mu y_m^2 + Q \left((a^2 - b^2\lambda^2)t - \frac{2(a-b\lambda)^2 \sin((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda))}{(n-n'\lambda)} \right) \quad (2.64)$$

$$impulse \times v_m = 2\mu y_m^2 \times v_m + Q \times v_m \left((a^2 - b^2\lambda^2)t - \frac{2(a-b\lambda)^2 \sin((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda))}{(n-n'\lambda)} \right) \quad (2.65)$$

$$Energy E = 2\mu y_m^2 v_m + Q v_m \left(\frac{(a^2 - b^2\lambda^2)(n-n'\lambda)t - 2(a-b\lambda)^2 \sin((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda))}{(n-n'\lambda)} \right) \quad (2.66)$$

The unit is in Joules or $\text{kgm}^2\text{s}^{-2}$ or Nm. However, in this work we assume the dynamic viscosity μ of the fluid medium where the constituted carrier wave is propagating as $\mu = 0.004 \text{ N s m}^{-2}$ and the elasticity of the wall of the narrow pipe $Q = 1.9048 \times 10^{-6} \text{ kgm}^{-1}\text{s}^{-1}$ note that for a non-elastic wall $Q = 0$.

2.8 Determination of the ‘host wave’ parameters (a, n, ε and k)

Let us now discuss the possibility of obtaining the parameters of the ‘host wave’ which were initially not known from the equation of the carrier wave. This is a very crucial stage of the study since there was no previous knowledge of the values. However, the carrier wave given by (2.23) can only have a maximum value provided the spatial oscillating phase is equal to one. Hence the non-stationary amplitude becomes

$$A = \left\{ \left(a^2 - b^2 \lambda^2 \right) - 2(a - b\lambda)^2 \cos \left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda) \right) \right\}^{\frac{1}{2}} \quad (2.67)$$

Using the boundary conditions that at time $t = 0$, $\lambda = 0$ and $A = a$, then

$$A = \left\{ a^2 - 2a^2 \cos(-\varepsilon) \right\}^{\frac{1}{2}} = a \left\{ 1 - 2 \cos(\varepsilon) \right\}^{\frac{1}{2}} \quad (2.68)$$

$$\left\{ 1 - 2 \cos(\varepsilon) \right\}^{1/2} = 1 \Rightarrow \varepsilon = \cos^{-1}(0) = 90^\circ \text{ (1.5708 rad.)} \quad (2.69)$$

Any slight variation in the combined amplitude A of the carrier wave y due to displacement with time $t = t + \delta t$ would invariably produce a negligible effect in the host amplitude a also under this situation $\lambda \approx 0$.

Hence we can write

$$\lim_{\delta t \rightarrow 0} \left\{ A + \frac{\delta A}{\delta t} \right\} = a \quad (2.70)$$

$$\lim_{\delta t \rightarrow 0} \left\{ \left(a^2 - 2a^2 \cos(n(t + \delta t) - \varepsilon) \right)^{1/2} + \frac{n a^2 \sin(n(t + \delta t) - \varepsilon)}{\left(a^2 - 2a^2 \cos(n(t + \delta t) - \varepsilon) \right)^{1/2}} \right\} = a \quad (2.71)$$

$$\left\{ \left(a^2 - 2a^2 \cos(nt - \varepsilon) \right)^{1/2} + \frac{n a^2 \sin(nt - \varepsilon)}{\left(a^2 - 2a^2 \cos(nt - \varepsilon) \right)^{1/2}} \right\} = a \quad (2.72)$$

$$\left(a^2 - 2a^2 \cos(nt - \varepsilon) \right) + n a^2 \sin(nt - \varepsilon) = a \left(a^2 - 2a^2 \cos(nt - \varepsilon) \right)^{1/2} \quad (2.73)$$

$$1 - 2 \cos(nt - \varepsilon) + n \sin(nt - \varepsilon) = \left(1 - 2 \cos(nt - \varepsilon) \right)^{1/2} \quad (2.74)$$

At this point of our work it may not be easy to produce a solution to the problem, this is due to the mixed sinusoidal wave functions. However, to get out of this complication we have implemented an unusual approximation technique to minimize the right hand side of (2.74). This approximation states that

$$\left(1 + \xi f(\phi) \right)^{\pm n} = \frac{d}{d\phi} \left(1 + n \xi f(\phi) + \frac{n(n-1)}{2!} (\xi f(\phi))^2 + \frac{n(n-1)(n-2)}{3!} (\xi f(\phi))^3 + \dots \right) \quad (2.75)$$

The general background of this approximation is the differentiation of the resulting binomial expansion of a given variable function. This approximation has the advantage of converging functions easily and also it produces minimum applicable value of result. Hence (2.74) becomes

$$1 - 2 \cos(nt - \varepsilon) + n \sin(nt - \varepsilon) = n \sin(nt - \varepsilon) \quad (2.76)$$

$$nt - \varepsilon = \cos^{-1}(0.5) = 60^\circ = 1.0472 \text{ rad.} \Rightarrow nt = 2.6182 \text{ rad.} \Rightarrow n = 2.6182 \text{ rad./s} \quad (2.77)$$

From (2.43), using the boundary conditions that for stationary state when

$$\delta t = 0, \lambda \approx 0, \theta = \pi - (\varepsilon - \varepsilon'\lambda) = \pi - \varepsilon = 3.142 - 1.5708 = 1.5712 \text{ rad, } E = \varepsilon = 1.5708 \text{ rad, then we}$$

$$\lim_{\delta t \rightarrow 0} \cos \left\{ (k - k'\lambda) r \cos \theta + (k - k'\lambda) r \sin \theta - (n - n'\lambda)(t + \delta t) - E \right\} = 1 \quad (2.78)$$

$$(k r (\cos \theta + r \sin \theta) - n t - \varepsilon) = 0 \quad (\text{since, } \cos^{-1} 1 = 0) \quad (2.79)$$

$$(k r (0.9996) - 2.6182 - 1.5708) = 0 \Rightarrow k r = 4.1907 \text{ rad} \Rightarrow k = 4.1907 \text{ rad} / m \quad (2.80)$$

The change in the resultant amplitude of the carrier wave is proportional to the frequency of oscillation of the spatial oscillating phase ϕ multiplied by the product of the variation with time t of the inverse of the oscillating phase with respect to the radial distance r and the variation with time t wave number $(k - k'\lambda)$. This condition would make by using (2.24) and (2.38) to write that

$$\frac{dA}{dt} = \frac{(n - n'\lambda)(a - b\lambda)^2 \sin((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda))}{\left((a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \right)^{1/2}} \quad (2.81)$$

$$\frac{d\phi}{dr} = -(k - k'\lambda)(\cos \theta + \sin \theta) \sin((k - k'\lambda)r (\cos \theta + \sin \theta) - (n - n'\lambda)t - E) \quad (2.82)$$

$$\frac{d\phi}{dt} = ((n - n'\lambda) + Z) \sin((k - k'\lambda)r (\cos \theta + \sin \theta) - (n - n'\lambda)t - E) \quad (2.83)$$

$$\frac{d\phi}{d(k - k'\lambda)} = (-r(\cos \theta + \sin \theta) - E) \sin((k - k'\lambda)r (\cos \theta + \sin \theta) - (n - n'\lambda)t - E) \quad (2.84)$$

$$\frac{dA}{dt} = \left(\frac{1}{2\pi} \frac{\partial \phi}{\partial t} \right) \left(\frac{1}{r} \frac{\partial r}{\partial \phi} \right) \left(\frac{\partial \phi}{\partial (k - k'\lambda)} \right) = f l \quad (2.85)$$

$$A = f l t \quad (2.86)$$

That is the time rate of change of the resultant amplitude is equal to the frequency f of the spatial oscillating phase multiplied by the length l of the arc covered by the oscillating phase. Under this circumstance, we refer to A as the instantaneous amplitude of oscillation.

The first term in the parenthesis of (2.85) is the frequency dependent term, while the combination of the rest two terms in the parenthesis represents the angular length or simply the length of an arc covered by the spatial oscillating phase. Note that the second term in the right hand side of (2.85) is the inverse of (2.82).

With the usual implementation of the boundary conditions

$$t = 0, \lambda = 0, \theta = \pi - (\varepsilon - \varepsilon'\lambda) = \pi - \varepsilon = 3.142 - 1.5708 = 1.5712 \text{ rad}, \quad E = \varepsilon = 1.5708 \text{ rad}, \quad dA/dt = a$$

We obtain the expression for the amplitude as

$$a = - \left(\frac{1}{2\pi} \right) \left(\frac{(\cos \theta + \sin \theta) - \varepsilon}{k \sin \varepsilon (\cos \theta + \sin \theta)} \right) = 0.0217 m \quad (2.87)$$

Note that $\cos(-\varepsilon) = \cos \varepsilon$ (even and symmetric function) and $\sin(-\varepsilon) = -\sin \varepsilon$ (odd and screw symmetric function). Thus generally we have established that the basic constituents parameters of the 'host wave are $a = 0.0217 m$, $n = 2.6182 \text{ rad} / s$, $\varepsilon = 1.5708 \text{ rad}$, and $k = 4.1907 \text{ rad} / m$ (2.88)

2.9 Determination of the 'parasitic wave' parameters (b, n', ε' and k')

Let us now determine the basic parameters of the 'parasitic wave' which were initially not known before the interference from the derived values of the resident 'host wave' using the below method. The gradual depletion in the physical parameters of the system under study would mean that after a sufficiently long period of time all the active constituents of the resident 'host wave' would have been completely attenuated by the destructive influence of the 'parasitic wave'. On the basis of these arguments, we can now write as follows.

$$\left. \begin{aligned} a - b\lambda &= 0 \Rightarrow 0.0217 = b\lambda \\ n - n'\lambda &= 0 \Rightarrow 2.6182 = n'\lambda \\ \varepsilon - \varepsilon'\lambda &= 0 \Rightarrow 1.5708 = \varepsilon'\lambda \\ k - k'\lambda &= 0 \Rightarrow 4.1907 = k'\lambda \end{aligned} \right\} \quad (2.89)$$

Upon dividing the sets of relations in (2.89) with one another with the view to eliminate λ we get

$$\left. \begin{aligned} 0.008288n' &= b \\ 0.013820 \varepsilon' &= b \\ 0.005178k' &= b \\ 1.6668 \varepsilon' &= n' \\ 0.6248k' &= n' \\ 0.3748k' &= \varepsilon' \end{aligned} \right\} (2.90)$$

However, there are several possible values that each parameter would take according to (2.90). But for a gradual decay process, that is for a slow depletion in the constituent of the host parameters we choose the least values of the parasitic parameters.

Thus a more realistic and applicable relation is when: $0.008288n' = 0.005178k'$. Based on simple ratio $n' = 0.00518 \text{ rad/s}$, $k' = 0.00829 \text{ rad/m}$, and eventually; $\varepsilon' = 0.00311 \text{ rad}$, $b = 0.0000429 \text{ m}$ (2.91)

Any of these values of the constituents of the ‘parasitic wave’ shall produce a corresponding approximate value of lambda $\lambda = 505$ upon substituting it into (2.89). Hence the interval of the multiplier is $0 \leq \lambda \leq 505$. Now, so far, we have systematically determined the basic constituents parameters of both the ‘host wave’ and those of the ‘parasitic wave’ both contained in the carrier wave.

2.10 Determination of the attenuation constant (η)

Attenuation is a decay process. It brings about a gradual reduction and weakening in the initial strength of the basic parameters of a given physical system. In this study, the parameters are the amplitude (a), phase angle (ε), angular frequency (n) and the spatial frequency (k). The dimension of the attenuation constant (η) is determined by the system under study. However, in this work, attenuation constant is the relative rate of fractional change (FC) in the basic parameters of the carrier wave. There are 4 (four) attenuating parameters present in the carrier wave. Now, if a, n, ε, k represent the initial basic parameters of the ‘host wave’ that is present in the carrier wave and $a - b\lambda, n - n'\lambda, \varepsilon - \varepsilon'\lambda, k - k'\lambda$ represent the basic parameters of the ‘host wave’ that survives after a given time. Then, the FC is

$$\sigma = \frac{1}{4} \times \left(\left(\frac{a - b\lambda}{a} \right) + \left(\frac{\varepsilon - \varepsilon'\lambda}{\varepsilon} \right) + \left(\frac{n - n'\lambda}{n} \right) + \left(\frac{k - k'\lambda}{k} \right) \right) \quad (2.92)$$

$$\eta = \frac{FC|_{\lambda=i} - FC|_{\lambda=i+1}}{\text{unit time (s)}} = \frac{\sigma_i - \sigma_{i+1}}{\text{unit time (s)}} \quad (2.93)$$

The dimension is *per second* (s^{-1}). Thus (2.93) gives $\eta = 0.001978s^{-1}$ for all values of λ ($i = 0, 1, 2, \dots, 505$).

2.11 Determination of the time (t)

We used the information provided in section 2.9, to compute the various times taken for the carrier wave to attenuate to zero. The maximum time the carrier wave lasted as a function of the raising multiplier λ is also calculated from the attenuation equation shown by (2.93). The reader should note that we have adopted a slowly varying regular interval for the raising multiplier since this would help to delineate clearly the physical parameter space accessible to our model. However, it is clear from the calculation that the different attenuating fractional changes contained in the carrier wave are approximately equal to one another. We can now apply the attenuation time equation given below.

$$\sigma = e^{-(2^\gamma \eta t) / \lambda} \quad (2.94)$$

$$t = - \left(\frac{\lambda}{2^\gamma \eta} \right) \ln \sigma \quad (2.95)$$

Where γ is the functional index of any physical system under study and here we assume $\gamma = 1$. The equation is statistical and not a deterministic law. It gives the expected basic intrinsic parameters of the ‘host wave’ that survives after time t . Clearly, we used (2.95) to calculate the exact value of the decay time as a function of the raising multiplier.

In this work, we used table scientific calculator and Microsoft excel to compute our results. Also the GNU PLOT 4.6 version was used to plot the corresponding graphs.

III. Presentation of results

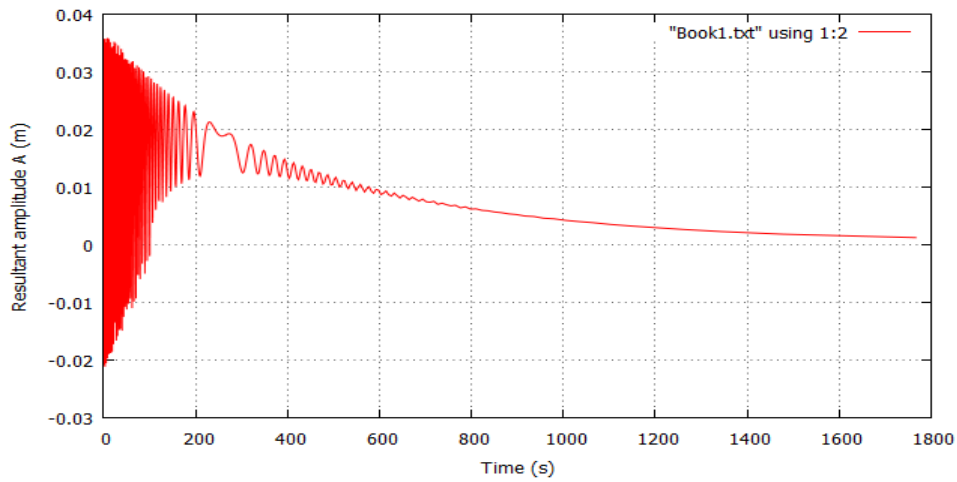


Fig. 1: Represents the resultant amplitude A of the carrier wave y as a function of time t .

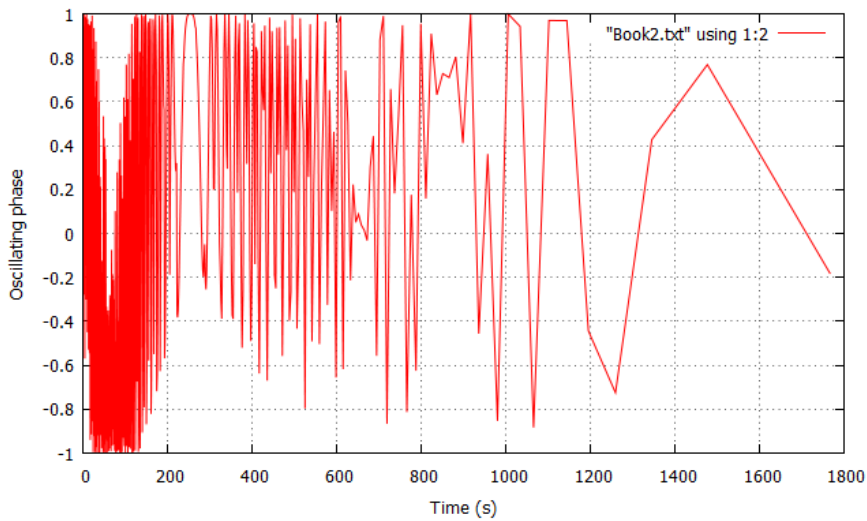


Fig. 2: Represents the spatial oscillating phase ϕ of the carrier wave y as a function of time t .

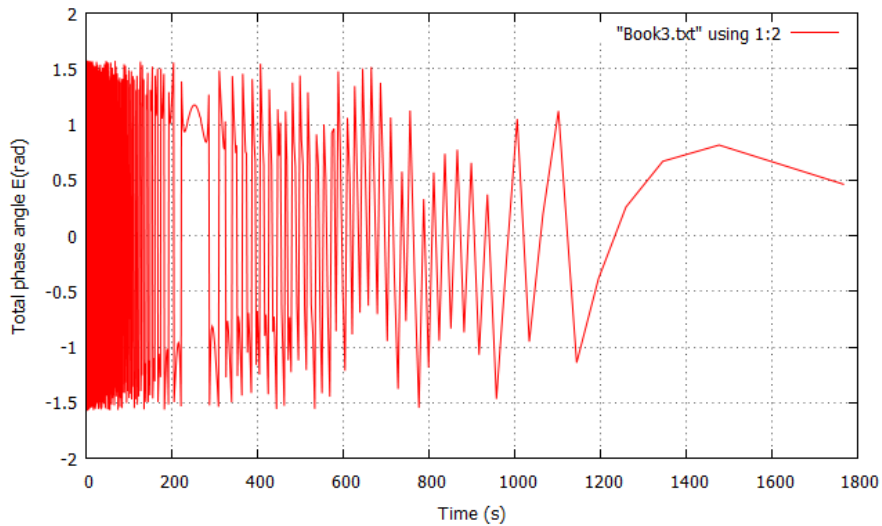


Fig. 3: Represents the total phase angle E of the carrier wave y as a function of time t .

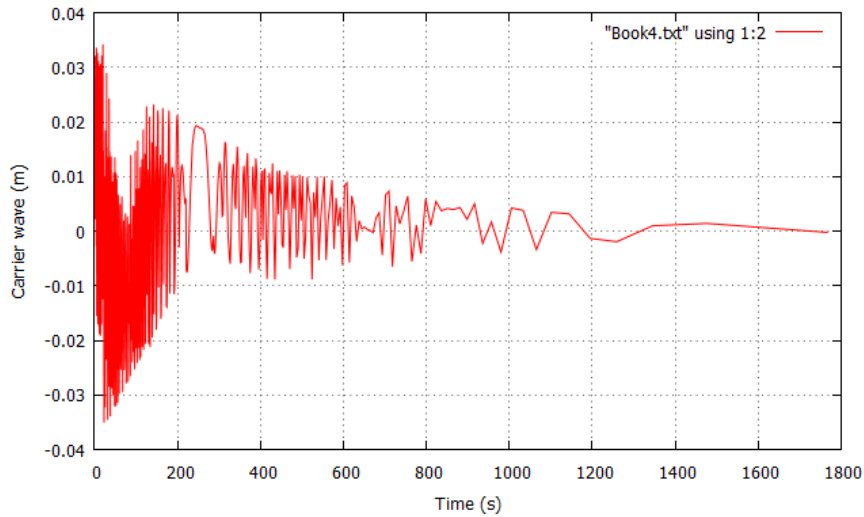


Fig. 4: Represents the carrier wave y as a function of time t .

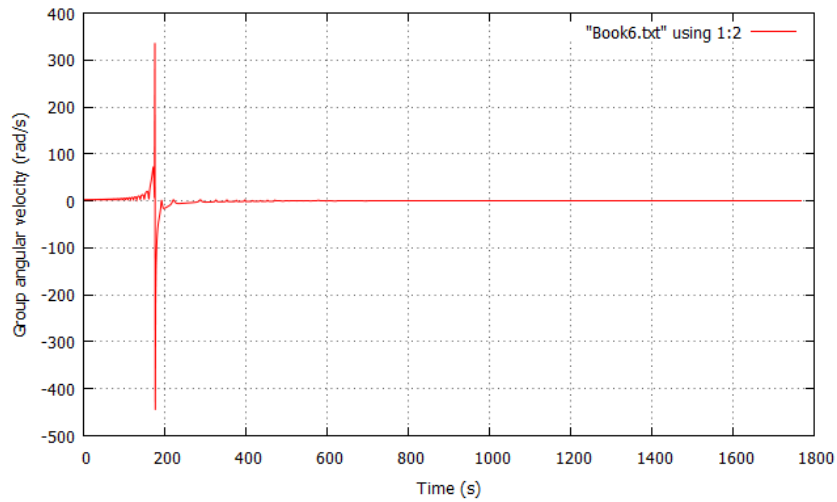


Fig. 5: Represents the group angular velocity w_g of the carrier wave y as a function of time t .

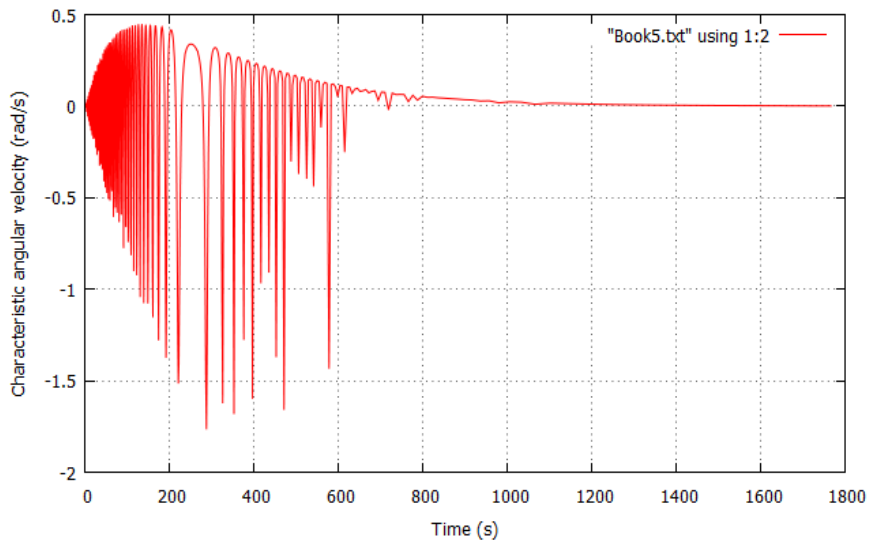


Fig. 6: Represents the characteristic angle velocity Z of the carrier wave y as a function of time t .

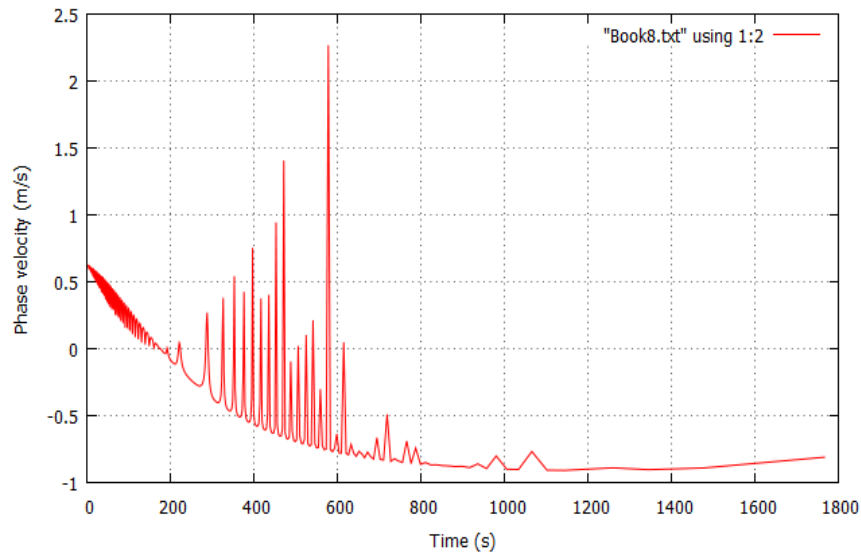


Fig. 7: Represents the phase velocity v_p of the carrier wave y as a function of time t .

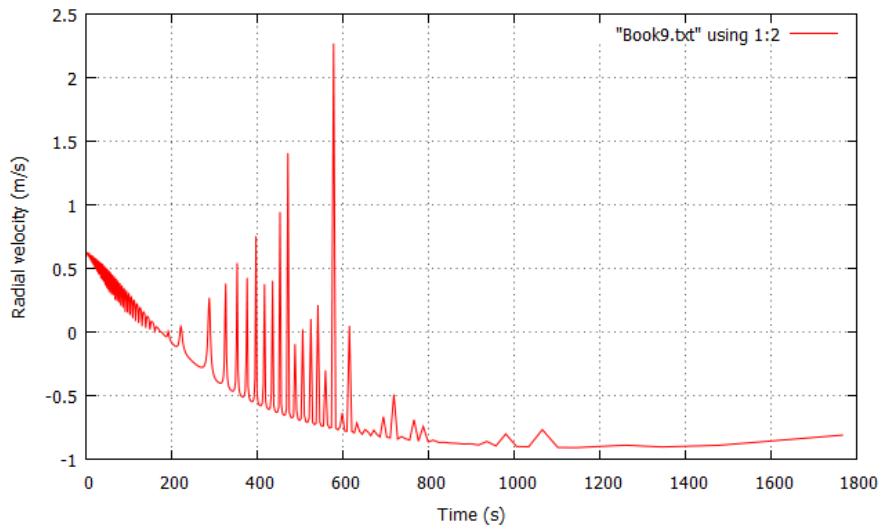


Fig. 8: Represents the radial velocity v_r of the carrier wave y as a function of time t .

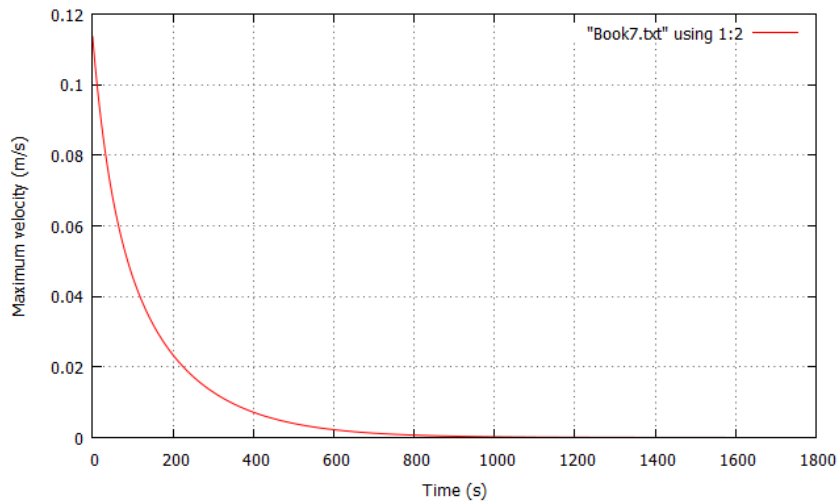


Fig. 9: Represents the maximum velocity v_m of the carrier wave y as a function of time t .

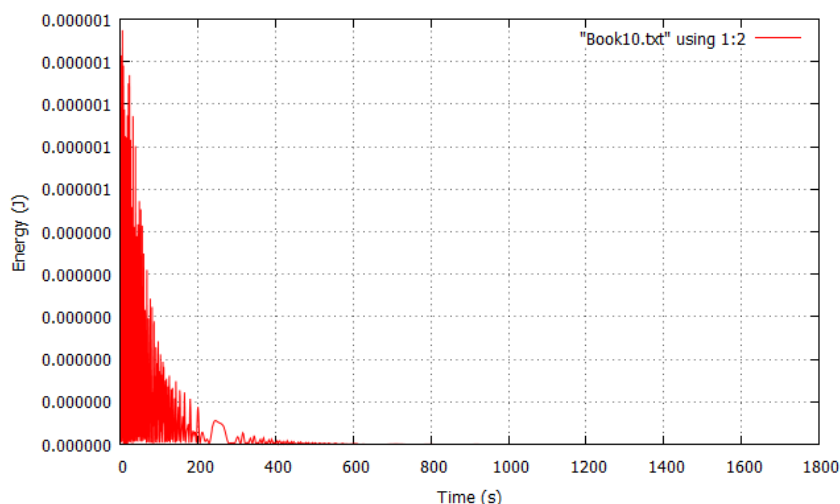


Fig. 10: Represents the Energy E of the carrier wave y as a function of time t .

IV. Discussion Of Results

It is clear from fig. 1, that the resultant amplitude of the constituted carrier wave shows blurred spectrum when $0 \leq t \leq 200s$ and this is when the multiplicative factor $0 \leq \lambda \leq 277$. In the interval of the multiplicative factor $278 \leq \lambda \leq 485$ is $3627 \leq \lambda \leq 6874$ and $200 \leq t \leq 810s$, the resultant amplitude oscillates monochromatically. After this time, it consistently attenuates with rapid damping to zero at the critical value of $\lambda=505$ and with a survival time of 1766s. Our calculation actually reveals that within the interval of the multiplicative factor $0 \leq \lambda \leq 166$ and the time $0 \leq t \leq 100s$, the amplitude of the carrier wave oscillates with both real and imaginary values. Consequently, the amplitude of the carrier wave is actually made up of the imaginary and real part, $A = A_1 + iA_2$. As a result the motion is actually two-dimensional (2D). Thus A_1 and A_2 are the components of the amplitude in x and y - directions, and A is tangential to the path of the moving amplitude in the carrier wave. In the regions where the amplitude of the carrier wave is greater than either of the amplitude of the individual wave, we have constructive interference that means the path difference is $(\varepsilon + \varepsilon'\lambda)$, otherwise, it is destructive in which case the path difference is $(\varepsilon - \varepsilon'\lambda)$.

The initial blurred spectrum is an indication of a constant agitation by the intrinsic parameters of the 'host wave' to resist, thereby suppressing the destructive influence of the interfering 'parasitic wave'. During this period, although unnoticeable as it may, but much imaginary harm would have been done to the constituent parameters of the physical system of the 'host wave'.

The graph of the spatial oscillating phase (SOP) of the carrier wave as shown in fig. 2, first experiences a temporary depletion when the time is about 50s with a high absolute value of 0.5336 rad. It also shows blurred behaviour in the interval $0 \leq \lambda \leq 277$. While beyond 200s and the in the interval $277 \leq \lambda \leq 505$ the spectrum displays a monochromatic behaviour. Thus the spectrum shows a blurred characteristic up to 200s followed by a gradual depletion of the wave form. The blurred nature of the spectra is an indication of the resistance of the intrinsic parameters of the 'host wave' to the destructive tendency of the interfering 'parasitic wave'. While the subsequent depleting behaviour involves a slow decay process, resulting to a gradual reduction and weakening in the initial strength of the intrinsic parameters of the spatial oscillating phase of the host system. The SOP oscillates between the maximum and minimum values of +1 and -1. In phasor language, for positive SOP, y_1 leads y_2 and ε leads ε' (or ε' lags ε) in the CCW, while for negative of SOP, y_2 leads y_1 and ε' leads ε (or ε lags ε'). Finally the SOP attains a value of 0.768538 rad after 1476s and decreases to zero in the limits of the multiplicative factor λ .

The graph of the total phase angle of the carrier wave as a function of time is shown in fig. 3. The total phase angle also shows blurred spectrum in the interval when $0 \leq t \leq 100s$ and thereafter the spectrum is purely monochromatic indicative of the predominance of the 'parasitic wave' as now purely the only.

The spectrum of the displacement vector of the CCW as a function of time shown in fig. 4 is similar to that of fig. 1 and 2, since the CCW is the product of the resultant amplitude A and the spatial oscillating phase ϕ . The exception to this similarity is that the CCW oscillates non-consistently with quadratic dispersion before it finally comes to rest. As we can generally observe in all the spectra, they show exemplary behaviour or a sharp transition from one system phase to another when the time is greater than 200s. But after a prolonged time, the

constituents of the carrier wave become monochromatic in nature signifying a predominance of the 'parasitic wave'. In this case, the group velocity of the carrier wave becomes equal to the phase velocity.

The graph of the spectrum of the group angular velocity (GAV) of the carrier wave as a function of time as shown in fig. 5, almost show undefined behaviour everywhere except at time $t = 90$ s with a maximum and minimum values of 340 rad/s and -440 rad/s. The undefined behaviour of the group angular velocity reveals that the two combining waves do not have affinity for one another. The predominant behaviour of the GAV of the carrier wave at this particular time is referred to as the wave packet. The wave packet is a localized pulse that is composed of both the 'host wave' and the 'parasitic wave' that cancel each other everywhere else except at this time during the interference. Since each component of the wave packet has different phase velocity in the medium, the modulation propagation number ($k - k'\lambda$) of the components of the carrier wave changes in the medium and consequently the group velocity changes. This results to a change in the width of the wave packet.

Where for positive values of the GAV means constructive interference between the two waves and negative values of the GAV means destructive interference between the two waves.

The graphs of the spectrum of the characteristic angular velocity (CAV), the phase velocity and the radial velocity of the CCW as a function of time are shown in figs. 6, 7 and 8. The spectrum for the CAV show more of negative values which is an indication of high repulsive behaviour between the two interfering waves and this invariably means destructive interference between them. The CAV experiences critical damping after $t \geq 600$ s. The CAV, the phase velocity and the radial velocity exhibit the same damping behaviour after $t \geq 600$ s. However, the wave mechanics of the phase and radial velocity are almost the inverse of the CAV. They initially show blurred behaviour up to 166s and thereafter, they all exhibit monochromatic property before they finally attenuate to zero in the limits of $\lambda = 505$.

The maximum velocity and the energy attenuation mechanism of the CCW are both represented by figs. 9 and 10. Our calculation shows that the maximum velocity consistently attenuates exponentially to zero after 745s when the multiplicative factor $\lambda = 479$. The properties of the energy of the CCW as it attenuates to zero is similar in explanation to those of the resultant amplitude and spatial oscillating phase shown in figs. 1 and 2. The nature of the energy wave mechanics which is represented by the spectrum of fig. 10 is a consequence of the highly viscous medium where the CCW is propagating. From the figure, the energy spectrum is initially blurred before 200s after the interference, and after this time, it experiences depletion, thereby becoming monochromatic in character. The average survival time for the energy possess by the CCW in the viscous fluid is about 600s instead of the usual 1766s for the other several properties of the carrier wave. However, we believe that for a less viscous and non-viscous fluid (air) the attenuation of the energy in such media would be very small, as a result the energy is expected to fluctuate with highly improved oscillating amplitude for a longer period of time.

V. Conclusion

The initial blurred attenuating spectra behaviour of the CCW and its several properties is a consequence of the fact that when a carrier wave is propagating in a viscous fluid under any given circumstance, its wave form do not steadily go to zero immediately, rather it fluctuates. The fluctuation is due to the constructive and destructive interference of both the 'host wave' and the 'parasitic wave' contained in the CCW.

Also the initial irregular behaviour exhibited by the CCW and its several properties during the damping process, is due to the resistance pose by the 'host wave' in an attempt to annul the destructive effects of the interfering 'parasitic wave'. The spectrum of the characteristic angular velocity and the group angular velocity converge to the same value when the raising multiplier is a maximum and both of them seem to be oppositely related.

The subsequent gradual depletion of the wave form of the CCW and its several properties characterizes a predominance of the 'parasitic wave' since all the active constituents of the 'host wave' would have been eroded. This situation leads to a steady decay process resulting to a gradual reduction and weakening of the initial strength

Of the intrinsic parameters of the host system. It is therefore the choice of the varying series interval of the multiplicative factor that would determine the life span of the CCW. The greater the value of the series interval the shorter the life span, and vice versa.

5.1 Suggestions for further work

This study in theory and practice can be extended to investigate wave interference and propagation in three-dimensional (3D) systems. The CCW we developed in this work can be utilized in the deductive and predictive study of wave attenuation in exploration geophysics and telecommunication engineering. This work can also be extended to investigate energy attenuation in a HIV/AIDS patient.

APPENDIX: Vector representation of the superposition of the ‘host wave’ and the ‘parasitic wave’. The amplitudes of the waves y_1 , y_2 and y are not constant with time but they oscillate at a given frequency.

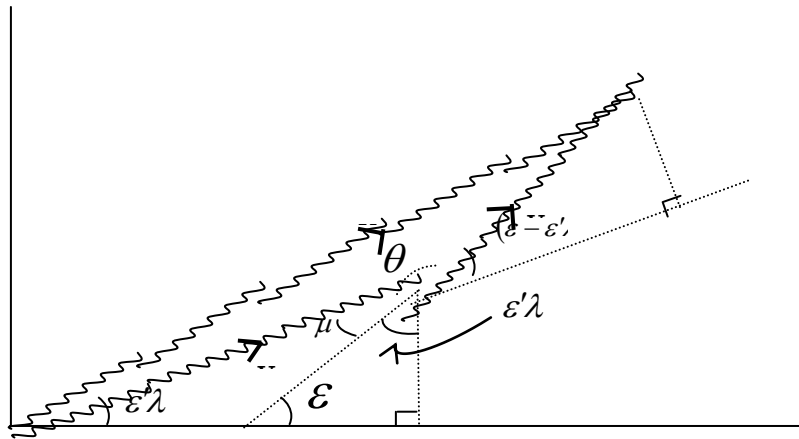


Fig. A 1: Represents the human ‘host wave’ y_1 and the HIV ‘parasitic wave’ y_2 after the interference. The superposition of both waves y_1 and y_2 is represented by the carrier wave displacement y . It is clear that from the geometry of the figure: $\mu + \varepsilon'\lambda + 180^\circ - \varepsilon = 180^\circ$; $\mu = \varepsilon - \varepsilon'\lambda$; $\theta = 180^\circ - (\varepsilon - \varepsilon'\lambda)$; and $\theta = \pi - (\varepsilon - \varepsilon'\lambda)$. Note that θ is the variable angle between y_1 and y_2 .

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